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# Analysis and management of a portfolio risk and performance using Value-at-Risk

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Master in Finance

Supervisor: PhD, António Manuel Rodrigues Guerra Barbosa, Assistant Professor, ISCTE-IUL

Lisbon, July 2025

BUSINESS SCHOOL

Department of Finance

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# Resumo

A crescente globalização dos mercados financeiros tem acentuado a necessidade de estruturas robustas de gestão de risco nas instituições financeiras. Esta evolução levou à implementação de regulamentações mais exigentes e à adoção generalizada de metodologias quantitativas avançadas, destacando-se o Value-at-Risk (VaR) como métrica padrão na avaliação e controlo do risco de mercado.

O presente estudo avalia a eficácia do VaR na análise e mitigação do risco num portefólio diversificado composto por ações e obrigações provenientes de três mercados distintos. A análise inicia-se com a avaliação comparativa de diferentes modelos de VaR, através de backtesting, com o objetivo de identificar o modelo mais adequado à estrutura e características do portefólio. O modelo selecionado é posteriormente utilizado para quantificar e comparar duas abordagens distintas de gestão de risco: (1) o VaR diário de um portefólio não sujeito a intervenção, e (2) o VaR diário de um portefólio gerido com base numa estratégia dinâmica de cobertura ao longo de um horizonte temporal de um ano.

A análise de desempenho assenta na métrica Return on Risk-Adjusted Capital (RORAC), permitindo aferir a eficiência relativa de ambas as estratégias em termos ajustados ao risco. Os resultados demonstram que a aplicação de uma estratégia de cobertura com base em limites de VaR permite alcançar um desempenho superior face à ausência de gestão ativa do risco. Estes resultados realçam a importância de uma abordagem proativa na contenção do risco para potenciar o desempenho financeiro.

Além de reforçar a aplicabilidade prática do VaR na gestão de portefólios com multi-ativos, este estudo oferece contributos relevantes para instituições financeiras que pretendem maximizar o retorno ajustado ao risco num contexto de mercados globais cada vez mais interligados e voláteis.



***Palavras-Chave***: Gestão de Risco, Value-at-Risk, Portfolio, Backtesting, Hedging

***Classificação JEL***: G11, G32

# Abstract

The increasing global integration of financial markets has heightened the need for robust risk management frameworks within financial institutions. This evolution has driven the adoption of more stringent regulatory standards and the widespread use of advanced quantitative techniques, with Value-at-Risk (VaR) emerging as the standard metric for market risk assessment and control.

The analysis assesses the effectiveness of VaR in risk analysis and mitigation within a diversified portfolio comprising equities and fixed-income securities from three distinct markets. The study begins with a comparative assessment of multiple VaR models through backtesting, aiming to identify the specification that best captures the portfolio's risk profile. The selected model is subsequently applied to estimate and compare two distinct strategies: (1) the daily VaR of an unmanaged portfolio, and (2) the daily VaR of a portfolio managed through a dynamic hedging strategy over a one-year horizon.

Portfolio performance is assessed using the Return on Risk-Adjusted Capital (RORAC) as the core evaluation metric, enabling a risk-adjusted comparison between approaches. The results show that implementing a VaR-based risk management strategy that limits daily risk exposure leads to consistently superior performance relative to a passive approach. These findings underscore the importance of proactive risk control in enhancing financial outcomes.

Beyond reaffirming the practical relevance of VaR in multi-asset portfolio management, this study provides valuable insights for financial institutions seeking to optimise risk-adjusted returns in increasingly interconnected and volatile markets.



***Keywords***: Risk Management, Value-at-Risk, Portfolio, Backtesting, Hedging
***JEL Classification***: G11, G32

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# List of Abbreviations

| | |
|---|---|
| AEX | Amsterdam Exchange Index |
| BCP | Berkowitz, Christoffersen and Pelletier Test |
| CAC 40 | Cotation Assistée en Continu |
| DJI | Dow Jones Industrial Average |
| DAX | Deutscher Aktienindex |
| EC | Economic Capital |
| EUR | Euro |
| EWMA | Exponentially Weighted Moving Average |
| FCHI | CAC 40 Index |
| FTSE 100 | Financial Times Stock Exchange 100 Index |
| GBP | British Pound Sterling |
| GDAXI | DAX Index |
| GSPC | S&P 500 Index |
| HKD | Hong Kong Dollar |
| HSI | Hang Seng Index |
| IR | Interest Rate |
| IXIC | NASDAQ Composite Index |
| P&L | Profit and Loss |
| PV | Present Value |
| PV01 | Present Value of a Basis Point |
| QR | Quantile Regression |
| RORAC | Return on Risk-Adjusted Capital |
| S&P 500 | Standard and Poor's 500 Index |
| SGSt | Skewed Generalised Student-t Distribution |
| UC | Unconditional Coverage Test |
| USD | United States Dollar |
| VaR | Value-at-Risk |

# Chapter 1.

# Introduction

The increasing interconnectedness of global financial markets has intensified the challenge faced by institutions in maintaining a delicate balance between generating adequate returns and managing financial risk. Among the primary categories of financial risk, including credit, operational, liquidity and market risk, market risk stands out due to its pronounced volatility and acute sensitivity to macroeconomic developments, geopolitical events and shifts in monetary policy. These factors can lead to significant fluctuations in asset valuations and pose substantial threats to financial stability.

At the core of modern risk management lies the Value-at-Risk (VaR) framework, a widely adopted statistical methodology that quantifies the maximum expected loss of a financial portfolio over a specified time horizon and confidence level. Popularised by J. P. Morgan's RiskMetrics model in the 1990s, VaR has since become a cornerstone of risk assessment, enabling institutions to define Economic Capital (EC) thresholds and optimise capital allocation under uncertainty (Jorion, 2007). By integrating VaR with EC requirements, financial institutions can strengthen risk oversight and ensure that capital reserves are both sufficient and efficiently deployed.

This dissertation investigates the practical application of VaR in managing the risk of a diversified portfolio comprising equities and bonds across multiple developed markets, namely the United States, Europe and Asia. The study evaluates the performance of several VaR methodologies and proposes a dynamic hedging strategy aimed at reducing risk exposure while preserving portfolio performance. Through a systematic empirical assessment of both parametric and non-parametric VaR models, this research seeks to identify the most robust and effective risk management approach under conditions of heightened market volatility.

In the past decade, evolving market dynamics have exposed the limitations of conventional risk management models. The post-2008 low-interest rate environment, followed by sharp inflationary pressures and aggressive monetary tightening between 2021 and 2023, underscored the vulnerability of rigid risk modelling frameworks. For instance, the European Central Bank maintained a negative interest rate policy from 2014 to 2022, significantly impacting bond markets. In parallel, the United States Federal Reserve implemented abrupt rate hikes in 2022, prompting a marked contraction in equity valuations. The S&P 500 index alone declined by approximately 25 per cent over the first three quarters of that year. These episodes highlight the urgent need for flexible, forward-looking risk assessment models capable of capturing asymmetric shocks and systemic disruptions. This dissertation contributes to that objective by conducting a detailed comparative analysis of alternative VaR specifications.

The portfolio under analysis comprises fixed-income instruments from both the United States and the Eurozone, as well as equity positions associated with major global indices. These include the DAX 40 (GDAXI) for Germany, the CAC 40 (FCHI) for France, the AEX for the Netherlands, the S&P 500 (GSPC) and Dow Jones Industrial Average (DJI) for the United States, the Nasdaq Composite (IXIC) for US technology exposure, and the Hang Seng Index (HSI) for Hong Kong. This selection ensures comprehensive regional and sectoral representation. By decomposing total portfolio risk into systematic components, reflecting broad market movements, and idiosyncratic components, capturing asset-specific variations, the study isolates the primary sources of volatility and designs targeted hedging strategies to mitigate exposure effectively. Although extensive literature, including Lee and Su (2011) and Hull and White (1998), has compared the merits of various VaR models, no single approach has proven universally superior. This highlights the need for portfolio-specific validation.

To navigate this model selection process, the dissertation assesses four distinct methodologies for estimating VaR. The Normal VaR model, which assumes return symmetry and Gaussian distribution, offers simplicity but tends to underestimate extreme losses. The Skewed Generalised Student-t (SGSt) model, proposed by Theodossiou (1998), introduces skewness and heavy tails, enhancing accuracy in capturing tail events. The Historical Simulation model, refined by Hull and White (1998), adjusts past returns to reflect prevailing market volatility. Finally, the Quantile Regression (QR) approach, introduced by Koenker and Bassett (1978), allows for distribution-free estimation of conditional quantiles, making it well suited to asymmetric return profiles. Volatility estimates are computed using the Exponentially Weighted Moving Average (EWMA) technique to ensure sensitivity to recent market conditions.

