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# Information disclosure and questionnaires in public tenders

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## Abstract

Public tenders typically involve uncertainty and unforeseen costs, which might not be fully known to the bidders themselves. This uncertainty may also concern adaptation costs to the procurer after the delivery, and information about such adaptation costs may influence the bidding strategies, even though the cost is paid by the procurer. Consequently, the availability of information matters, and if it is obtained by the procurer, withholding information from bidders may be better than sharing all of it. To investigate the methods of gathering information, we consider pre-qualification stage questionnaires designed to obtain information from the bidders. If suitably designed, they can be used to disclose information of which the bidders are themselves unaware and can thereby serve as a tool for revealing the inherent tendency of the cost for the bidders, so that the unforeseen future cost can be estimated.

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## 1. Introduction

It is common practice that public authorities run competitive tenders to procure goods and services, usually via an auction. However, tenders are often uncertain, particularly when involving complex projects such as the construction of a bridge or an offshore wind farm. The uncertainty arises from varying economic or geographical conditions, such as unforeseen underground complications, which can result in post-default occurrences. It is difficult for the public procurer – and, to a lesser extent, for the bidders themselves – to predict the likelihood of such occurrences at the time of launching the tender. These types of occurrences can take the form of general disturbances, giving rise to additional costs (hereafter adaptation costs), which often become the

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responsibility of the procurer unless otherwise contractually agreed upon. The bidders may be privy to information about potential adaptation costs to the likely benefit to the procurer should they obtain it.

Directly asking the bidder to reveal the adaptation cost will make sense only if it is already known to them, and even so, the bidder would be induced to communicate unrealistically small values of the cost, making the information valueless. This is because the bidder does not itself pay for the (adaptation) cost. Consequently, if the information must come from the bidder, it should be obtained in an indirect way.

In practice, one of the most widespread and straightforward indirect ways of obtaining information about bidders is the questionnaire, conducted before the bidding process (auction) starts. While each screening process differs from state to state in the United States, the European Union (EU) is known for its common rules across its member states. Although the EU procurement code itself does not explicitly mention the use of the pre-qualification questionnaire (PQQ), their public procurement guide for practitioners highlights the PQQ as a toolkit to use to obtain information (European Commission, 2015). The best-known example of the use of the PQQ is from the UK, given their widespread use in the public and private procurement sectors (Ancarani et al., 2017), with a similar use of the PQQ in, for example, Australia, Canada, and South Africa (Shire of Exmouth, 2016; State of Queensland, 2025; Government of Canada, 2025; Public Investment Corporation, 2019). For the present paper, the questionnaire we have in mind is the one used by the UK energy regulator Ofgem to procure offshore transmission assets that link offshore wind farms to the onshore electricity grid, containing rules regarding general disturbance costs (Ofgem, 2023). Similar examples are from Australia for the procurement of transportation or infrastructure (Shire of Exmouth, 2016; State of Queensland, 2025), from Canada for the construction of housing (Government of Canada, 2025), or from South Africa for an office building (Public Investment Corporation, 2019).

However, questionnaires may also place demands upon the bidder. In surveys conducted on the experience of participating in public tenders, one of the most highlighted issues is deadlines that are too short (South Australian Productivity Commission, 2019; Cabinet Office and Crown Commercial Service, 2022). However, and presumably in line with the first issue, another common issue raised is the use of questionnaires that are too long. For example, a UK tender for an estimated value of £7.5 million contained almost 1,100 pages (Laryea, 2011). Participating firms stressed afterwards that the extensive amount of information to be delivered was one of the main challenges in the tender.

Although the questionnaire is time-consuming for a bidder in tenders, asking them to supply details about their former experience, management, etc., it may provide useful supplementary knowledge relating to the construction

cost and, to a certain extent, on the adaptation cost, particularly if these costs can be anticipated or at least be subject to contingency funds.

We study a procurer that uses a first-score auction to allocate a complex project.<sup>1</sup> The cost of the project is not known in advance and unexpected costs are likely to occur as the project unfolds, as well as after the completion of the construction phase. We distinguish between the construction cost, which is paid to the winning bidder for the project, and the adaptation costs, stemming from initiating the project and then needing to adapt it to conform to its intended purpose. The adaptation cost is, to some extent, specific to each bidder, because projects are bidder-specific, with individualities in relation to both construction and completion of the projects. We consider a pre-qualification stage where the procurer can obtain additional information on the likely adaptation costs from the bidders. Here the procurer can choose to disclose its assessment on that information to the bidders before the bidding stage starts. Because the bids submitted in the bidding stage depend on the construction cost and on the anticipated adaptation cost of each of the bidders (evaluated in the pre-qualification stage), the information available to the procurer and the bidders will have relevance to the equilibrium strategies. The question that then arises is whether this information increases the payoff of the procurer. Under certain assumptions, we show that the procurer benefits from information on the adaptation costs, but is better off withholding some information from the bidders rather than sharing all of it.

An additional question is how to implement the transfer of information to the procurer on the adaptation cost. For the model, we study the use of questionnaires, but consider them together with what shall be called a random error model of adaptation cost. With this approach, small deviations during the project delivery add up to a final discrepancy, giving rise to a subsequent cost to the procurer.<sup>2</sup> Under these circumstances, we show that the questionnaire can be designed as a tool for revealing the inherent tendency of the bidder to diverge from the planned or desired action, so that the (future) adaptation cost may be estimated. Further, as the focus is on standard deviation of answers rather than on means, the incentive for misinformation on the side of the bidder is largely eliminated.

The literature on auctions in general and on procurement auctions in particular is large. Those considered in the present paper are multi-dimensional (or score) auctions, where bidders can submit bids on several attributes; in our

<sup>1</sup>The preferred auction format used by Ofgem has been the first-score function (Ofgem, 2017). As is the case for Ofgem, the other mentioned tenders also use a first-score auction.

<sup>2</sup>This is also the case for Ofgem. For example, according to Ofgem (2014, pp. 18–19), they write: “[t]he drill became stuck. Subsequent attempts to complete HDD failed due to drill holes collapsing.” The developer estimated the adaptation cost to be £10 million. Ofgem agreed to cover £7 million.

case, in the bidding stage, they submit a bid combined of a level of quality with a cost of delivering the project. Che (1993) studies score auctions, including the first-score auction, in a procurement setting that induces bidders to submit truthful costs; see also the papers by Branco (1997) or Asker and Cantillon (2010). An extension of this literature in the direction of the present paper discusses score auctions with reserve prices. While Che (1993) and Asker and Cantillon (2010) use reserve prices in an indirect way by setting a ceiling on the quality level, Hanazono et al. (2013) show that a reserve price results in a lower expected score under the first-score auction than under the second-score auction. Other contributions to this literature are Ding and Wolfstetter (2011) and Albano et al. (2024).

There are only a few contributions dealing with a procurer's cost after completion of the project. A closely related situation is treated in the literature on auctions with defaults, also known as *ex post* shocks, and includes Wachrer (1995), Parlane (2003), Board (2007), Ganuza (2007), Burguet et al. (2012), and Chillemi and Mezzetti (2014). In this literature, the studies that come closest to this present study are perhaps the work of Birulin (2020) and Birulin and Izmalkov (2022), who study a situation where a procurer runs a tender for the procurement of a complex project, and who also suggest tools to overcome a potential cost overrun. While Birulin (2020) studies the use of a surety bond (i.e., compensation to pay the procurer in case of default), Birulin and Izmalkov (2022) suggest a split in the payment (of the winner) into two – one part to be paid immediately after the end of the auction and the other part upon the completion of the project. We study the use of a pre-auctioning questionnaire that has the purpose of detecting/obtaining information on the likely adaptation cost. See also Chillemi and Galavotti (2025) for the use of audit.

In the literature on the use of questionnaires to extract information from an agent, a key paper is by Glazer and Rubinstein (2014), who study a principal–agent model within the framework of Bayesian persuasion – where the principal uses questionnaires to reduce the probability of dishonest reporting from the agent – and they show that a complex questionnaire will incentivize the agent to report truthfully about its type. This is in line with what emerges from the random error model of adaptation cost, where the complexity of the questionnaire will improve the procurer's estimate of the future cost. However, and compared with Glazer and Rubinstein (2014), we use an auction with a pre-qualification stage where a questionnaire is used and information (from the questionnaires) can be shared with the bidders pre-auctioning. Further, their agent is bounded rational, which is not the case for our bidders. For related literature, see Glazer and Rubinstein (2012), De Clippel et al. (2019), and Jakobsen (2020), where the 2019 and 2020 papers study mechanism design within the framework of behavioral economics.

We also contribute to the literature on information disclosure in auctions, where a key paper is by Milgrom and Weber (1982). They show that information in an affiliated value setting is revenue-increasing (see also Esó and Szentes, 2007). The closest paper to ours is by Bergemann et al. (2022), who study a second-price auction where bidders are unaware of their values while the auctioneer has access to information about them and can choose whether to reveal all or some of this information to the bidders. They show that optimal information sharing implies the withholding of some but not all information. Their results are not immediately transferable to our context as adaptation cost may be observed but construction cost remains unknown to the procurer. Other papers on information disclosure are Ganuza (2004), Bergemann and Pesendorfer (2007), Bergemann et al. (2026), Catonini and Stepanov (2025), and Ashkenazi-Golan et al. (2023), where the latter study the optimal information to disclose to the bidders to increase a seller's revenue in first-price two-stage auctions.