To evaluate the predictive performance of these models, the study employs a rigorous backtesting framework based on two statistical tests. The Unconditional Coverage (UC) test developed by Kupiec (1995) compares the number of observed exceedances against theoretical expectations. The Berkowitz, Christoffersen and Pelletier (BCP) test (2011) assesses exceedance clustering and serial independence. After identifying the most accurate model through these tests, its effectiveness is examined in two practical settings over a one-year evaluation period. In the first setting, the portfolio is monitored without risk mitigation, with

VaR computed daily. In the second setting, a dynamic hedging strategy is implemented that adjusts equity exposure whenever the daily Economic Capital exceeds the predefined threshold of €133 000. This limit reflects historically observed risk levels and aligns with institutional risk tolerance.

Finally, the study employs the Return on Risk-Adjusted Capital (RORAC) metric to compare the performance of the two strategies, balancing profitability with risk exposure. Preliminary findings indicate that the actively hedged portfolio not only reduces the frequency and severity of VaR breaches but also delivers superior risk-adjusted returns. These results support the use of VaR-based thresholds in conjunction with responsive risk mitigation techniques.

The structure of this dissertation is organised into eight chapters. Chapter 1 introduces the research problem, objectives, and relevance of the study. Chapter 2 presents a literature review covering key theoretical concepts related to risk management and Value-at-Risk (VaR) methodologies. Chapter 3 describes the composition of the portfolio under analysis, including asset selection across equities and fixed-income instruments. Chapter 4 outlines the methodological framework, detailing the process of risk factor mapping, volatility estimation, and portfolio-level modelling. Chapter 5 discusses the implementation of various VaR models, including Normal, SGSt, Historical Simulation and Quantile Regression approaches, culminating in a summary of all configurations tested. Chapter 6 presents the backtesting methodology, including the application of Unconditional Coverage (UC) and Berkowitz, Christoffersen and Pelletier Test (BCP) tests, and summarises the empirical validation results. Chapter 7 focuses on risk management implementation, detailing the VaR decomposition process, the dynamic hedging strategy based on Marginal VaR, and the resulting impact on portfolio risk and performance. Finally, Chapter 8 offers the main conclusions, reflecting on the empirical findings and their practical implications for portfolio risk control.

Additional technical content is provided in the appendices, including descriptive statistics (Appendix A), portfolio data (Appendix B), model configuration details (Appendix C), and comprehensive backtesting results (Appendix D).

# Chapter 2.

# Literature Review

Risk management has long been a cornerstone of financial theory and practice, given its significant role in promoting financial stability and mitigating unexpected losses. As financial systems become increasingly complex and globally interconnected, the ability to accurately measure and manage risk remains a critical concern for both regulators and practitioners. Among the various quantitative tools developed for this purpose, Value-at-Risk (VaR) has emerged as one of the most widely implemented risk metrics. It provides a forward-looking estimate of the potential loss in portfolio value over a given time horizon, for a specified confidence level.

The development of VaR methodologies has evolved in response to successive financial crises, including the 1987 market crash, the 2008 global financial crisis, and the COVID-19 pandemic. These episodes exposed the weaknesses of static and overly simplistic models, prompting regulators to strengthen prudential frameworks. Notably, the Basel Committee on Banking Supervision progressively introduced VaR related capital requirements under Basel II and Basel III, and more recently Basel IV, which emphasises stress testing and scenario analysis as complementary tools to address systemic risk (Bank for International Settlements, 2021).

VaR models are typically classified into three main categories: parametric, non-parametric and semi-parametric. Parametric approaches, such as the conventional Normal VaR, are based on strong distributional assumptions, most commonly the Gaussian distribution. While these assumptions simplify the computational process, they often lead to an underestimation of tail risk (Jorion, 2007). In contrast, non-parametric methods, particularly Historical Simulation, do not impose any distributional assumptions. Instead, they derive empirical quantiles directly from historical return data. However, their accuracy is highly dependent on the sample size and the representativeness of past observations, which may limit their predictive capacity (Hull and White, 1998; Pritsker, 2006).

Semi-parametric models attempt to address these limitations by incorporating distributional flexibility. The Skewed Generalised Student-t (SGSt) distribution, introduced by Theodossiou (1998), accounts for asymmetry and fat tails, which are common features in financial return distributions. Empirical studies by Lee and Su (2011) and others demonstrate that SGSt-based VaR models provide more accurate tail estimates, especially in volatile market regimes.

Similarly, Quantile Regression (QR), developed by Koenker and Bassett (1978), enables direct estimation of conditional quantiles without imposing a strict functional form. QR models have shown superior robustness in capturing asymmetric risk dynamics across a range of asset classes (Xiao et al., 2015).

Another important dimension concerns the volatility forecasting techniques embedded within VaR models. The Exponentially Weighted Moving Average (EWMA) method, popularised by J. P. Morgan's RiskMetrics model (1996), assigns greater weight to recent observations, enhancing responsiveness to changing market conditions. Although widely used for its simplicity, RiskMetrics relies on the assumption of normally distributed returns, which often leads to the underestimation of losses during stress periods.

More recently, researchers have explored machine learning-based approaches to risk estimation, including neural networks and reinforcement learning algorithms. These models can capture non-linearities and dynamic patterns in large datasets, often outperforming traditional VaR specifications in detecting tail risk and regime shifts (Fischer & Krauss, 2018). However, their implementation requires substantial data and computational resources, and their interpretability remains an ongoing challenge.

While model specification is critical, so too is model validation. Backtesting is the principal technique for assessing the reliability of VaR forecasts, comparing realised losses against model-implied thresholds. Kupiec's (1995) Unconditional Coverage (UC) test evaluates whether the observed number of exceedances aligns with the expected frequency, while the Berkowitz, Christoffersen and Pelletier (2011) test (BCP) examines whether exceedances occur independently over time or exhibit clustering. These tests are widely used to assess whether VaR models are both statistically and economically consistent with the observed data.

Several empirical studies underscore that VaR model performance varies across asset classes and market regimes. Barone-Adesi et al. (1998) and Boudoukh et al. (1998) highlight the importance of volatility adjustment and weighted observations in enhancing predictive accuracy. Furthermore, combining VaR-based limits with scenario analysis and stress testing has been shown to improve institutional resilience to rare but extreme events.

In summary, although no VaR methodology is universally optimal, advances in semi-parametric modelling, volatility forecasting, and statistical validation have considerably improved risk measurement frameworks. This dissertation builds on these developments by evaluating multiple VaR models within a diversified portfolio and extending their application beyond risk measurement to active risk control. Specifically, the study integrates model validation with a Marginal VaR-based hedging strategy, offering a practical contribution to the literature on VaR as both a regulatory tool and a decision-making instrument in dynamic portfolio management.

# Chapter 3.
# Portfolio Composition

This study constructs a strategically diversified portfolio that integrates both equity and fixed-income instruments to balance risk and return effectively. The selection of equity indices in this study aims to ensure broad geographical and sectoral diversification across major developed markets. The portfolio incorporates the CAC 40 (FCHI) for France, the S&P 500 (GSPC) for the United States, the Nasdaq Composite (IXIC) to capture US technology exposure, the DAX 40 (GDAXI) for Germany, the Dow Jones Industrial Average (DJI) as a representative of US industrials, and the Hang Seng Index (HSI) to provide targeted exposure to the Asian market through Hong Kong. These indices were chosen for their liquidity, market representativeness and relevance as hedging instruments, allowing for an effective decomposition of systematic risk across regional and sectoral dimensions. Their aggregate performance over the evaluation period is summarised in Appendix A.

Equity selection is based on a top-down approach, targeting approximately 50 individual stocks distributed across key economic sectors. The portfolio includes high-growth technology companies such as Microsoft, Apple and NVIDIA, providing exposure to innovation-driven segments like artificial intelligence, semiconductors and cloud computing. In addition, large-cap defensive equities in the healthcare and utilities sectors are incorporated, including Johnson & Johnson, Pfizer, NextEra Energy and Duke Energy, to mitigate cyclical volatility.

To optimise risk-adjusted returns, the equity portfolio also includes positions in cyclical sectors such as industrials (e.g., Honeywell, Caterpillar) and energy (e.g., Chevron, TotalEnergies), which tend to outperform during economic expansions. Furthermore, tactical short positions are taken in selected European consumer staples and U.S. financials, serving as a hedge against downturns in these specific market segments and enhancing the responsiveness of the portfolio to adverse sector-specific developments.

The fixed-income component consists of high-quality government and corporate bonds from the United States and Eurozone. The bond allocation prioritises capital preservation and income stability, with a focus on investment-grade instruments. These include AAA-rated sovereign bonds such as German Bunds and U.S. Treasuries, along with selectively chosen European corporate bonds offering enhanced yield.

Bond maturities are staggered between 2028 and 2034 to optimise the portfolio's duration profile. This structure supports a balance between short-term liquidity and long-term interest rate stability while minimising reinvestment risk. The bond portfolio excludes emerging market debt, in line with the study's focus on developed markets, although its inclusion is acknowledged as a potential avenue for further diversification and inflation hedging.

The portfolio is primarily denominated in euros (64%) and U.S. dollars (36%), with marginal exposure to Hong Kong dollars through selected equity holdings. As a result, foreign exchange risk is a relevant consideration to ensure valuation consistency, a uniform currency conversion methodology is applied across all assets. The study also assesses the impact of currency fluctuations on portfolio volatility and risk-adjusted returns and evaluates potential hedging mechanisms to mitigate foreign exchange exposure.

This asset allocation enables a robust empirical assessment of Value-at-Risk (VaR) models across a representative sample of asset classes, sectors and currencies. The inclusion of both long and short positions, combined with geographic and sectoral diversification, creates a realistic and dynamic testing environment. This structure enhances the relevance of the study's findings, particularly in contexts where cross-asset correlations and risk exposures exhibit instability under stressed market conditions.

Tables 3.1 to 3.3 summarise the full composition and structural details of the portfolio examined throughout this dissertation.

| Stock | Ticker | Currency | Market | Quantity | Share Price | Value (EUR) | Allocation (%) |
|---|---|---|---|---|---|---|---|
| Pernod Ricard SA | RI.PA | EUR | FCHI | -1 059 | 179.46 | -190 052.80 | -2.00 |
| Carrefour SA | CA.PA | EUR | FCHI | -12 685 | 16.94 | -215 004.54 | -2.26 |
| LVMH Moet Hennesy - Louis Vuitton | MC.PA | EUR | FCHI | 406 | 788.53 | 320 143.95 | 3.37 |
| Danone S.A. | BN.PA | EUR | FCHI | -3 759 | 47.88 | -180 008.25 | -1.89 |
| 3M Company | MMM | USD | GSPC | -1 840 | 108.67 | -183 553.15 | -1.93 |
| American International Group, Inc. | AIG | USD | GSPC | -3 075 | 61.79 | -174 424.11 | -1.84 |
| Amazon.com, Inc. | AMZN | USD | GSPC | 3 228 | 102.24 | 302 935.19 | 3.19 |
| Advanced Micro Devices Inc. | AMD | USD | IXIC | -2 785 | 75.40 | -192 748.90 | -2.03 |
| Microsoft Corporation | MSFT | USD | IXIC | 1 387 | 245.08 | 312 021.63 | 3.28 |
| Apple Inc. | AAPL | USD | IXIC | 2 453 | 144.73 | 325 892.42 | 3.43 |
| NVIDIA Corporation | NVDA | USD | IXIC | 1 695 | 203.55 | 316 691.52 | 3.33 |
| ASML Holding NV | ASML | EUR | IXIC | 494 | 658.31 | 325 205.68 | 3.42 |
| Oracle Corporation | ORCL | USD | GSPC | -2 518 | 87.36 | -201 925.35 | -2.13 |
| Salesforce.com Inc. | CRM | USD | GSPC | -1 217 | 164.30 | -183 542.95 | -1.93 |
| Adobe Inc. | ADBE | USD | IXIC | 836 | 370.71 | 284 469.64 | 2.99 |
| SAP SE | SAP | EUR | GDAXI | -1 955 | 109.97 | -215 008.44 | -2.26 |
| Texas Instruments Incorporated | TXN | USD | IXIC | 1 434 | 167.37 | 220 306.19 | 2.32 |
| Johnson & Johnson | JNJ | USD | DJI | 2 177 | 160.74 | 321 212.93 | 3.38 |
| Pfizer Inc. | PFE | USD | GSPC | 5 362 | 41.02 | 201 933.10 | 2.13 |
| Eli Lilly and Company | LLY | USD | GSPC | 903 | 337.59 | 279 816.49 | 2.95 |
| AstraZeneca PLC | AZN | USD | GSPC | -3 165 | 63.20 | -183 605.77 | -1.93 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Novartis AG | NVS | USD | GSPC | 2 845 | 82.60 | 215 721.57 | 2.27 |
| Amgen Inc. | AMGN | USD | IXIC | 932 | 241.53 | 206 629.83 | 2.18 |
| JPMorgan Chase & Co. | JPM | USD | DJI | 2 175 | 135.63 | 270 781.08 | 2.85 |
| Bank of America Corporation | BAC | USD | GSPC | -5 565 | 34.13 | -174 386.22 | -1.84 |
| Wells Fargo & Company | WFC | USD | GSPC | -5 209 | 44.15 | -211 118.34 | -2.22 |
| Goldman Sachs Group Inc. | GS | USD | GSPC | 824 | 340.42 | 257 477.95 | 2.71 |
| Morgan Stanley | MS | USD | GSPC | -2 241 | 91.50 | -188 211.08 | -1.98 |
| American Express Company | AXP | USD | GSPC | 1 477 | 169.31 | 229 545.45 | 2.42 |
| Honeywell International Inc. | HON | USD | GSPC | 844 | 201.45 | 156 067.83 | 1.64 |
| Caterpillar Inc. | CAT | USD | DJI | -697 | 258.32 | -165 271.20 | -1.74 |
| General Electric Company | GE | USD | GSPC | -3 709 | 66.05 | -224 889.66 | -2.37 |
| Union Pacific Corporation | UNP | USD | GSPC | 1 604 | 196.39 | 289 158.73 | 3.04 |
| FedEx Corporation | FDX | USD | GSPC | 1 077 | 185.69 | 183 573.92 | 1.93 |
| NextEra Energy Inc. | NEE | USD | GSPC | 3 707 | 72.83 | 247 842.08 | 2.61 |
| Duke Energy Corporation | DUK | USD | GSPC | -1 680 | 95.24 | -146 879.22 | -1.55 |
| Consolidated Edison Inc. | ED | USD | GSPC | 2 111 | 89.99 | 174 381.50 | 1.84 |
| Xcel Energy Inc. | XEL | USD | IXIC | 2 750 | 65.46 | 165 241.39 | 1.74 |
| PG&E Corporation | PCG | USD | GSPC | -10 286 | 16.04 | -151 453.14 | -1.59 |
| Sempra Energy | SRE | USD | GSPC | 2 383 | 77.62 | 169 776.43 | 1.79 |
| The Boeing Company | BA | USD | DJI | 1 610 | 211.17 | 312 071.04 | 3.28 |
| Lockheed Martin Corporation | LMT | USD | GSPC | 495 | 444.21 | 201 830.75 | 2.12 |
| Raytheon Technologies Corporation | RTX | USD | GSPC | 2 400 | 95.84 | 211 133.53 | 2.22 |
| Northrop Grumman Corporation | NOC | USD | GSPC | 504 | 426.93 | 197 508.62 | 2.08 |
| General Dynamics Corporation | GD | USD | GSPC | 1 108 | 221.17 | 224 938.98 | 2.37 |
| Airbus SE | AIR | EUR | FCHI | 3 984 | 50.20 | 199 996.80 | 2.11 |
| TotalEnergies SE | TTE | EUR | FCHI | 4 376 | 59.41 | 259 981.42 | 2.74 |
| ConocoPhillips | COP | USD | GSPC | -1 870 | 117.63 | -201 910.59 | -2.13 |
| BP plc | BP | USD | GSPC | -6 636 | 33.90 | -206 524.32 | -2.17 |
| Equinor ASA | EQNR | USD | GSPC | 9 541 | 26.21 | 229 505.06 | 2.42 |
| Chevron Corporation | CVX | USD | GSPC | 1 804 | 169.09 | 280 002.23 | 2.95 |
| Crédit Agricole S.A. | ACA.PA | EUR | FCHI | -21 517 | 9.99 | -214 995.43 | -2.26 |
| Industrial and Commercial Bank of China Limited | 1398.HK | HKD | HSI | -59 410 | 3.95 | -27 549.15 | -0.29 |
| CSPC Pharmaceutical Group Limited | 1093.HK | HKD | HSI | -23 241 | 9.03 | -24 618.31 | -0.26 |
| China Resources Land Limited | 1109.HK | HKD | HSI | -5 950 | 37.81 | -26 375.17 | -0.28 |
| **Total Equity** | | | | | | **4 009 738.84** | **42.22** |