The paper is organized as follows. Section 2 contains the basic model of a procurement auction with adaptation cost, as well as the situation where the procurer and the bidders do not have any information about the adaptation costs. In Section 3, we analyze the situations where either the procurer alone, the procurer, and the individual bidder, or all participants have full information about the (adaptation) costs. In Section 4, knowledge about the cost is incomplete to both the procurer and each bidder. Section 5 introduces the questionnaire and the random error model of adaptation cost, followed by some final comments in the last section. Proofs of propositions stated in the text are collected in an appendix. The completion of an example is marked with a circle.

## 2. A model of procurement auctions with adaptation cost

We consider a situation where a buyer (i.e., procurer) invites  $n$  firms (hereafter bidders) to submit bids for carrying out a project. A bid essentially consists of a description of the quality of the project to be delivered, here assumed to be described by a single variable  $q$ , together with a payment  $b$ , which should cover the cost of delivering the project at the quality level  $q$ . The construction cost for the bidder is given by a function  $c(q, \theta)$ , which depends on the planned quality  $q$  of the project and a productivity parameter  $\theta$ . We assume that  $c$  is continuously differentiable with positive partial derivatives  $c'_q(q, \theta)$  and  $c'_\theta(q, \theta)$ . The parameter  $\theta$  of the bidders is assumed to be identically and independently distributed in an interval  $[\underline{\theta}, \bar{\theta}]$  with a distribution function  $F$  which admits a density function  $f$ .

In addition to the quality of the project and the cost associated with acquiring it, there is an additional aspect of the project; this is related to its functioning during or after the construction phase, which is assumed to be

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important to the procurer. Depending on the particular case, it may consist of delays in the proper functioning of the project or unexpected discrepancies between the project as contemplated by the procurer and the one that was contracted and delivered. A main feature of these discrepancies is that they cannot easily be predicted or even described in detail, so that it is difficult or even impossible to take them into consideration in the contract. Instead, they take the form of general disturbances giving rise to additional cost, which eventually must be covered by the procurer. It is assumed throughout that the cost is not verifiable, so that it cannot be made an object of contracting between procurer and bidder. We refer to this as the “adaptation cost” associated with the project as constructed by a given bidder, and denote it by  $\tau$ . It may differ among bidders, and we assume that the possible adaptation cost  $\tau$  is distributed in an interval  $[0, \bar{\tau}]$  with distribution function  $G$ , which again allows for a density function  $g$ . We denote by  $\tau^*$  the expected value of  $\tau$ , that is  $\tau^* = \int \tau dG$ .

Adaptation cost may originate from different sources, and this origin may matter when we turn to the possibility of the procurer acquiring information about its size. A particular case is the “random error model” of adaptation cost, which is generated by small deviations from the plans during the construction period. We return to this random error model later when we discuss the role of questionnaires.

Adaptation cost is encountered when the project is in possession and also in the course of its operation, so that there is a time span, in some cases substantial, between submission of bids and outlays for adaptation cost, and it would therefore be reasonable to factor in a suitable discounting of the future outlays. We have chosen not to use explicit discounting of future payments in the model, which amounts to selecting a zero discounting rate of interest. Alternatively, we can assume that all adaptation cost  $\tau$  should be interpreted as the present value of the future flow of payments for adaptation cost.

We define the payoff of the procurer as

$$U(q, b, \tau) = u(q, \tau) - b, \quad (1)$$

where  $u$  is the utility function of the procurer, assumed to be twice continuously differentiable, concave in  $q$ , and linear in  $\tau$ , with  $u'_q > 0$ ,  $u'_\tau > 0$ . In some cases, we shall consider the special case of a separable utility function

$$u(q, \tau) = v(q) - \tau, \quad (2)$$

where the adaptation cost takes the form of a monetary outlay to be covered by the procurer.

From a social point of view, the surplus achieved by implementing the project in quality  $q$  by a bidder of type  $(\theta, \tau)$  is

$$S(q, \theta, \tau) = u(q, \tau) - c(q, \theta), \quad (3)$$

The payment  $b$  in (1) then determines the split of this surplus between the procurer and the winning bidder.

In order to allocate the project to a bidder, the procurer uses an auction. The auction used is assumed to be a first-score auction, where the bidders deliver a bid consisting of a proposed project quality and a payment. The bidders are assumed to know the procurer's objectives as specified in (1), and all the involved parties know the cost function  $c(\cdot, \cdot)$ .<sup>3</sup> The type parameters  $\theta$  and  $\tau$  are private knowledge of the bidder, but their distributions are common knowledge. We write arrays of parameters using boldface notation, as  $\theta = (\theta_1, \dots, \theta_n)$ ,  $\tau = (\tau_1, \dots, \tau_n)$ .

For the adaptation cost parameter  $\tau$ , we shall be more specific about the information of the participants in each of the cases considered below. Indeed, our analysis pertains to the way in which information about the adaptation cost connected with the bidder will influence the bidding behavior, the procurer's payoff, and later the auction design.

As a first step, we consider the most straightforward case where the procurer has no specific information about the adaptation cost  $\tau$ , knowing only its distribution  $G$ . In this case, the cost of adaptation becomes irrelevant to the bidder. The unknown adaptation cost will show up only in the utility function of the procurer, which will take the form  $u(\cdot, \tau^*)$  as a function of  $q$  only, and only the productivity type  $\theta$  matters for the bids.

A bidding strategy in this procurement auction is a pair of functions  $(q, \beta)$  which to each parameter value  $\theta$  assigns a quality level and a payment. The following result is due to Che (1993), and it will serve as a starting point for our discussion of the role of information about adaptation cost. For later reference, we state also the payoff of the procurer.

**Proposition 1.** *Assume that procurer and bidders have no information about the adaptation cost. Then the bidding strategy in a symmetric equilibrium is  $(q^\circ, \beta^\circ)$ , where*

$$q^\circ(\theta) = \operatorname{argmax}_q S(q, \theta, \tau^*), \tag{4}$$

$$\beta^\circ(\theta) = c(q^\circ(\theta), \theta) + \int_{\theta}^{\bar{\theta}} c'_\theta(q^\circ(s), s) \left[ \frac{1 - F(s)}{1 - F(\theta)} \right]^{n-1} ds. \tag{5}$$

The procurer's payoff at the array  $\theta = (\theta_i)_{i=1}^n$  of construction cost parameters is

$$W_0(\theta) = \min_i \{u(q^*(\theta_i), \tau^*) - \beta^*(\theta_i)\}, \tag{6}$$

and the ex ante expected payoff is  $\mathbb{E}_\theta W_0(\theta)$ .

<sup>3</sup>As a consequence, the procurer will not accept bids  $(q, b)$  for which payment  $b$  exceeds maximal possible construction cost  $c(q, \theta)$ .

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The quality component of the equilibrium bidding strategy is seen from (4) to be determined by maximization of the social surplus, a feature often referred to as Che's lemma, and which will reappear also under other assumptions about the information available to the procurer. The procurement auction can be seen as a usual first-score auction, where bidders propose to leave to the procurer part of the surplus  $S(q, \theta, \tau)$  as payment for winning the contract. Rather than proposing the full surplus, a bidder will shade their bid, so that the payment exceeds the cost to an amount dependent on the probability of winning. We shall use this interpretation of the bid repeatedly in the following.

In many procurement auctions, the auctioneer uses a reserve price, which restricts the possible payment of the procurer to the winner of the auction. In our case, where the bids have both a quality and a price component, the restriction could in principle take the form of a functional relationship between quality levels and the maximal acceptable payment for this quality. In a simpler and more realistic version, the reserve price is just an upper bound  $b_{\max}$  for the procurer's payment, no matter which quality may be selected. The restriction pertains only to the payment of construction cost, by the very nature of the adaptation cost its size cannot be part of a contract between procurer and deliverer.

**Proposition 2.** *Assume that the procurer has set a reserve price of the size  $b_{\max}$ . Then the bidding strategy in a symmetric equilibrium is*

$$q^{\circ\circ}(\theta) = \operatorname{argmax}_{q: c(q, \theta) \leq b_{\max}} S(q, \theta, \tau^*)$$

$$\beta^{\circ\circ}(\theta) = c(q^{\circ\circ}(\theta), \theta) + \int_{\theta}^{\hat{\theta}} c'_{\theta}(q^{\circ\circ}(s), s) \left[ \frac{1 - \widehat{F}(s)}{1 - \widehat{F}(\theta)} \right]^{n-1} ds,$$

where  $\hat{\theta} = \max\{\theta | c(q^{\circ\circ}(\theta), \theta) \leq b_{\max}\}$  and  $\widehat{F}$  is the conditional distribution of  $F$  on  $[\theta, \hat{\theta}]$ .

It is seen that, formally, the optimal bidding strategies under reserve prices differ only slightly from those found in Proposition 1, the main change being that the previous surplus maximizing quality level is replaced by one which maximizes surplus under the payment constraint given by the reserve price. This difference may well be substantial in practice, but for our subsequent analysis of adaptation cost and the role of information, we may abstract from possible reserve prices.

### 3. Information about adaptation cost

So far, the future adaptation cost of each bidder has been assumed to be unknown not only to the procurer, but also to the bidder in question. We

shall be interested in the extent to which improved information will change the bidding strategies and the outcomes of the procurement auction. For this, we begin with the other extreme, that of full information. This is admittedly unrealistic, but it serves as an indication of what is at stake when considering alternative methods of obtaining additional information, given that it is inherently incomplete.