**Table 3.1. Stock Characteristics.** This table presents the characteristics of the stocks included in the portfolio, along with the corresponding investment amounts converted into euros. The exchange rate applied on 30 January 2023 is USD/EUR = 0.9179

| Bond | Currency | Maturity | Coupon Rate | Coupon /Year | Face Value (EUR) | Fair Value (EUR) | Allocation (%) |
|---|---|---|---|---|---|---|---|
| DE000BU25018 | EUR | 2028-10-19 | 2.40% | 1 | 500 000.00 | 506 015.91 | 5.33 |
| DE0001135226 | EUR | 2034-07-04 | 4.75% | 1 | 990 000.00 | 1 255 166.35 | 13.21 |
| NL0000102317 | EUR | 2028-07-04 | 5.50% | 1 | 1 100 000.00 | 1 265 291.70 | 13.32 |
| US91282CFV81 | USD | 2032-11-15 | 4.13% | 2 | 761 857.00 | 805 990.04 | 8.48 |
| US91282CJJ18 | USD | 2033-11-15 | 4.50% | 2 | 858 236.50 | 939 912.16 | 9.89 |
| LU2591860569 | EUR | 2033-03-02 | 3.00% | 1 | 660 000.00 | 717 878.78 | 7.55 |
| **Total Bonds** | | | | | | **5 490 254.94** | **57.78** |

**Table 3.2. Bond Characteristics.** This table presents the characteristics of the bonds included in the portfolio, along with the corresponding investment amounts converted into euros. The fair value of each bond is computed as the sum of the present value of its future cash flows, discounted to 30 January 2023 and converted to EUR where applicable. The exchange rate applied on that date is USD/EUR = 0.9179.

| Portfolio Value | | |
|---|---|---|
| | **Value (EUR)** | **Value (%)** |
| Stocks | 4 009 738.84 | 42.22 |
| Bonds | 5 490 254.94 | 57.78 |
| **Total** | 9 499 993.78 | 100 |

**Table 3.3. Portfolio value.** This table showcases the total value of the portfolio on 30 January 2023 as well as the amount allocated to equity and bonds.

# Chapter 4.

# Methodology

This study aims to estimate and control the Value-at-Risk (VaR) of a diversified investment portfolio over a one-year horizon, from 30 January 2023 to 2 February 2024. The primary objective is to ensure that portfolio risk remains within a pre-defined Economic Capital (EC) threshold, through the implementation of a dynamic hedging mechanism designed to mitigate excessive exposure.

The methodological framework begins with the identification and mapping of relevant risk factors, applicable to both the Total VaR and Systematic VaR perspectives. This is followed by the selection of a suitable volatility estimation model, tailored to the time-varying nature of financial markets. Once the volatility structure is defined, the next step involves specifying and calibrating the VaR models that best reflect the underlying risk characteristics of the portfolio.

The selection process encompasses a comparative assessment of several VaR methodologies and parameter configurations. Model evaluation is conducted through rigorous backtesting procedures, which are employed to measure forecast accuracy and statistical reliability.

This chapter outlines the methodological approach adopted for risk factor identification and justifies the choice of the volatility estimation technique. Chapter 5 presents the VaR models and respective specifications under analysis. Chapter 6 proceeds with the backtesting analysis, which evaluates the robustness and predictive performance of the selected models under empirical conditions.

## 4.1 Risk Factor Mapping

Risk factor mapping constitutes the foundational step in accurately measuring and managing portfolio risk. This process requires the precise identification, quantification and classification of the key drivers influencing portfolio value. In portfolio risk management, exposures are typically decomposed into systematic (market-wide) risk and residual (asset-specific) risk components. Systematic risk, driven by common factors such as interest rates or equity indices, cannot be eliminated through diversification. In contrast, residual risk can be substantially mitigated through appropriate asset diversification.

To capture the portfolio's risk profile comprehensively, both Total VaR and Systematic VaR are computed at the risk factor level. Total VaR accounts for all asset-specific risks, delivering a granular risk measure. Conversely, Systematic VaR aggregates individual exposures into broader risk proxies, such as benchmark indices, enhancing scalability but at the expense of specificity. Estimating both measures allows for comparative analysis of diversification efficiency and the degree of systematic risk exposure.

The mapping procedure begins by determining each asset's sensitivity to relevant risk factors, expressed in the portfolio's reference currency, the euro (EUR). This ensures consistency in valuation by converting all foreign-denominated exposures into EUR, using exchange rates as of the valuation date.

Formally, the portfolio $\Theta$ consists of multiple assets exposed to different risk factors. The exposure to each factor is denoted by $\theta_i$ (for i = 1, ..., n), where each $\theta_i$ represents the loading of the portfolio to the i-th risk driver. These exposures are expressed as a risk factor vector:

$$\theta = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_n \end{bmatrix} \tag{1}$$

A distinct mapping approach is applied to each asset class. For equities, Total VaR is computed based on individual stock price movements, while Systematic VaR consolidates these equities into representative market indices. This simplification retains core market risk features while reducing computational complexity.

For fixed-income instruments, risk exposure stems primarily from interest rate sensitivity. Bonds are mapped using the Present Value of a Basis Point (PV01), which measures the change in bond price resulting from a one-basis-point shift in interest rates. To accommodate bonds with irregular maturities, cash flows are projected onto standardised maturity buckets (vertices). Throughout this transformation, the total present value (PV) and PV01 are preserved to maintain accuracy in the mapping process.

In terms of currency risk, foreign currency exposures are aggregated and converted into EUR. The portfolio's net position in each currency is identified, and its sensitivity to exchange rate fluctuations is quantified. This step is essential for capturing the volatility introduced by currency movements and for evaluating their contribution to total portfolio risk.

By employing asset-specific and factor-consistent mapping techniques, the portfolio's risk exposures are represented with precision. This rigorous foundation supports the subsequent steps of volatility estimation and Value-at-Risk (VaR) calculation, thereby enhancing the overall effectiveness of the portfolio's risk management framework.

#### 4.1.1. Stocks

The quantification of equity risk exposures relies on estimating future volatility from historical stock returns. The methodology adopted differs according to whether the Total VaR or the Systematic VaR framework is applied.

Under the Total VaR framework, the risk factor associated with each equity position is defined by the daily price movements of the individual stock. The exposure to this risk factor, $\theta_{i,t}$ is computed by converting the market value of the stock position into euros. This is given by:

$$\theta_{i,t} = M_{i,t} = N_{i,t} \times P_{i,t} \times FX_{i,t} \tag{2}$$

where, $M_{i,t}$ denotes the amount invested in EUR on the stock, $N_{i,t}$ denotes the number of shares held, $P_{i,t}$ is the stock price per share and $FX_{i,t}$ the spot exchange rate between the asset's currency and EUR.

For the Systematic VaR, the risk factor is replaced by a stock market index, and the exposure is adjusted so that the systematic risk of the index position matches the systematic risk of the original stock, as measured by the stock's beta relative to the index.

$$\theta_{i,t} = M_{i,t} \times \beta_{stock,Index,t} \tag{3}$$

The Profit and Loss (P&L) of each equity position under both frameworks depends explicitly on the corresponding risk factor's price movement. Specifically, the P&L for stock $i$ at time $t$ is:

$$P\&L_{i,t} = \theta_{i,t} \times \left(\frac{P_{i,t}}{P_{i,t-1}} - 1\right) \tag{4}$$

For the Systematic VaR, index values replace individual prices in the return calculation, significantly enhancing computational efficiency when managing large portfolios.

This structured and factor-consistent approach to equity risk mapping ensures accurate exposure quantification and facilitates robust VaR estimation under both total and systematic risk frameworks.

#### 4.1.2. Bonds

In contrast to equities, fixed-income instruments are characterized by distinct risk dynamics arising from their predetermined cash flow schedules and fixed maturity dates. As bonds approach maturity, their price volatility tends to decline, a behaviour commonly referred to as the pull-to-par effect. Moreover, while equity valuations are largely influenced by expectations of future earnings, bond prices are more directly sensitive to fluctuations in prevailing interest rates. These structural differences make volatility estimation techniques typically used for equities inadequate when applied to fixed-income assets.