Now, there are three cases to be considered, numbered I–III, where we refer to the case from the previous section – neither the procurer or the individual bidder has information about the adaptation cost – as Case 0. In the following first case, only the procurer is informed about the adaptation cost of each of the bidders; in the second case, each of the bidders also know their own adaptation cost parameter; and finally in the third case, every participant knows the adaptation cost of each of the bidders.

### **Case I. Procurer alone has full information about adaptation parameters.**

If only the procurer has access to information about adaptation cost, whereas the bidders know only its distribution, then the bidding behavior must be as given in (4) and (5), because the probability of winning as perceived by the bidder depends only on the construction cost. The situation in which bidders themselves do not know their own adaptation cost may seem strange, but might possibly occur due to the particular nature of the (adaptation) cost. It should be noticed that although bids take the same form as in the case of no information, the outcome of the auction is influenced. Indeed, because for each  $(\theta, \tau)$  the procurer will choose the bidder  $i$  for which the payoff  $U(q^\circ(\theta_i), \beta^\circ(\theta_i), \tau_i)$  is maximal, we find that the expected payoff is at least as large as with no information, and larger if the distribution of  $\tau_i$  is not too concentrated around  $\tau^*$ .

### **Case II. Procurer and bidder have full knowledge of the adaptation parameters.**

In this case, each bidder knows its own adaptation cost, but not that of the other bidders. The bidders know that the procurer has this knowledge and take this into account when determining their bids, so that bids now depend not only on  $\theta$  but also on  $\tau$ . The bids will reflect the combination of construction and adaptation costs, even though the latter are paid by the procurer. Indeed, a bidder with adaptation parameter larger than average may be inclined to reduce the shading of the cost parameter, knowing that the procurer chooses the bid  $(q_i, b_i)$  for which the payoff  $U(q_i, b_i, \tau_i)$  is maximal.

The bidding strategy is now a function to which each bidder with type  $(\theta, \tau)$  assigns a pair  $(q(\theta, \tau), \beta(\theta, \tau))$  consisting of a proposed quality and a payment. This must be taken into consideration when formulating the bid strategy, as bid strategies should reflect the competition of bidders in both

parameters. We introduce the notation

$$v(\theta, \tau) = \max_q S(q, \theta, \tau) \quad (7)$$

for the maximal social surplus achieved when the bidder has type  $(\theta, \tau)$ , and we let  $H$  denote the distribution of  $v$ ,  $H(t) = \mathbb{P}\{v(\theta, \tau) \leq t\}$ .

**Proposition 3.** *Assume that the procurer has full information about the adaptation cost of each of the bidders, and that each bidder knows its own adaptation cost. The bidding strategy in a symmetric equilibrium is  $(q^\circ, \beta^\circ)$ , where*

$$q^\circ(\theta, \tau) = \operatorname{argmax}_q S(q, \theta, \tau) \quad (8)$$

$$\beta^\circ(\theta, \tau) = c(q^\circ(\theta, \tau), \theta) + \int_0^{v(\theta, \tau)} \left[ \frac{H(t)}{H(v(\theta, \tau))} \right]^{n-1} dt. \quad (9)$$

The expected payoff of the procurer is  $\mathbb{E}_{\theta, \tau} \max_i U(q^\circ(\theta_i, \tau_i), \beta^\circ(\theta_i, \tau_i), \tau_i)$ .

While the quality bid (8) is largely identical to that of (4), modified only to take  $\tau_i$  into account, the payment part (9) of the optimal bid reflects that each individual bidder  $i$  is aware that its adaptation cost parameter  $\tau_i$  matters. Using the same interpretation of the procurement auction as an ordinary auction where the participant with parameter  $(\theta, \tau)$  bids for the surplus with value  $S(q^\circ(\theta, \tau), \theta, \tau)$ , we see that the parameter  $\tau_i$  may indeed influence the bid shading.

In the particular case of separable procurer utility (2), it would in principle be possible that the bidder also covers the adaptation cost. The equilibrium bidding strategies for this case turn out to be equivalent to those stated in Proposition 3, and this holds also for the payoff of the procurer.

**Proposition 4.** *Assume that the procurer's utility is separable and the winning bidder bears the adaptation cost. If the procurer has full information about the adaptation costs, and each bidder knows its own adaptation cost, then the bidding strategy in a symmetric equilibrium takes pairs  $(\theta, \tau)$  to bids  $(q^\circ(\theta, \tau), \beta^\circ(\theta, \tau) + \tau)$ , where  $q^\circ$  and  $\beta^\circ$  are given in (8) and (9). The procurer's expected payoff is*

$$\mathbb{E}_{\theta, \tau} \max_i U(q^\circ(\theta_i, \tau_i), \beta^\circ(\theta_i, \tau_i), \tau_i).$$

We notice that  $q^\circ(\theta, \tau)$  in Proposition 3 does not depend on  $\tau$  when procurer's utility is separable, so that the quality component of the bid given  $(\theta_i, \tau_i)$  will be the same no matter who pays the adaptation cost. The total cost to be paid by the procurer will, however, differ. We should expect the

procurer's cost to be lower when it takes the form of a single payment for both types of cost, since in this case the bidders will compete with regard to this payment, whereas in the second case the procurer must expect to pay the average adaptation cost  $\tau^*$  and bidders compete only on the construction cost. We indicate this with the following example.

**Example 1.** Assume that both  $\theta$  and  $\tau$  are uniformly distributed in  $[0, 1]$ , and that the cost function takes the form

$$c(q, \theta) = \theta q.$$

We assume that the procurer's utility function is separable, and that  $v(q)$  satisfies  $v'(q) > 1$  for  $0 \leq q < 1$ , but is constant for  $q \geq 1$ , so that  $q^*(\theta, \tau) = 1$  for all  $\theta$  and  $\tau$ .

We consider first the case of the previous section where neither the procurer nor the bidders know  $\tau$  (i.e., Case 0). Then each bidder  $i$  bids the share of the surplus  $v(1) - \theta_i$  that it will grant to the procurer in order to get the contract. Because the expected payment in a first-score auction is the expectation of the second highest value, the procurer's payment will be the expected second-smallest of  $n$  realizations of the uniform variable  $\theta$ , which is  $2/(n + 1)$ . To this should be added the expected adaptation cost, which is  $1/2$ . For  $n = 2$ , the total cost is 1.17, for  $n = 3$ , it is 1, and for  $n = 4$ , we get 0.9.

Next we consider Case I, where the procurer knows the bidders' adaptation parameters, but the bidders themselves do not know them. Then the procurer chooses the bidder for which the sum of the adaptation and construction costs is minimal. The bid function is  $\beta^\circ(\theta) = \theta + [(1 - \theta)/n]$  with distribution function

$$\mathbb{P}(\beta^\circ(\theta) \leq b) = \frac{n}{n-1}b - \frac{1}{n-1}.$$

Using this, we find the density  $\varphi$  of  $w = \tau + b$  over the interval  $[0, 2]$  as

$$\varphi(w) = \begin{cases} 0 & w \leq \frac{1}{n}, \\ \frac{n}{n-1} \left( w - \frac{1}{n} \right) & \frac{1}{n} \leq w \leq 1, \\ 1 & 1 \leq w \leq \frac{n+1}{n}, \\ \frac{n}{n-1} (2-w) & \frac{n+1}{n} \leq w \leq 2. \end{cases}$$

The expected value of the minimum over  $n$  draws from this distribution is 1.065 (for  $n = 2$ ), which is clearly an improvement over the previous result. For  $n = 3$ , the value is 0.868.

Finally, we consider Case II where also the bidders know their own adaptation parameter. The bidding strategies (8) and (9) are simplified considerably as quality is constant and maximization of social surplus reduces to minimization of total cost  $v = \theta + \tau$ . The sum of  $\theta$  and  $\tau$  has density and distribution functions

$$h(v) = \begin{cases} v & 0 \leq v \leq 1 \\ 2 - v & 1 < v \leq 2 \end{cases}; \quad H(v) = \begin{cases} \frac{v^2}{2} & 0 \leq v \leq 1 \\ 1 - \frac{(2 - v)^2}{2} & 1 < v \leq 2. \end{cases}$$

The second smallest of  $n$  realizations of  $v$  has density function  $n(n - 1)h(v)H(v)(1 - H(v))^{n-2}$ . Its expected value is 1.23 for  $n = 2$ , which is larger than what was found in Case 0 (as well as Case I). For  $n = 3$ , it is 0.8728, which is much smaller than the result in Case 0, but still exceeds that obtained in Case I. ○

**Case III. Procurer and all bidders can observe the adaptation parameters of all bidders.** Here, the auction pertains only to construction cost, and the bids are assessed by the procurer only after adding the adaptation cost. Each bidder knows all the individual adaptation costs and the bidding strategies take this into account.

**Proposition 5.** Assume that all participants have perfect knowledge of the bidders' adaptation parameters as given by  $\tau = (\tau_i)_{i=1}^n$ . Then the equilibrium bidding strategies are  $(q^\star, \beta_i^\star)$ ,  $i = 1, \dots, n$ , where

$$q^\star(\theta_i, \tau_i) = \underset{q}{\operatorname{argmax}} S(q, \theta_i, \tau_i), \tag{10}$$

$$\beta_i^\star(\theta_i, \tau) = c(q^\star(\theta_i, \tau_i), \theta_i) + \int_0^{\theta_i} \prod_{j \neq i} \frac{H[\tau_j](t)}{H[\tau_j](v_i)} dt, \tag{11}$$

and  $H[\tau_j](t) = \mathbb{P}\{v(\theta, \tau_j) \leq t\}$ . If the procurer's utility is separable, then  $q^\star(\theta_i, \tau_i) = q^\star(\theta_i)$  is independent of  $\tau_i$ , the mapping  $\phi(\theta) = \max_q [v(q) - c(q, \theta)]$  is invertible, and (11) can be written as

$$\beta_i^\star(\theta_i, \tau) = c(q^\star(\theta_i), \theta_i) + \int_{\theta_i}^{\bar{\theta}} c'_\theta(q^\star(s), s) \prod_{j \neq i} \left[ \frac{1 - F(\phi^{-1}(\phi(s) - \tau_i + \tau_j))}{1 - F(\phi^{-1}(\phi(\theta_i) - \tau_i + \tau_j))} \right] ds. \tag{12}$$

The equilibrium bidding strategies have a less simple form here than in the previous cases considered. It is seen that the bids relate only to the

construction cost, but are suitably affected by the adaptation parameters, which are now common knowledge. The informational rent that bidders can obtain in auctions is strictly confined to construction cost, whereas it would involve also the adaptation cost in the case where bidders knew only their own parameters. It might therefore be expected that the procurer's expected cost will be smaller when all bidders are aware of their adaptation parameters.

In order to get some intuition about the effects of full information, we consider a simplified version of Example 1.

**Example 2.** We keep the model of Example 1, but add a further simplification: the adaptation parameter  $\tau$  will have a discrete distribution, taking the values 0 or 1 with probability 1/2. As the expected value  $\tau^*$  is unchanged, the expected cost of the procurer when neither procurer nor bidders have any information is unchanged from Example 1, amounting to  $(1/2) + [2/(n+1)]$ , which gives 1.17 for  $n = 2$ . If only the procurer knows the adaptation cost parameters (Case I), then expected construction cost is  $2/3$  independent of the values of  $\tau$ , but the procurer will choose a bidder with  $\tau = 0$  whenever possible. Because this occurs with probability  $3/4$ , we get an expected total cost of  $(1/4) + (2/3) = 0.917$ , which as expected is much smaller.

In Case II, the procurer knows all adaptation cost parameters and the bidders know only their own. If  $n = 2$ , then a bidder with  $\tau = 0$  will be selected if the other bidder has  $\nu = 1$ . If the competitor also has  $\nu = 0$ , then the auction is symmetric and we may look for a symmetric bid function. The probability of winning given  $\theta$  is  $(1/2)(1 - \theta) + (1/2)$ , giving an optimal bid of  $1 - (\theta/4)$ . If  $\tau = 1$ , then the bid is relevant only if the other bidder has  $\nu = 1$ , and then the auction is again symmetric. Because the probability of winning here is  $(1 - \theta)$ , the optimal bid will be  $1 - (\theta/2)$ . The winning bid for the combination  $(0, 0)$  is  $1 - (1/4) \max\{\theta_1, \theta_2\}$  with average value  $5/6$ ; for  $(1, 1)$ , it is  $1 - (1/2) \max\{\theta_1, \theta_2\}$  with average value  $2/3$ ; and for  $(0, 1)$  and  $(1, 0)$ , average payment is the average of the bid, which is  $1 - (1/8) = (7/8)$ . Adding the adaptation cost and taking the average over the four possible situations, we obtain an *ex ante* expected cost of 1.063. As was to be expected, the result is better for the procurer than having no information whatsoever, but it is inferior to Case 1 as considered above. When bidders learn that their adaptation cost is below the average, they take advantage of this knowledge to inflate their bids.

In Case III, each bidder knows the adaptation parameters of all bidders. Consider first the case  $n = 2$ . There are two possible scenarios, depending on whether the bidders have the same or different values of  $\tau$ . In the first case, the expected payment for construction by the procurer is  $2/3$ . In the second case only the bidder with  $\tau = 0$  will participate and bid with the payment 1.

Collecting the cases, the *ex ante* expected cost to the procurer will be

$$\frac{1}{4} \cdot \frac{2}{3} + \frac{1}{4} \left(1 + \frac{2}{3}\right) + \frac{1}{2} \cdot 1 = \frac{13}{12} = 1.083,$$

which is greater than the 1.063 found in Case I. The knowledge of the competitor's adaptation cost makes it possible to increase the information rent coming from the construction cost. The difference may be smaller if there are more than two bidders, but the general picture will be the same, showing that the procurer is better off restricting the information flow to the bidders.  $\circ$

The results of Example 2 were obtained under rather specific assumptions on distributions and technology, and they may not necessarily reflect a more general pattern. Some additional insights can be gathered from studying the situation from the point of view of the bidders, each of which is affected by the possible knowledge of the competitors' adaptation costs. Define the expected information rent  $R_i$  of bidder  $i$  at  $(\theta_i, \tau_i)$  as the excess of expected payment over expected construction cost, which is  $R_i^{II} = \int_0^{\nu(\theta_i, \tau_i)} H(t)^{n-1} dt$  if bidders know only their own adaption cost, and  $R_i^{III} = \int_0^{\theta_i} [\prod_{j \neq i} H[\tau_j](t)] dt$  if they know all adaption costs. The *ex ante* information rent  $\mathbb{E}_\tau R_i$  of bidder  $i$  at  $\theta_i$  is the expected information rent before the competitor's adaptation cost is revealed (i.e., the expectation of  $R_i$  over all values of  $\tau_j$ ).

The information rent is closely related to the notion of stochastic dominance: if  $H_1$  and  $H_2$  are distribution functions defined on  $[\underline{\nu}, \bar{\nu}]$ , then  $H_1$  (second-order) stochastically dominates  $H_2$ , written  $H_1 >_{SD} H_2$ , if  $\int_{\underline{\nu}}^{\nu} H_1(t) dt \leq \int_{\underline{\nu}}^{\nu} H_2(t) dt$  for all  $\nu$  (cf. e.g., Rothschild and Stiglitz, 1970). Using this terminology, if the distribution  $H^{n-1}$  of winning with unknown competitor cost stochastically dominates  $\prod_{j \neq i} H[\tau_j]$ , where competitors' adaptation cost is known, that is if  $\prod_{j \neq i} H[\tau_j] >_{SD} H^{n-1}$ , then  $R_i^{II} \geq R_i^{III}$  at all  $(\theta_i, \tau_i)$ , meaning that the informational rent obtained by bidder  $i$  is at least as large without any information as with information about adaptation costs.

To see why no information can be better than information, we use a fundamental property of stochastic dominance. If a distribution  $H_2$  is stochastically dominated by another distribution  $H_1$  (i.e., if  $H_1 >_{SD} H_2$ ), then the random variable with distribution  $H_2$  can be seen as obtained from a variable distributed according to  $H_1$  after adding some random noise, giving rise to increased riskiness. In our case, this type of increased riskiness means that bidders expect their competitors to have occasional very high adaptation costs so that bids with a higher information rent may win.

We explore this in the following proposition. Here the assumption of separability is essential, and the conclusions do necessarily carry over general utility functions of the procurer, depending on adaptation cost in a nonlinear

way, something that may well occur in practice. The restriction of the case to only two bidders has been made to avoid additional formalism, as the case considered indicates the type of results that can be obtained.

**Proposition 6.** *Assume that the procurer's utility is separable and that  $n = 2$ . Let  $i$  be any bidder and  $j \neq i$  the competitor. Then, (a)  $H[\tau^*] >_{SD} H$ , (b) if  $\tau_j < \tau^*$ , then  $R_i^{II} \geq R_i^{III}$  at all  $(\theta_i, \tau_i)$ , and (c) the ex ante informational rent of bidder  $i$  satisfies  $\mathbb{E}_\tau R_i^{III} \geq \mathbb{E}_\tau R_i^{II}$  at all  $(\theta_i, \tau_i)$ .*

Part (a) of the proposition confirms the intuition about stochastic dominance in the particular case where all competitors have exactly the average adaptation cost, as the knowledge of this adaptation cost does not benefit the bidder. Part (b) states that if the adaptation cost of the competitor is below average, then the bidder will be better off in Case II (bidders know only their own adaptation cost), in the sense that the upwards shading of the bid is larger than in Case III, where the strong position of the competitor is taken into account. After taking expectation over all the possible adaptation costs of the competitor, the ranking is reversed. The shading changes in an upwards or downwards direction with the competitor's adaptation cost, but (c) shows that, on average, the changes in the upwards direction exceed those pointing downwards. This can be seen as a concavity property of the information rent, which is satisfied under the specific assumptions of the proposition.

Summing up the findings of this section, we notice that information about adaptation cost, and in particular the availability of this information for procurer or for bidders, matters for the procurer as well as for the bidders. As was to be expected, the case where the procurer has this information and the bidders know nothing is the best for the procurer. When bidders know their own adaptation cost, but not those of the competitor, the bids will be adapted to this situation, and the advantages for the procurer will be diminished. This also goes for the case where bidders have information on all adaptation costs, but the ranking of the two last cases depends on further details on the data of the problem.

#### 4. Acquiring information about a bidder's adaptation cost

In the previous sections, the relevant adaptation parameters were either fully known or fully unknown to the decision-makers. In a more realistic approach, the adaptation parameter  $\tau_i$  of bidder  $i$  will be known only imperfectly, not only by the procurer but also by the individual bidders. We must therefore keep track of the knowledge available to either the procurer or each of the bidders, taking into account that this knowledge is incomplete.