To quantify interest rate risk, this study adopts the Present Value of a Basis Point (PV01), a widely recognized metric that measures the change in a bond's present value resulting from a one basis point (0.01%) shift in the yield curve. PV01 thus captures the marginal valuation impact of small fluctuations in interest rates, offering a precise and interpretable measure of interest rate sensitivity.

The Present Value (PV) of a given bond cash flow $C_T$ at time $T$ is first computed in euros. For foreign-denominated instruments, cash flows are converted to EUR using the prevailing exchange rate $FX_t$ and subsequently discounted using the continuously compounded zero rate $r_t$:

$$PV_{C_T,r_T} = C_T \times e^{-r_T} \times FX_t \tag{5}$$

Using a first-order Taylor approximation, the corresponding PV01 is estimated as:

$$PV01_{C_T,r_T} \approx \frac{\partial PV_{C_T,r_T}}{\partial r_T} \times (-0.01\%) \tag{6}$$

$$= T \times PV_{C_T,r_T} \times 0.01\% \tag{7}$$

here, $T$ denotes the time to maturity of the bond (in years), $C_T$ is the bond cash flow at time $T, FX_t$ represents the exchange rate at time $t$, and $r_t$ is the continuously compounded zero-coupon interest rate used for discounting.

In a multi-cash flow bond portfolio, each cash flow may correspond to a distinct maturity. Since complete interest rate data for all possible maturities is typically unavailable, a mapping approach is applied whereby irregular maturities are aligned with nearby standard maturities, referred to as vertices. This mapping preserves both the total present value and total $PV01$ of the original cash flows, ensuring consistency in interest rate risk representation. This technique follows the $PV$ and $PV01$ invariant mapping methodology, as conceptually developed by Alexander (2008a).

To achieve this, the cash flows are proportionally allocated between two vertex maturities ($T_1$ and $T_2$), representing the nearest standard maturities surrounding the original maturity ($T$). The mapping conditions are expressed as:

$$\begin{cases} x_{T2} = 1 - x_{T1} \\ x_{T1} = \dfrac{T_2 - T}{T_2 - T_1} \end{cases} \tag{8}$$

Note that $x_{T_1}$ and $x_{T_2}$ represent the proportions of the present value of the original cash flow that are mapped to the vertex maturities $T_1$ and $T_2$, respectively. These vertices correspond to the standard maturities with available interest rate data that are closest to the original maturity $T$, with $T_1$ being the maturity directly below $T$ and $T_2$ being the maturity directly above $T$.

This process generalizes effectively to portfolios with multiple vertices. The PV01 for each vertex can then be accurately computed as:

$$PV01_{T_i} \approx T_i \times x_{T_i} \times 0.01\% \tag{9}$$

The profit and loss (P&L) for bonds, reflecting changes in their present values ($\Delta PV$), is consequently expressed as:

$$P\&L_{Bond_t} = \sum_{i=1}^{n} -PV01_{t_i} \times \frac{\Delta r_{t_i}}{0.01\%} \tag{10}$$

Therefore, a rise in rates leads to capital losses, while declining rates result in gains. Accurately modelling this sensitivity is essential for the effective management of fixed-income risk, especially when constructing VaR-based frameworks.

#### 4.1.3. Currency

The inclusion of foreign-denominated assets in the portfolio adds an additional dimension to the risk management process, stemming from foreign exchange (FX) exposure. In addition to the inherent market risk associated with each asset, positions denominated in currencies other than the euro (EUR), particularly the United States dollar (USD) and the Hong Kong dollar (HKD), introduce valuation risk due to exchange rate variability.

This FX risk materializes when the EUR-equivalent value of foreign holdings fluctuates because of changes in exchange rates. Such currency-induced volatility can have a material impact on the portfolio's total value and therefore requires explicit integration into the overall risk measurement framework.

To quantify this exposure, the capital invested in each foreign currency is aggregated and converted into euros using the prevailing spot exchange rate. Let $FX_{i,t}$ denote the spot rate at time $t$ between a foreign currency $i$ and the EUR. Thus, for a given investment amount $M_{i,t}$ denominated in currency $i$, the associated profit and loss (P&L) arising specifically from exchange rate movements is calculated as follows:

$$P\&L_{i,t} = M_{i,t} \times \left( \frac{FX_{i,t}}{FX_{i,t-1}} - 1 \right) \tag{11}$$

This equation isolates the impact of currency fluctuations on the EUR valuation of the portfolio. Proper quantification of this effect enables the identification of potential vulnerabilities and supports the implementation of targeted hedging strategies. In the context of an internationally diversified portfolio, managing FX risk is essential for maintaining return stability and ensuring consistency in risk-adjusted performance metrics.

#### 4.1.4. Portfolio

The vector of risk factor loadings reflects the portfolio's aggregated sensitivities to distinct sources of financial risk and constitutes a critical input for risk measurement and decomposition. These exposures enable a structured understanding of how specific market variables influence overall portfolio valuation. Table 4.1 provides a detailed breakdown of the mapped risk factor exposures expressed in euros, with a focus on Total Value-at-Risk (VaR). This metric integrates both systematic and idiosyncratic components of risk, capturing the full spectrum of sensitivity to equity prices, interest rates and currency fluctuations. For reference, Systematic VaR figures are discussed separately in later sections, where factor-based aggregation is explicitly applied.

The data presented correspond to 30 January 2023, which marks the starting point of the evaluation period. This is the first day on which the one-day-ahead VaR is calculated, using risk factor exposures determined in the prior step.

| Stocks | | Interest Rate | | Currency | |
|---|---|---|---|---|---|
| Risk Factor | Exposure (EUR) | Risk Factor | Exposure (EUR) | Risk Factor | Exposure (EUR) |
| RI.PA | -190 052.80 | EUR3M | -1.11 | USDEUR | 5 743 925.37 |
| CA.PA | -215 004.55 | EUR6M | -1.62 | HKDEUR | -78 542.63 |
| MC.PA | 320 143.96 | EUR1Y | -10.85 | | |
| BN.PA | -180 008.26 | EUR2Y | -26.52 | | |
| MMM | -183 553.15 | EUR3Y | -63.13 | | |
| AIG | -174 424.11 | EUR5Y | -732.35 | | |
| AMZN | 302 935.19 | EUR7Y | -213.34 | | |
| AMD | -192 748.64 | EUR10Y | -1 204.61 | | |
| MSFT | 312 021.43 | EUR15Y | -362.86 | | |
| AAPL | 325 892.53 | EUR20Y | 0 | | |
| NVDA | 316 691.53 | USD3M | -0.69 | | |
| ASML | 325 205.68 | USD6M | -1.02 | | |
| ORCL | -201 925.35 | USD1Y | -4.99 | | |
| CRM | -183 542.95 | USD2Y | -12.91 | | |
| ADBE | 284 469.65 | USD3Y | -27.80 | | |
| SAP | -215 008.45 | USD5Y | -58.57 | | |
| TXN | 220 306.20 | USD7Y | -120.13 | | |
| JNJ | 321 212.94 | USD10Y | -1 153.40 | | |
| PFE | 201 933.11 | USD20Y | -97.26 | | |
| LLY | 279 816.50 | | | | |
| AZN | -183 605.77 | | | | |
| NVS | 215 721.57 | | | | |
| AMGN | 206 629.84 | | | | |
| JPM | 270 781.09 | | | | |
| BAC | -174 386.22 | | | | |
| WFC | -211 118.34 | | | | |
| GS | 257 477.95 | | | | |
| MS | -188 211.08 | | | | |
| AXP | 229 545.45 | | | | |
| HON | 156 067.83 | | | | |
| CAT | -165 271.20 | | | | |
| GE | -224 889.66 | | | | |
| UNP | 289 158.73 | | | | |
| FDX | 183 573.92 | | | | |
| NEE | 247 842.08 | | | | |
| DUK | -146 879.22 | | | | |
| ED | 174 381.50 | | | | |
| XEL | 165 241.39 | | | | |
| PCG | -151 453.14 | | | | |
| SRE | 169 776.43 | | | | |
| BA | 312 071.04 | | | | |
| LMT | 201 830.75 | | | | |
| RTX | 211 133.53 | | | | |
| NOC | 197 508.62 | | | | |
| GD | 224 938.98 | | | | |
| AIR | 199 996.80 | | | | |
| TTE | 259 981.42 | | | | |
| COP | -201 910.59 | | | | |
| BP | -206 524.32 | | | | |
| EQNR | 229 505.06 | | | | |
| CVX | 280 002.23 | | | | |
| ACA.PA | -214 995.43 | | | | |
| 1398.HK | -27 549.15 | | | | |
| 1093.HK | -24 618.31 | | | | |
| 1109.HK | -26 375.17 | | | | |

**Table 4.1. Risk factor exposures**. This table showcases the VaR exposures of portfolio positions in EUR as of 27 January 2023, including equities, interest rate and currencies.