If all the above distributions are given and not subject to change, then the bidding behavior will be largely as in the previous sections, where the

observed parameter  $\tau_i$  is replaced by a subjective probability distribution over its possible values, reflecting the beliefs of either the procurer or the bidder. A fundamentally new situation will occur if we allow for changes in the subjective distributions as a result of observations. We assume that there is an initial pre-auction phase during which the procurer collects observations about each of the bidders. Based on these observations, the procurer updates the beliefs about the adaptation parameters of the bidders.

Formally, an observation method is a pair  $(\Sigma, \delta)$ , where  $\Sigma$  is a finite set of signals and  $\delta$  is a map which to each (“true”) value  $\tau$  of the adaptation parameter assigns a probability distribution  $\delta(\sigma|\tau)$  over signals  $\sigma \in \Sigma$ . A signal  $\sigma$  induces a (posterior) probability density function  $g(\cdot|\sigma)$  over the adaptation parameters in  $[0, \bar{\tau}]$  of the bidder in question, found by Bayesian updating as  $g(\tau|\sigma) = \delta(\sigma|\tau)g(\tau)/\mathbb{P}(\sigma)$ , where  $\mathbb{P}(\sigma) = \int_0^{\bar{\tau}} \delta(\sigma|t)g(t)dt$  is the probability of observing the signal  $\sigma$ . A participant, procurer, or bidder, who observes the signal  $\sigma$  related to a particular bidder, will have a revised subjective distribution  $g(\cdot|\sigma)$  over the adaptation parameters of this bidder. We let  $\tau^*(\sigma) = \int_0^{\bar{\tau}} \tau g(\tau|\sigma)$  denote the expectation of  $\tau$  with respect to the distribution  $g(\cdot|\sigma)$ .

We now proceed to consider how the use of an observation method will influence the outcome of the auction and in particular the payoff of the procurer. Because the use of an information method may be costly, it will matter for the procurer whether it will increase the payoff on average, and it may matter whether or not the information about a given bidder is shared with the bidder or perhaps with all bidders. As in the previous section, there are several different scenarios to consider.

**Case I. Only the procurer has access to the information.** In this case, the signals  $\sigma_i$  pertaining to the bidder  $i = 1, \dots, n$  will be unknown to the bidders themselves and are used only to select among the bids submitted.

**Proposition 7.** *Assume that the procurer uses the information method  $(\Sigma, \delta)$  and observes signals  $\sigma_i$  for bidder  $i, i = 1, \dots, n$ . If bidders are ignorant of their adaptation cost and do not observe their signals, then symmetric equilibrium bid strategies are  $(q^\circ, \beta^\circ)$  as defined in (4) and (5), and for given arrays  $\theta = (\theta_i)_{i=1}^n$  and  $\sigma = (\sigma_i)_{i=1}^n$ , the procurer’s expected utility*

$$W_1(\theta, \sigma) = \max_i \left\{ \int_0^{\bar{\tau}} u(q^\circ(\theta_i), \tau)g(\tau|\sigma_i)d\tau - \beta^\circ(\theta_i) \right\}$$

satisfies  $W_1(\theta, \sigma) \geq W_0(\theta)$ .

The result stated is quite intuitive. Because bidders have no information about adaptation costs, neither their own nor those of the other bidders, they

use the bidding strategy adapted to this situation, and as the procurer may choose in each situation according to what will give the highest payoff, it is to be expected that the overall expected payoff will increase.

Define *ex ante* expected utility of the procurer as  $W_1^* = \sum_{\sigma \in \Sigma^N} \mathbb{P}(\sigma) \mathbb{E}_\theta W_1(\theta, \sigma)$ , where  $\mathbb{P}(\sigma) = \prod_{i=1}^n \mathbb{P}(\sigma_i)$ . Similarly, we let  $W_0^* = \mathbb{E}_\theta W_0(\theta)$ , and we have that  $W_1^* \geq W_0^*$ .

**Example 3.** Consider an observation method with two possible signals *L* (Low) and *H* (High), where  $\delta$  is given by the columns in the following table.

	0	1
<i>L</i>	3/4	1/4
<i>H</i>	1/4	3/4

Assume that  $n = 2$  and the procurer uses the method on each of the bidders. In the case that the observation method is not used, the expected payoff is 1.17 as in Example 2. If the procurer chooses to use the observations, the optimal choice is selecting a bidder with signal *L* if possible, and if so, choosing the one with the lowest bid. To find the expected payment of the procurer, we consider the three possible combinations of signals.

- (i) No *L* observed. With probability 9/16, both bidders have  $\tau = 1$ ; with probability 6/16, one bidder has  $\tau = 1$ ; and with probability 1/2, this is the bidder chosen. Finally, with probability 1/16, both have  $\tau = 0$ . Expected adaptation cost is therefore 12/16, to which comes construction payment 2/3, in total 17/12.
- (ii) One *L* and one *H* observed. Here the bid is found from (5) as  $\beta^*(\theta) = \theta + [(1 - \theta)/2]$ , and its expected value is 3/4. The bidder selected has  $\tau = 0$  with probability 3/4 and  $\tau = 1$  with probability 1/4, giving an expected payment of 1.
- (iii) Two instances of *L*. Here, both bidders have  $\tau = 1$  with probability 9/16, and one of them has  $\tau = 1$  with probability 6/16. The expected payment can be found as 507/480.

Weighing the expected payments in the three cases with their probabilities (1/4, 1/2, 1/4), one gets an overall expected payment of 1.12, which shows that the use of observations improves the payoff of the procurer. In the terminology introduced above, we have that  $W_1^* > W_0^*$ , with strict inequality in the case considered. ○

**Case II. Each bidder gets access to the information about their own adaptation cost.** In this case, bidder  $i$  knows the signal  $\sigma_i$  resulting from the use of the information method. With respect to the competing bidders, bidder  $i$  knows only that their adaptation parameters are distributed according to  $G$ . The bid will depend on the signal  $\sigma_i$  received and on the construction cost parameter.

**Proposition 8.** *Assume that the procurer performs an observation of the adaptation cost, and that bidders have access to their own observation result. Then, the bidding strategy in a symmetric equilibrium assigns to each pair  $(\theta, \sigma)$  the decisions  $(q^\circ(\theta, \tau^*(\sigma)), \beta^\circ(\theta, \tau^*(\sigma)))$ . For given arrays  $\theta$  and  $\sigma$ , the procurer's expected utility*

$$W_2(\theta, \sigma) = \max_i \left\{ \int_0^{\bar{\tau}} u(q^\circ(\theta_i, \tau)g(\tau|\sigma_i))d\tau - \beta^\circ(\theta_i, \tau^*(\sigma_i)) \right\}$$

satisfies  $W_2(\theta, \sigma) \geq W_0(\theta)$ .

The proof follows that of Proposition 7 and is omitted. Proceeding as above, we define the *ex ante* expected utility as  $W_2^* = \sum_{\sigma \in \Sigma^N} \mathbb{P}(\sigma) \mathbb{E}_\theta W_2(\theta, \sigma)$ , and we have that  $W_2^* \geq W_0^*$ .

The information acquired by the procurer and communicated to the bidder matters because it influences the bids, but additional knowledge about the adaptation cost may influence the bidding in both directions. A bidder who realizes that the adaptation cost is high will be inclined to reduce the allowance for construction cost in order to win the auction, and bidders knowing that their adaptation cost is low may increase the bid for the construction part. This in its turn may induce the procurer to transmit only partial information or even no information at all.

**Example 4.** Using the model of Example 2 together with the information method of Example 3, we find the bidding strategies, given the information received, and compare them with the case of no information provided to bidders.

If the bidder's signal is  $L$ , then the expected adaptation cost to the procurer will be  $1/4$ . As the result for the other bidder is not known, the bidder will use the average value  $1/2$  of  $\tau$  for the competing bidder, and the bidder with construction cost  $\theta$  will win the auction if  $(1/4) + \theta < (1/2) + \theta'$ , where  $\theta'$  is the construction cost of the competitor. Using the fact that the optimal bidding strategy at  $\theta$  is the expected value of  $\theta'$  over the values at which  $\theta$  is winning, namely the interval  $[\theta - 1/4, 1]$ , we find that the bid is

$$\beta^\circ\left(\theta, \frac{1}{4}\right) = \theta + \frac{1}{2} \left(1 - \theta + \frac{1}{4}\right) = \frac{\theta}{2} + \frac{5}{8} \quad (13)$$

for  $\theta \leq 3/4$ , and  $\beta^\circ(\theta, 1/4) = 1$  otherwise. If the signal is  $H$ , the same argumentation can be used to find the bidding strategy

$$\beta^\circ\left(\theta, \frac{3}{4}\right) = \frac{\theta}{2} + \frac{3}{8}. \tag{14}$$

Having found the bid strategies, we proceed to the payment of the procurer. If the observations are  $(L, L)$ , then both bidders use the bid strategy (13) and the winning bid is  $(1/2) \min\{\theta, \theta'\} + (5/8)$  when either  $\theta$  or  $\theta'$  are below  $3/4$ , an event happening with probability  $1 - (1/16)$ , and 1 otherwise, giving an average of 0.789, to which should be added the expected adaptation cost of  $1/4$ , given a total of 1.039. Proceeding in the same way if observations are  $(H, H)$ , where the winning bid is  $(1/2) \min\{\theta, \theta'\} + (3/8)$ , we get an average of 0.542; adding  $3/4$  for expected adaption cost, we have a total of 1.292.