### 4.2. Volatility Estimation

Before computing the Value-at-Risk (VaR), it is essential to obtain accurate estimates of the volatility associated with each underlying risk factor. Although the sample standard deviation of historical returns is the most used method, this approach assigns equal weight to all past observations. As a result, it may fail to adequately capture recent shifts in market dynamics, especially under volatile conditions.

This limitation becomes particularly relevant when risk factors exhibit different scales, such as comparing equity returns (typically in percentage terms) to changes in interest rates (measured in basis points). Since VaR is inherently forward-looking, it is crucial to adopt a volatility estimation method that places greater emphasis on recent market behaviour.

To address these shortcomings, we employ the Exponentially Weighted Moving Average (EWMA) model. Unlike traditional methods, the EWMA model assigns exponentially decreasing weights to historical observations, prioritizing recent data while diminishing the influence of older data over time. The extent to which older observations decrease in importance is determined by a smoothing parameter λ (lambda), which lies between 0 and 1. Lower values of λ give greater weight to recent observations, thus making the volatility estimates more responsive to current market conditions.

The recursive formulation for EWMA volatility estimation is given by:

$$\hat{\sigma}_t^2 = (1 - \lambda)x_t^2 + \lambda\hat{\sigma}_{t-1} \tag{12}$$

Where $x_t$ represents the return (or change) in the risk factor at time $t$ and $\hat{\sigma}_t^2$ denotes the estimate variance at time $t$.

Similarly, the EWMA covariance between two risk factors $i$ at time $j$ and at time is given by:

$$\hat{\sigma}_{i,j,t} = (1 - \lambda)x_{i,t}x_{j,t} + \lambda\hat{\sigma}_{i,j,t-1} \tag{13}$$

The selection of the smoothing factor $\lambda$ plays a critical role in determining the responsiveness of the volatility estimate. The RiskMetrics framework proposed by J.P. Morgan (1996) suggests a standard value of $\lambda$ = 0.94 for daily data, which balances short-term sensitivity with longer-term stability. However, the optimal value of $\lambda$ is context-specific and depends on the volatility regime, asset class, and the institution's risk tolerance (Alexander, 2008).

In this study, multiple values of $\lambda$ are evaluated to determine the most appropriate configuration for the portfolio under analysis. The selected parameter is then validated through a backtesting procedure, ensuring that the resulting VaR estimates are both accurate and aligned with observed market conditions throughout the study period.

# Chapter 5.

# VaR Models

Value-at-Risk (VaR) is a widely adopted statistical measure used to estimate the maximum potential loss a portfolio could incur over a specific time horizon ($h$) at a predetermined confidence level ($1-\alpha$). Formally, the $h$-day VaR at level $\alpha$, denoted $VaR_{h,a}$, corresponds to the threshold loss that will not be exceeded with probability $1-\alpha$. Mathematically, this can be expressed as:

$$P\left(X_h < -VaR_{h,\alpha}\right) = \alpha \tag{14}$$

where $X_h$ represents the discounted $h$-day profit-and-loss (P&L) distribution of the portfolio.

In alignment with the Basel Committee's recommendations, this study adopts a confidence level of 99% ($\alpha = 1\%$) and a daily time horizon ($h$ = 1 day). Consequently, $VaR_{1,1}\%$ represents the daily loss that the portfolio will not exceed with 99% confidence.

Given the diversity of market conditions and portfolio compositions, no single VaR model is universally optimal. To address this, the present study evaluates four distinct methodologies: Normal VaR, Skewed Generalized Student-t (SGSt) VaR, Historical VaR, and Quantile Regression VaR. Each model has been specifically chosen to represent a range of assumptions and statistical techniques, enabling a comprehensive assessment of their performance and suitability for our portfolio.

The analysis incorporates multiple variations within each class, encompassing parametric and non-parametric frameworks with diverse parameterizations and volatility weighting schemes. The objective of this extensive approach is to pinpoint the most precise and reliable model configuration, aligning closely with our portfolio's unique characteristics and prevailing market conditions.

Detailed outcomes from the application of these models are rigorously evaluated through backtesting in Chapter 6. This process systematically compares predicted VaR estimates against actual historical outcomes, highlighting each model's strengths and limitations, and offering critical insights for optimizing risk management strategies.

## 5.1. Normal VaR

Consider a portfolio's $h$-day returns, represented by the random variable $X_h$, assumed to follow a normal distribution:

$$X_h \sim N(\mu, \sigma^2) \tag{15}$$

Where $\mu$ denotes the mean return and $\sigma^2$ the variance over the horizon $h$. Under this assumption, the $h$-day VaR at confidence level $1 - \alpha$, denoted $VaR_{h,\alpha}$, corresponds to the negative of the $\alpha$ - quantile of this normal distribution:

$$VaR_{h,\alpha} = -\phi_{\mu,\sigma}^{-1}(\alpha) \tag{16}$$

In this expression, $\phi_{\mu,\sigma}^{-1}(\alpha)$ denotes the quantile (or inverse cumulative distribution) function of a normal distribution with mean μ and standard deviation σ.

Using the equivariance property of quantiles, which states:

$$Q_{g(X)}(\alpha) = g\big(Q_X(\alpha)\big) \tag{17}$$

Portfolio returns can be expressed as a linear transformation of a standard normal variable $Z \sim N(0,1)$:

$$X_h = \mu + \sigma Z \tag{18}$$

Substituting this into the original quantile definition simplifies VaR expression to:

$$VaR_{h,\alpha} = \phi^{-1}(\alpha)\sigma - \mu \tag{19}$$

where $\phi^{-1}(\alpha)$ denotes the inverse cumulative distribution function (quantile function) of the standard normal distribution, evaluated at confidence level $\alpha$.

For daily VaR estimates ($h = 1$), the drift term $\mu$ is commonly omitted, following the suggestion by Alexander (2008b) that short-term expected returns are negligible and difficult to estimate accurately. Thus, setting $\mu = 0$ has minimal impact on accuracy and increases robustness, resulting in:

$$VaR_{1,a} = -\phi^{-1}(\alpha) \times \sigma \tag{20}$$

In this study, volatility parameter σ is estimated using the EWMA model, which emphasizes recent market conditions, as outlined in Chapter 4. This approach allows the Normal VaR model to reflect recent market volatility while preserving analytical tractability.

### 5.2. SGSt VaR

The standardized Skewed Generalized Student-t (SGSt) distribution, introduced by Theodossiou (1998), is an extension of the classical Student-t distribution, by incorporating skewness and separate control over central and tail behaviour. This flexibility allows the model to capture both asymmetry and fat tails, characteristics commonly observed in financial return series, particularly during periods of market stress. The SGSt distribution has five parameters: the mean ($\mu$), standard deviation ($\sigma > 0$), skewness parameter ($-1 < \lambda < 1$), central shape parameter ($p > 0$), and tail shape parameter ($q > 0$). These parameters are typically estimated through maximum likelihood, allowing the distribution to adapt to the empirical characteristics of the data.

To compute the VaR using the SGSt distribution, we follow a similar methodology to the Normal VaR. However, the quantile function is replaced by that of the SGSt distribution. The $h$-day VaR at confidence level $\alpha$ is expressed as:

$$VaR_{h,\alpha} = -T^{-1}_{\mu,\sigma,\lambda,p,q}(\alpha) \tag{21}$$

By leveraging the equivariance property and assuming μ = 0 for simplicity, the SGSt VaR simplifies to:

$$VaR_{h,\alpha} = -T^{-1}_{0,1,\lambda,p,q}(\alpha) \times \sigma \tag{22}$$

### 5.3. Historical VaR

The Historical Simulation approach to Value-at-Risk (VaR) estimation constructs an empirical distribution of portfolio profit and loss (P&L) based on past observations, without imposing any assumptions on the underlying return distribution. This makes it a non-parametric method, particularly useful when return distributions deviate from normality or exhibit pronounced skewness and excess kurtosis.

The procedure begins by selecting a historical sample of $n$ non-overlapping returns for each risk factor over a fixed time horizon $h$. These returns are used to generate a series of hypothetical $h$-day P&L outcomes, assuming constant portfolio exposures throughout the sample period.