If one bidder has  $(\theta, L)$  and the other  $(\theta', H)$ , then the  $L$ -bidder wins when  $\theta \leq \theta' + (1/4)$ , happening with probability  $7/8$ , and the winning bid is  $(1/2)\theta + (5/8)$ , found as in (13). Otherwise, the  $H$ -bidder wins with the bid  $(1/2)\theta + (1/4)$ . Adding the adaptation cost and averaging over  $\theta$  and  $\theta'$ , we obtain a payment of 1.112. Finally, taking the average over the signal outcomes  $(L, L)$ ,  $(H, H)$ ,  $(L, H)$ , and  $(H, L)$ , we get an average payment of 1.139. This exceeds the payment that we found in Example 2, so that  $W_2^* < W_1^*$  in this case, showing that bidders may take advantage of the additional information. ○

As a particular case, we note that with only two bidders it is better to hide the observation results rather than communicate them to the bidders. This corresponds to the case of no information considered above, so that partial or full communication of observations may not necessarily be advantageous for the procurer.

**Case III. All participants have access to the information obtained.**

Because each bidder is now aware of the signals of all the other bidders, the bidding strategies must use this information, so that they will depend on the whole array  $\sigma$  of signals. As for the previous case, what matters for the dependence on signals is the expected value of the adaptation parameter given the signal, that is,  $\tau^*(\sigma_i)$ , for  $i = 1, \dots, n$ .

**Proposition 9.** *Assume that the procurer performs an observation of the adaptation cost, and that bidders have access to all the observation results. Then for each observation  $\sigma$  the bidding strategies  $(q^\bullet(\cdot, \tau^*(\sigma_i)), \beta^\bullet(\cdot, (\tau^*(\sigma_i))_{i=1}^n))$ ,  $i = 1, \dots, n$ , constitute an equilibrium,*

and for given arrays  $\theta$  and  $\sigma$ , the procurer's utility

$$W_3(\theta, \sigma) = \min_i \left\{ \int_0^{\bar{\tau}} u(q^\bullet(\theta_i, \sigma_i)\tau^*(\sigma_i))g(\tau|\sigma_i)d\tau - \beta^\bullet(\theta, (\tau^*(\sigma_i))_{i=1}^n) \right\}$$

satisfies  $W_3(\theta, \sigma) \geq W_0(\theta)$ .

Once again, the proposition is an easy consequence of previous results (i.e., proof omitted), with the inequality for the procurer's utility following from  $g(\tau|\sigma_i)$  being the result of an observed signal. Also, we define procurer's *ex ante* expected utility as

$$W_3^* = \sum_{\sigma \in \Sigma^N} \mathbb{P}(\sigma) \mathbb{E}_\theta W_3(\theta, \sigma)$$

and notice that  $W_3^* \geq W_0^*$ .

**Example 5.** From the case considered in previous examples, the bidding strategies can now be seen to depend on the observations for both bidders. If the observations are  $(L, L)$  or  $(H, H)$ , the bid function will be  $(\theta/2) + (1/2)$ , so that average payment is  $(1/6) + (3/4)$  and  $(1/6) + (5/4)$ , respectively, so that the average over these two observations is  $(1/6) + 1 = 1.167$ . If observations are  $(L, H)$ , then the  $L$ -bidder has bid function  $(\theta/2) + (3/4)$ , whereas the  $H$ -bidder uses  $(\theta/2) + (1/4)$ . The bidder with  $(\theta, L)$  will win over  $(\theta', H)$  when  $\theta' \geq \theta - 1/2$ , something that happens with probability  $7/8$ , and the payment, including adaptation cost, is  $(\theta/2) + 1$ . Otherwise, the bidder with  $(\theta', H)$  wins, and the payment is again  $(\theta'/2) + 1$ . Averaging over  $(\theta, \theta')$  again yields 1.167, which therefore is the expected total cost of the procurer.

Comparing with the results in the previous examples, we see that cost has increased, so that  $W_3^* < W_2^*$  in the case considered.  $\circ$

As is seen in this and the previous proposition, the effects of bidders' information when obtained through the use of an observation method are similar to those encountered when information is exact, not surprisingly as full information can be seen as a special case of what we have treated in this section. The use of different methods for information retrieval may, however, add some new aspects to the procurement auction, and we consider a particular case in the next section.

## 5. The random error model and the use of questionnaires

In the previous section, we considered abstract observation methods and their possible impact on the procurement auction without specifying details of the methods. Observations may range from inspection of historical records to direct questioning of the bidder. This latter method has several difficulties:

the bidder may not know its own adaptation cost and, if knowing, may be reluctant to furnish detailed information. This is because the cost is paid by the procurer and its size will matter only for the bid strategy.

It follows that the relevant information from the bidder must be obtained in an indirect way. Whether this is at all possible will depend upon the nature of the adaptation cost, which, as already mentioned, may originate from different sources, and the observation methods should correspond to the nature of the cost of project adaptation. In the following, we consider a particular version of an adaptation cost driver.

We assume that the adaptation cost is generated by small independent random deviations from planned actions, adding up in the course of the construction period to a deviation that eventually gives rise to a cost to the procurer. It should be underlined that the deviations are not errors in the legal sense, meaning that the bidder delivering the project could be held responsible for the cost. The deviations may be positive, which in our case means that they contribute to the cost of adapting the project, but they may also be negative or cost-reducing.

In our application of the random error model, the small deviations taking place during the construction are independent and identically distributed with mean zero and standard deviation  $\sigma$ , but the distribution may otherwise be arbitrary. The errors cumulate over time to form the final deviation, and to find the distribution of this aggregate or final deviation, we can appeal to the law of large numbers. Given that errors can occur at any moment during the construction period of length  $T$ , the appropriate version of this law is Donsker's theorem (see Lemma A3 in the Appendix). It states that when errors are small but frequent, the error process can be described approximately as a Brownian motion. We can use this to find the distribution of adaptation costs.

**Proposition 10.** *Let  $T$  be the length of the construction period for the project. In the random error model of adaptation cost, the cumulated deviations at the end of the period are approximately normally distributed with mean 0 and standard deviation  $\sigma\sqrt{T}$ , and the expected adaptation cost is  $\tau = \sigma\sqrt{T/2\pi}$ .*

Returning to the problem of observing adaptation cost, we notice that the cost driver is the propensity to diverge from the directly or indirectly specified project. If this propensity is an inherent property of bidders, it may show up also in other contexts, possibly very different from that of project construction. This means that observation of bidder behavior in other contexts may reveal the size of future adaptation cost.

This is where questionnaires may be useful. As mentioned in the introduction, questionnaires play an important role in procurement auctions. There is a considerable literature on the design of questionnaires, (see, e.g., Bradburn et al., 2004; Brace, 2008; Arundel, 2023). Though

mainly developed for surveys or market research, the design principles may be useful also for a procurer seeking preliminary knowledge of the bidder's project and the ability to deliver it as proposed. In our current context of random errors, we shall concentrate on adaptation cost and the way in which questionnaires may be used to prompt the bidders to reveal their tendency to generate this type of cost, even when they are not themselves aware of it.

To explain the method of observing deviations, we consider it first in a very simple and abstract setting, returning afterwards to its possible use in practice. Formally, we define a questionnaire as a collection  $Q^m = (q_1, \dots, q_m)$  of  $m$  questions. Each question  $q_h$ ,  $h = 1, \dots, m$ , consists of an interval  $[\underline{a}_h, \bar{a}_h]$  of possible answers together with a specific element  $a_h^0 \in [\underline{a}_h, \bar{a}_h]$ , to be considered as a "typical" or "desired" answer, where  $\underline{a}_h, \bar{a}_h$  are chosen so as to accommodate all expected answers. A response to the questionnaire is an array  $a = (a_1, \dots, a_m)$  specifying an answer to each question. If the deviations  $a_h - a_h^0$  from the designated typical answer are generated by random errors, then the sample variance  $s_m^2 = (m - 1)^{-1} \sum_{h=1}^m (a_h - a_h^0)^2$  can be used to estimate the variance of the random error process and through this the expected future adaptation cost. We state this as a separate proposition.

**Proposition 11.** *Assume that deviations in the questionnaire are generated by the random error process with parameter  $\sigma$ . For each  $m$ , let  $Q^m$  be a questionnaire with  $m$  queries. Then, the sample variance  $s_m^2$  is a consistent estimator of  $\sigma^2$ , and  $s_m \sqrt{T/2\pi}$  is an estimator of the adaptation cost.*

The proposition states that an ideal questionnaire of the type outlined above can be used to detect the error-proneness of the bidder and thereby to forecast the adaptation cost to which the bidder's project may give rise. It is, however, far from easy to set up a real-life questionnaire with the properties of the ideal questionnaire.

First of all, the questions should pertain to something to which the bidder has an answer and which can be predicted rather accurately, so that deviations from the predicted answer become meaningful. Care should be taken to avoid questions where the predicted or "correct" answer reflects the subjective view of the procurer, something that might penalize unconventional or innovative bidders. Deviations from the predicted answers should reflect lack of concentration or insufficient goal-orientedness rather than differences in points of view or opinion. For this to make sense, the questions of this type should constitute only part of the questionnaire, possibly scattered between questions pertaining to the project and details of construction cost.

Secondly, for the purposes of estimating standard errors, the units of measurement in the deviations registered in different questions should be compatible, something that could be achieved if they all pertain to details of cost calculations. Thirdly, the questions should be selected so as to avoid

embarrassing repetitions while being sufficiently numerous to compute a standard error.