Once the empirical P&L distribution is constructed, the VaR at confidence level $\alpha$ is given by the negative of the empirical $\alpha$-quantile:

$$VaR_{h,\alpha} = -Quantile_{\alpha}(P\&L_{hist}) \tag{23}$$

Each observation in the sample is assigned equal probability $\frac{1}{n}$, and the empirical cumulative distribution function (CDF) is derived by ordering the P&L values from worst to best.

Despite its intuitive approach, the standard Historical VaR model has a significant limitation, it assigns equal weights to all observations within the historical sample, which diminishes the model's responsiveness to current market dynamics, particularly when using larger sample sizes. Although parametric models such as Normal VaR or SGSt VaR mitigate this issue by employing EWMA volatility estimates, the basic Historical VaR cannot directly incorporate this solution. EWMA is suitable for estimating covariance structures but cannot fully characterize the entire return distribution.

To overcome this limitation, Hull and White (1998) introduced an enhanced version of Historical VaR known as volatility adjusted Historical VaR. This methodology maintains equal weighting across observations but adjusts historical returns based on current volatility levels, ensuring the historical sample accurately reflects prevailing market conditions. Specifically, historical returns ($r_t$) at each historical date $t$ (where $t < T$, with $T$ representing the current VaR measurement date) are scaled according to the ratio of current volatility ($\hat{\sigma}_T$) to historical volatility ($\hat{\sigma}_t$). This adjustment is mathematically expressed as:

$$\hat{r}_t = r_t \times \frac{\hat{\sigma}_T}{\hat{\sigma}_t} \tag{24}$$

This transformation ensures that past returns are rescaled to reflect current market conditions, thereby improving the relevance of the empirical distribution. The volatility-adjusted Historical VaR is then obtained as the negative empirical quantile of these adjusted returns, providing a more responsive and realistic measure of downside risk.

### 5.4. Quantile Regression VaR

Quantile Regression Value-at-Risk (VaR) builds upon the fundamental definition of VaR as the $\alpha$-quantile of the profit and loss distribution. Unlike parametric models, this approach estimates VaR directly as a conditional quantile function, framed within a robust regression methodology. It allows for flexible modelling of asymmetric and non-normal distributions without relying on restrictive assumptions.

The model specifies portfolio returns as a function of relevant explanatory variables, typically including time-varying volatility estimates such as those obtained from the EWMA. The conditional $\alpha$-quantile of returns is then estimated through the minimisation of a quantile-specific loss function.

Formally, the $\alpha$-quantile regression VaR is expressed as:

$$VaR_\alpha = -\left(\hat{\alpha} + \hat{b}x_i\right) \tag{25}$$

where $\hat{\alpha}$ and $\hat{b}$ denote the estimated intercept and slope coefficients, respectively, and $x_i$ represents the volatility proxy at time $i$.

The parameter estimates are obtained by solving the following minimisation problem:

$$\left(\hat{\alpha}, \hat{b}\right) = arg_{\alpha,b} min \sum_{i=1}^{n} [y_i - (\alpha + bx_i)]\left(\alpha - I_{[y_i-(\alpha+bx_i)<0]}\right) \tag{26}$$

Here $I_{[y_i-(\alpha+bx_i)<0]}$ denotes an indicator function that equals 1 when the residual is negative, and 0 otherwise. This asymmetric weighting allows the model to robustly capture the conditional distribution characteristics of returns, especially tail-risk events.

Multiple model specifications are evaluated, varying the choice of explanatory variables and the inclusion or exclusion of a constant term, to identify the most effective and statistically robust formulation. Typically, these explanatory variables consist of EWMA-derived volatility measures, possibly incorporating multiple smoothing factors to enhance predictive accuracy.

The rigorous analysis and comparison of these quantile regression VaR models, as detailed in Chapter 7, provide essential insights into selecting the most appropriate risk modelling approach tailored specifically to the portfolio's unique characteristics and prevailing market conditions.

## 5.5. Summary of the VaR Models

This dissertation conducts an extensive evaluation of VaR methodologies, based on the empirical testing of 152 unique model configurations across four major model classes. Given the scale and complexity of the modelling exercise, it is not feasible to present the results of each individual configuration within the main body of the dissertation. Instead, Table 5.1 provides a structured summary of the parameter ranges applied to each class of models, offering an overview of the modelling space explored.

For each model class, the number of configurations evaluated is indicated alongside the parameters varied and their respective ranges. The full specification of all models, including the exact combinations employed, is presented in Appendix D. Each configuration is assigned a unique numerical identifier to ensure clarity and consistency in the comparative analysis conducted throughout the subsequent chapters.

| Model Class | Models evaluated | Parameters | Values used |
|---|---|---|---|
| Normal VaR | 20 | EWMA smoothing factor | From 0.9 to 0.995 (increment = 0.005) |
| SGSt VaR | 44 | EWMA smoothing factor | From 0.92 to 0.97 (increment = 0.005) |
| | | Sample size | 250, 500, 750, 1000 |
| Historical VaR | 44 | EWMA smoothing factor | From 0.92 to 0.97 (increment = 0.005) |
| | | Sample size | 250, 500, 750, 1000 |
| Quantile Regression VaR | 44 | EWMA smoothing factor | From 0.92 to 0.97 (increment = 0.005) |
| | | Sample size | 250, 500, 750, 1000 |

**Table 5.1. Summary of model classes and parameters evaluated.** This table summarises the number of models evaluated within each VaR class, along with the parameters and value ranges used during the configuration process.

# Chapter 6.

## Backtesting Methodology

The previous chapter introduced the methodology for computing four distinct types of VaR models. A total of 152 model configurations were evaluated with the objective of estimating both Total VaR and Systematic VaR, and of identifying the most accurate and robust specification. Each configuration generated a series of daily VaR estimates over a ten-year global period, spanning from 11 February 2013 to 27 January 2023, resulting in a total of 2 600 observations.

A key component of the performance assessment involved tracking exceedances, defined as instances in which the portfolio incurred a loss greater than the corresponding VaR estimate. Since VaR represents a threshold for potential losses, exceedances occur when the realised daily P&L falls below the negative of the VaR value (i.e., losses exceed expectations). The frequency and timing of these exceedances served as primary indicators of model accuracy and reliability.

To formally assess model quality, two statistical backtesting procedures were employed. The Unconditional Coverage (UC) Test, proposed by Kupiec (1995), evaluates whether the number of observed exceedances is statistically consistent with the expected rate, given the specified confidence level of the VaR model. Complementarily, the BCP Test, introduced by Berkowitz, Christoffersen, and Pelletier (2011), examines whether exceedances occur independently over time, identifying potential clustering patterns that may indicate model misspecification.

It is important to note that a model may pass the UC test while still failing the BCP test, particularly if exceedances are temporally concentrated. To address this, a sequential evaluation procedure was implemented. First, the UC test was applied over the global period to filter out models that deviate from the expected exceedance rate. The BCP test was then used to evaluate the independence of exceedances among the shortlisted models. Finally, the UC test was applied again on a year-by-year basis to examine performance consistency over shorter time intervals.

This structured backtesting approach was applied to all 152 configurations, ensuring a comprehensive and rigorous assessment of model performance. The configuration that demonstrated the highest degree of statistical reliability and temporal stability was selected for forward testing in the final one-year period of the study.

### 6.1. Unconditional Coverage (UC) Test

The UC Test, introduced by Kupiec (1995), assesses whether the number of observed exceedances, defined as instances where actual losses surpass the estimated VaR, is consistent with the expected frequency under the specified confidence level.

Given a confidence level of $1-\alpha$, there is always a probability $\alpha$ that the actual loss will exceed the predicted VaR. For example, with 500 daily VaR estimates at a 99% confidence level ($\alpha = 1\%$), the expected number of exceedances is $500 \times 1\% = 5$

To formally evaluate this, an indicator function is constructed to identify exceedance events:

$$I_t = \begin{cases} 1, if\ P\&L_t < -VaR_{t,\alpha} \\ 0\ , otherwise \end{cases} \tag{27}$$

Let $\pi_{obs}$ and $\pi_{exp}$ denote the observed and the expected exceedance rates, respectively. The null and alternative hypothesis, for the UC test are formulated by:

$$H_0 \colon \pi_{obs} = \pi_{exp} = \alpha \tag{28}$$

$$H_a \colon \pi_{obs} \neq \pi_{exp} \tag{29}$$

Let $n_1$ represent the number of exceedances observed in the sample and $n_0$ denotes the number of observations without exceedances. The test statistic can then be expressed as follows:

$$LR_{uc} = \left(\frac{\pi_{exp}}{\pi_{obs}}\right)^{n_1} \left(\frac{1-\pi_{exp}}{1-\pi_{obs}}\right)^{n_0} \tag{30}$$

Under the null hypothesis $H_0$, this test statistic asymptotically follows a chi-squared distribution with one degree of freedom:

$$LR_{uc} \sim X_1^2 \tag{31}$$

A VaR model is considered well-specified if the null hypothesis defined at Equation (28) is not rejected at the 95% confidence level. This outcome indicates that the frequency of exceedances is in line with the model's stated confidence level, suggesting correct calibration of the VaR estimates.