To set up a questionnaire in which a certain number of questions satisfies the above demands would be next to impossible, and we should not expect a numerical calculation of the bidder's adaptation cost parameters. Nevertheless, the idealized questionnaire may be roughly approximated using subjective assessment of the answers given by the bidders, where the propensity to deviate may be revealed by inconsistencies in the project information delivered. The longer and more detailed the questionnaire, the greater the possibility of spotting errors. Questions with answers that can be verified by the procurer will similarly be useful when it comes to detection of deviations. This points to the phenomenon of long and complex questionnaires already mentioned in the introduction, hinting that the specific features of questionnaire design described above may indeed have been adapted in practice.

## 6. Conclusions

Public projects differ in their complexity. Even in cases where the construction and delivery phases of the project are relatively transparent, the adaptation costs occurring after initiation of the project may be difficult to forecast. The adaptation cost of a simple project, such as purchasing office pencils and paper, is often predictable. However, this type of cost is more unpredictable when dealing with complex and unique projects, such as military equipment and large infrastructure, where the cost may not even be known to the bidders themselves.

In the preceding sections, we have considered procurement auctions where the adaptation cost must be paid by the procurer so that, at least formally, they do not figure in the bids. However, if information about adaptation cost is available to some of the involved parties, then this information will influence their behavior and thereby the outcome of the auction. Intuitively, the procurer will have an interest in having the information, but concealing it from the bidders if possible, and the findings confirm this intuition, at least to some extent.

To extract information about future adaptation costs from the bidders, a procurer may use questionnaires that can help them to determine the expected cost of a project. Indirectly, these questionnaires can also be used to influence the bidding strategies of the bidders. In this paper, we have studied a situation where questionnaires can be designed and used to disclose information, not necessary targeted at the construction cost of the project, but rather the subsequent adaptation cost of putting the project into operation.

Our results provide some input to the study of best practices in the use of questionnaires in public procurement tenders, where core principles are

non-discrimination, transparency, relevance, clarity, and simplicity (European Commission, 2015). These principles are not always met in practice, and questionnaires that are too lengthy and resource intensive have been mentioned as a main issue. We have shown that this issue can be an advantage to the buyer who may use it to assess the (uncertain) adaptation cost connected with each of the bidders.

Throughout this paper, we have been concerned principally with adaptation cost, arising from discrepancies between what the procurer considered as the intended or planned functioning of the project, and its actual adaptability to these purposes, in most cases of a nature that could not be formulated in the contract and to which the delivering bidder cannot be held responsible. The influence of such costs in the auction, giving rise to changes in the optimal bidding strategies, might be of interest also in a more general context of ordinary auctions, where the auctioneer has to cover an additional cost that depends on the buyer; a situation that occurs more often than one would usually expect, but which has so far been outside our scope.

We have seen that when the procurer collects additional information about the bidders' adaptation cost parameters, it may not be advantageous to transmit this information to the bidders. Future research may clarify whether the optimal information disclosure is similar to that of ordinary auctions, and given that the information is communicated fully, whether one method of measurement is better than another in terms of the induced bidder behavior. Further, we have had our focus on the pre-bidding and the bidding stages. However, some real-life tenders also add a negotiation stage that follows the bidding stage, where aspects of the bids can be negotiated upon. An alternative approach would be to analyse a process, where the procurer can influence both quality and cost at the same time with the opportunity of having a trade-off between the two parameters. This again would be a matter for future research.

## Appendix. Proofs of propositions

In this appendix, we provide proofs of the propositions that appear in the main text. It is convenient to treat the quality dimension of the bids separately in the following version of what is known as Che's lemma (see Che, 1993).

**Lemma A1.** *Assume that all bids  $(q, b)$  must satisfy  $b \leq b_{\max}$ . Let  $(q^*(\theta, \tau), b^*(\theta, \tau))$  be an optimal bid for a bidder with type  $(\theta, \tau)$ . Then,*

$$q^*(\theta, \tau) = \operatorname{argmax}_{q:c(q, \theta) \leq b_{\max}} \{u(q, \tau) - c(q, \theta)\}.$$

*Proof:* Assume to the contrary that the bid  $(q', b')$  with  $c(q', \theta) \leq b'$  is optimal for a bidder with type  $(\theta, \tau)$ , and that the probability of winning with this bid is positive, but that  $q' \neq q^*(\theta, \tau) = \operatorname{argmax}_{q:c(q, \theta) \leq b_{\max}}$

$\{u(q, \tau) - c(q, \theta)\}$ . Let  $b''$  be such that

$$u(q^*(\theta, \tau), \tau) - b'' = u(q', \tau) - b',$$

Then  $b'' \leq b^*(\theta, \tau) \leq b_{\max}$ , so that  $(q^*(\theta, \tau), b'')$  is at least as advantageous to the procurer as  $(q', b')$ . Consequently, the probability of winning the auction with the bid  $(q^*(\theta, \tau), b'')$  is at least as large as with the bid  $(q', b')$ , and from the definition of  $q^*(\theta, \tau)$  we have that the payoff to the bidder when winning satisfies

$$\begin{aligned} b'' - c(q^*(\theta, \tau), \theta) &= u(q^*(\theta, \tau), \tau) - u(q', \tau) \\ &\quad + b' - c(q^*(\theta, \tau), \theta) > b' - c(q', \theta). \end{aligned}$$

So, the bidder's expected payoff from  $(q^*(\theta, \tau), b'')$  exceeds that from  $(q', b')$  if the probability of winning with  $(q', b')$  is positive, giving a contradiction.  $\square$

*Proof of Proposition 1:* The expression for  $q^\circ(\theta)$  follows from Lemma A1 with  $b_{\max}$  chosen sufficiently large. Because  $\tau$  has the constant value  $\tau^*$ , the quality bid depends only on  $\theta$ .

To find  $\beta^\circ(\theta)$ , let the random variable  $v$  be given by

$$v = \phi(\theta) = \max_q \{u(q, \tau^*) - c(q, \theta)\} = u(q^\circ(\theta), \tau^*) - c(q^\circ(\theta), \tau^*),$$

taking values in an interval  $[\underline{v}, \bar{v}]$ . By the envelope theorem,  $\phi$  is differentiable with  $\phi'(\theta) = -c'_\theta(q^\circ(\theta), \theta) < 0$ ; in particular,  $\phi$  is invertible. The probability distribution function of  $v$  can then be written as

$$H(v) = \mathbb{P}\{\phi(\theta) < v\} = \mathbb{P}\{\theta > \phi^{-1}(v)\} = 1 - F(\phi^{-1}(v)). \tag{A1}$$

Considering the procurement auction as an ordinary auction where bidders propose a part of the surplus of the size  $v - b$  to the procurer in order to obtain the contract, leaving the remaining part to themselves, we can use results in first-price auctions (see, e.g., Riley and Samuelson, 1981, Proposition 2) to find the optimal bidding function  $\beta$  as

$$\beta(v) = v - \int_{\underline{v}}^v \left[ \frac{H(t)}{H(v)} \right]^{n-1} dt. \tag{A2}$$

The payment to the bidder can then be found as the sum of construction cost and the remaining part of the surplus, so that

$$\begin{aligned} \beta^*(\theta) &= c(q^*, \theta) + [v - \beta(v)] = c(q^*, \theta) + \int_{\underline{v}}^v \left[ \frac{H(t)}{H(v)} \right]^{n-1} dt \\ &= c(q^*, \theta) + \int_{\theta}^{\bar{\theta}} c'_\theta(q^*(s), s) \left[ \frac{1 - F(s)}{1 - F(\theta)} \right]^{n-1} ds, \end{aligned} \tag{A3}$$

where we have used (A1) and a change of integration variable to obtain the expression in (5).

The remaining statements of the proposition are straightforward. □

*Proof of Proposition 2:* The quality part of the strategy  $q^{\circ\circ}$  is given by Lemma A1. To find the payment part  $\beta^{\circ\circ}$ , we use the same reasoning as in the proof of Proposition 1 but with  $v = \phi(\theta) = u(q^{\circ\circ}(\theta), \tau^*) - c(q^{\circ\circ}(\theta), \theta)$ . In the final transformation from (A2) to (A3) we use that the lower bound  $\underline{v}$  for  $v$  is attained for  $\hat{\theta}$  such that  $c(q^{\circ\circ}(\hat{\theta}), \hat{\theta}) = b_{\max}$ . □

*Proof of Proposition 3:* As before, the expression in (8) follows from Lemma A1. Using the same reasoning as in the proof of Proposition 1, but with  $v$  as given in (7), we can identify bids with a proposal for leaving the part  $v - b$  to the procurer, so that the bids take the form

$$\beta(v) = v - \int_0^v \left[ \frac{H(t)}{H(v)} \right]^{n-1} dt.$$

Inserting  $v$ , we obtain the expression in equation (9). □

*Proof of Proposition 4:* The bidding strategies are found as in the proof of Proposition 3 with  $v$  replaced by  $\tilde{v} = \phi(\theta) - \tau$ . To find the distribution of  $\tilde{v}$ , we use that as a sum of two independent random variables it can be obtained as the convolution of the distribution of  $\phi(\theta)$ , having density  $-c'_\theta(q^*(\theta), \theta)f(\theta)$ , with the distribution of  $-\tau$ . This gives us the density

$$h(\tilde{v}) = \int_0^{\bar{\theta}} c'_\theta(q^*(\theta), \theta)f(\theta)g(\phi(\theta) - \tilde{v})d\theta.$$

The optimal bid of the bidder with parameters  $(\theta, \tau)$  can now be found as

$$\beta^*(\theta, \tau) = c(q^*(\theta), \theta) + \tau + (\tilde{v} - \beta(\tilde{v})) = c(q^*(\theta), \theta) + \tau + \int_0^{\tilde{v}} \left[ \frac{H(t)}{H(\tilde{v})} \right]^{n-1} dt,$$

with  $H(t) = \int_0^t h(s)ds$ . □

*Proof of Proposition 5:* As previously, (10) follows from Lemma A1. For each bidder  $i$  and each value of  $\theta_i$ , the maximal surplus achieved by bidder  $i$  can be written as  $v_i = \phi(\theta_i) - \tau_i$ , where  $\phi(\theta_i) = \max_q [u(q, \tau_i) - c(q, \theta_i)]$ . The probability that another bidder  $j$  has maximal surplus  $v_j$  below  $v_i$  is  $H[\tau_j](v_i)$ , so that the probability that  $i$  has the highest surplus is  $\prod_{j \neq i} H[\tau_j](v_i)$ . Consequently, in the auction where buyers bid for the contract paying the procurer a share of the surplus, the optimal bid with asymmetric bidders is

$$b(\theta_i, \tau_i) = v_i - \int_0^{v_i} \prod_{j \neq i} \frac{H[\tau_j](t)}{H[\tau_j](v_i)} dt,$$

(see, e.g., Kirkegaard, 2009). Transforming back to the procurement auction and inserting, we obtain the expression in (11).

For the second part of the proposition, we use

$$\mathbb{P}\{v_j \leq t\} = \mathbb{P}\{\phi(\theta_j) \leq v_j + \tau_j\} = \mathbb{P}\{\theta_j \geq \phi^{-1}(t + \tau_j)\} = 1 - F(\phi^{-1}(v + \tau_j)), \tag{A4}$$

so that  $H[\tau_j](v_i) = 1 - F(\phi^{-1}(\phi(\theta_i) - \tau_i + \tau_j))$ , and inserting we obtain equation (12).  $\square$

For the proof of Proposition 6 we shall need a lemma.

**Lemma A2.** *Under the assumption of separability, the family  $(H[\tau])_{\tau \in [0, \bar{\tau}]}$  of distributions has the property  $H[\lambda\tau + (1 - \lambda)\tau'] \succ_{SD} \lambda H[\tau] + (1 - \lambda)H[\tau']$  for all  $\tau, \tau'$  and  $\lambda \in [0, 1]$ .*

*Proof:* First of all, we notice that under separability the distributions functions  $H[\tau]$  and  $H[\tau']$  differ only by a horizontal translation of size  $\tau' - \tau$ . Assume that  $\tau' > \tau$ , and suppose first that both are one-point distributions, so that  $H[\tau] = \mathbb{1}_{[\tau, \bar{v}]}$ ,  $H[\tau'] = \mathbb{1}_{[\tau', \bar{v}]}$  (where  $\mathbb{1}_A$  is the indicator function for the set  $A$ ). If  $\tau_\lambda = \lambda\tau + (1 - \lambda)\tau'$ , then

$$\int_0^v H[\tau_\lambda](t) dt = \begin{cases} 0 & v < \tau_\lambda \\ v - \tau_\lambda & v \geq \tau_\lambda \end{cases}$$

$$\int_0^v [\lambda H[\tau] + (1 - \lambda)\tau'](t) dt = \begin{cases} 0 & v < \tau \\ \lambda(v - \tau) & \tau \leq v < \tau' \\ \lambda(\tau' - \tau) + (v - \tau') & v \geq \tau' \end{cases}$$

and an easy check gives that  $\int_0^v H[\tau_\lambda](t) dt \leq \int_0^v [\lambda H[\tau] + (1 - \lambda)\tau'](t) dt$  for all  $v$ .

Next, suppose that  $H[\tau]$  is derived from a discrete distribution assigning probabilities  $p_1, \dots, p_n$  to points  $v_1, \dots, v_n \in [0, \bar{v}]$  with  $v_1 < v_2 < \dots < v_n$ . Then  $H[\tau] = \sum_{i=1}^n p_i \mathbb{1}_{[v_i, \bar{v}]}$ ,  $H[\tau'] = \sum_{i=1}^n p_i \mathbb{1}_{[v'_i, \bar{v}]}$ , where  $v'_i = v_i + (\tau' - \tau)$ ,  $i = 1, \dots, n$ . Using the fact that the property holds for all one-point distributions, and exploiting linearity of the integral, we find that it holds for all discrete distributions.

For general distributions  $H[\tau]$ , we use the fact that it can be obtained as a limit of a sequence of discrete distributions for which the property holds, so that it holds also in the limit.  $\square$

*Proof of Proposition 6:*

(a) The random variable  $v(\theta) - \tau$  can be written as

$$v(\theta) - \tau = v(\theta) - \tau^* + \epsilon,$$

where  $\epsilon$  is an independent random variable with values in  $[0, \bar{\tau}]$  and mean value 0. Thus,  $H$  is a mean-preserving spread of  $H[\tau^*]$ , and it follows that  $H[\tau^*] \succ_{SD} H$ .

- (b) If  $\tau_j < \tau^*$ , then  $H[\tau_j](\theta) \leq H[\tau^*](\nu)$ , all  $\nu$ , so that  $\int_{\underline{\nu}}^{\nu} H[\tau_j](t) dt \leq \int_{\underline{\nu}}^{\nu} H[\tau^*](t) dt$  for all  $\nu$ , and  $H[\tau_j] \succ_{SD} H[\tau^*]$ . Using transitivity of  $\succ_{SD}$  we find that  $H[\tau_j] \succ_{SD} H$ , and we conclude that  $\int_{\underline{\nu}}^{\nu} H[\tau_j](t) dt \leq \int_{\underline{\nu}}^{\nu} H(t) dt$ .
- (c) Let  $(\tau^1, \dots, \tau^S)$  be an array of values of the parameter  $\tau$  with associated probabilities  $(p^1, \dots, p^S)$  so that  $\tau^* = \sum_{s=1}^S p^s \tau^s$ . By Lemma A2

$$H[\tau^*] = H \left[ \sum_{s=1}^S p^s \tau^s \right] \succ_{SD} \sum_{s=1}^S p_j^s H[\tau^s],$$

and it follows that

$$\sum_{s=1}^S p^s \int_{\underline{\nu}}^{\nu} H[\tau_j^s](t) dt = \int_{\underline{\tau}}^{\nu} \sum_{s=1}^S p_s H[\tau_j^s](t) dt \geq \int_{\underline{\tau}}^{\nu} H(t) dt$$

for all  $\nu$ . Choose a sequence of discrete probability distributions on  $[0, \bar{\tau}]$  given by arrays  $(\tau_n^1, \dots, \tau_n^{S_n}, p_n^1, \dots, p_n^{S_n})$  as above and converging to the distribution of  $\tau$  given by  $G$ . Then

$$\int_{\underline{\nu}}^{\nu} \left[ \int_0^{\bar{\tau}} H[\tau](t) g(\tau) d\tau \right] dt \geq \int_{\underline{\nu}}^{\nu} H(t) dt$$

for all  $\nu$ , proving the last part of the proposition. □

*Proof of Proposition 7:* Optimality of bidding strategies follows as in Proposition 1. The inequality  $W_1(\theta, \sigma) \geq W_0(\theta)$  is a consequence of the Blackwell–Sherman–Stein theorem – see Blackwell (1951, Theorem 4) or Le Cam (1996, Theorem 1) – using the fact that  $u(q, \theta)$  is concave in  $\theta$ . □

For the proof of Proposition 10 we use the following lemma, which is a reformulation of a result in the theory of stochastic processes known as Donsker’s theorem (see, e.g., Whitt, 2002; Kallenberg, 2021).

**Lemma A3.** *Let  $e_1, e_2, \dots$  be random variables, which are independent and identically distributed with mean 0 and standard deviation 1. Let*

$S_m = e_1 + \dots + e_m$  be the sum of the first  $m$  instances. Then, the stochastic process defined by

$$S_m(t) = \frac{1}{\sqrt{m}} S_{\lfloor mt \rfloor}, t \in [0, 1]$$

converges weakly for  $m \rightarrow \infty$  to a standard Brownian motion on  $[0, 1]$ .

Here  $\lfloor mt \rfloor$  is the largest integer, which is  $\leq mt$ , so that  $S_m$  can be interpreted as the sum of errors at time  $t$  if they occur at the rate  $m$  per unit of time.

*Proof of Proposition 10:* We apply Lemma A3 to the error process running over the time interval from 0 to  $T$ , which is seen to be a Brownian motion with parameter  $\sigma$ , so that the cumulated error at  $T$  has a normal distribution with mean 0 and variance  $\sigma^2 T$ .

To find the expected cumulated error, given that only positive values count, we must find the expectation over the positive values of the cumulated error, which is

$$\int_0^\infty \frac{x}{\sqrt{2\pi}\sigma\sqrt{T}} e^{-(x^2/2\sigma^2 T)} dx = \frac{\sqrt{T}}{\sqrt{2\pi}} \sigma \int_0^\infty e^{-(x^2/2\sigma^2 T)} d\left(\frac{x^2}{2\sigma^2}\right) = \sigma \sqrt{\frac{T}{2\pi}},$$

which is the value of  $\tau$  stated in the proposition.  $\square$

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