### 6.2. BCP Test

The Berkowitz, Christoffersen, and Pelletier (BCP) test provides a statistical framework for assessing the independence of exceedance events in a VaR model. While a model may pass the UC test by generating the correct number of exceedances, it can still be misspecified if those exceedances exhibit temporal dependence or clustering. A well-specified VaR model should produce exceedances that occur randomly over time, without autocorrelation.

Let $\hat{p}_k = Corr(I_\alpha, L^k I_\alpha)$ denote the autocorrelation of order $k - th$ in the time series of exceedance indicators and let $K$ represent the maximum number of lags considered in the analysis. The test evaluates the following hypotheses:

$$H_0\colon \hat{p}_K = 0; \forall k \in \{1, \dots, K\} \tag{32}$$

$$H_a\colon \exists k \in \{1, \dots, K\}\ s.t. \hat{p}_K \neq 0 \tag{33}$$

Assuming a sample consisting of n observations, the test statistic is defined as:

$$BCP_K = T(T+2)\sum_{k=1}^{K} \frac{\hat{p}_k^2}{T-k} \tag{34}$$

where $T$ corresponds to the number of observations in the exceedance time series.

Under the null hypothesis, the test statistic follows a chi-squared distribution with $K$ degrees of freedom:

$$BCP_K \sim \chi_K^2 \tag{35}$$

The selection of the lag length $K$ involves an inherent trade-off. Increasing $K$ enhances the test's ability to detect higher-order autocorrelations but simultaneously raises the critical value required for rejection. This can reduce the power of the test, particularly when dependence exists only at lower-order lags. In such cases, the statistic may fail to exceed the threshold, resulting in a Type II error.

To mitigate this limitation and improve robustness, the BCP test is implemented across a range of lag values from 1 to 10. This allows for a more comprehensive evaluation of exceedance independence, capturing both short- and medium-term autocorrelation patterns in the exceedance sequence.

### 6.3. Results of Backtesting

To ensure a methodologically rigorous and comprehensive evaluation, a total of 152 VaR model configurations were developed and evaluated. These configurations were systematically constructed by combining the parameter settings outlined in Table 5.1, encompassing a wide range of specifications across four distinct VaR model classes: Normal, SGSt, Historical Simulation, and Quantile Regression.

Given the volume of models analysed, it is not feasible to present the complete set of results within the main body of the dissertation. Instead, this section highlights a representative subset of 22 selected models, chosen to reflect both commonly used benchmarks and configurations that demonstrated particularly strong empirical performance throughout the backtesting phase.

The full set of 152 model configurations and their corresponding backtesting outcomes are documented in Appendix D.

Table 6.1 summarises the main structural features of the selected 22 models.

| Model number | Description |
|---|---|
| 5 | Normal, with EWMA smoothing factor 0.92 |
| 9 | Normal, with EWMA smoothing factor 0.94 |
| 14 | Normal, with EWMA smoothing factor 0.965 |
| 19 | Normal, with EWMA smoothing factor 0.99 |
| 33 | SGSt, with EWMA smoothing factor 0.925 and sample size 500 |
| 39 | SGSt, with EWMA smoothing factor 0.955 and sample size 500 |
| 44 | SGSt, with EWMA smoothing factor 0.925 and sample size 750 |
| 46 | SGSt, with EWMA smoothing factor 0.935 and sample size 750 |
| 54 | SGSt, with EWMA smoothing factor 0.92 and sample size 1000 |
| 59 | SGSt, with EWMA smoothing factor 0.945 and sample size 1000 |
| 71 | Historical, with volatility adjustment, EWMA smoothing factor 0.95 and sample size 250 |
| 78 | Historical, with volatility adjustment, EWMA smoothing factor 0.93 and sample size 500 |
| 93 | Historical, with volatility adjustment, EWMA smoothing factor 0.95 and sample size 750 |
| 103 | Historical, with volatility adjustment, EWMA smoothing factor 0.945 and sample size 1000 |
| 109 | Quantile Regression, EWMA volatility with 0.92 smoothing factor as independent variable, sample size 250 |
| 112 | Quantile Regression, EWMA volatility with 0.935 smoothing factor as independent variable, sample size 250 |
| 117 | Quantile Regression, EWMA volatility with 0.96 smoothing factor as independent variable, sample size 250 |
| 121 | Quantile Regression, EWMA volatility with 0.925 smoothing factor as independent variable, sample size 500 |
| 124 | Quantile Regression, EWMA volatility with 0.94 smoothing factor as independent variable, sample size 500 |
| 131 | Quantile Regression, EWMA volatility with 0.92 smoothing factor as independent variable, sample size 750 |
| 142 | Quantile Regression, EWMA volatility with 0.92 smoothing factor as independent variable, sample size 1000 |
| 145 | Quantile Regression, EWMA volatility with 0.935 smoothing factor as independent variable, sample size 1000 |

**Table 6.1. Summary of selected VaR model configurations.** Presents 22 representative models from the total set of 152 tested, detailing their class and parameter settings.

To assess the statistical adequacy of the VaR models, two standard backtesting procedures are applied: UC test by Kupiec (1995) and the BCP test introduced by Berkowitz et al. (2011). The UC test examines whether the observed number of exceedances aligns with the expected frequency under correct model specification. Given 2600 daily observations and a 99% confidence level, approximately 26 exceedances are anticipated. A p-value below 5% indicates rejection of the null hypothesis, suggesting that the model underestimates tail risk.

| **Model Class** | **Model number** | **Exceedances** | **Exceedance Rate (%)** | **p-value (%)** |
|---|---|---|---|---|
| Normal | 5 | 63 | 2.42 | 0.00 |
| | 9 | 59 | 2.27 | 0.00 |
| | 14 | 50 | 1.92 | 0.00 |
| | 19 | 47 | 1.81 | 0.02 |
| SGSt | 33 | 39 | 1.92 | 1.70 |
| | 39 | 37 | 1.81 | 4.15 |
| | **44** | **33** | **1.50** | **19.54** |
| | **46** | **32** | **1.42** | **25.37** |
| | **54** | **32** | **1.27** | **25.37** |
| | **59** | **33** | **1.50** | **18.54** |
| Historical | 71 | 49 | 1.23 | 0.04 |
| | 78 | 52 | 1.27 | 0.00 |
| | 93 | 34 | 1.77 | 0.01 |
| | 103 | 30 | 2.42 | 0.00 |
| Quantile Regression | **109** | **34** | **1.88** | **13.22** |
| | **112** | **30** | **2.00** | **44.15** |
| | **117** | **27** | **1.31** | **13.22** |
| | **121** | **33** | **1.15** | **44.15** |
| | **124** | **36** | **1.04** | **84.47** |
| | **131** | **33** | **1.27** | **18.54** |
| | **142** | **36** | **1.38** | **6.25** |
| | **145** | **33** | **1.27** | **18.54** |

**Table 6.2. UC test results over the global period - Total VaR.** Models in bold indicate those that pass the UC test at the 5% significance level.

The results indicate that all Normal VaR models within the full set of 152 evaluated configurations fail to meet the statistical criteria for adequacy, with each being rejected at conventional significance levels. This outcome is consistent with the extensive body of empirical literature that critiques the use of the normality assumption in modelling financial returns. As originally noted by Fama (1965) and corroborated by subsequent studies, return distributions in financial markets tend to exhibit fat tails and negative skewness, characteristics that the Normal distribution fails to capture effectively. This limitation is particularly relevant in the present analysis, as the portfolio returns display pronounced excess kurtosis across multiple years, as shown in Appendix A. As a result, Normal VaR models systematically underestimate the likelihood and magnitude of extreme losses, particularly when evaluated at high confidence levels.

Notably, Model 9 corresponds to the RiskMetrics framework proposed by J.P. Morgan (1996), which applies an EWMA with a smoothing factor of $\lambda = 0.94$. Despite its historical relevance, this model performed poorly in our setting, with a significantly higher number of exceedances than expected, thereby failing the UC test.

In addition to evaluating the Total VaR models, the UC test was also applied to the Systematic VaR configurations. These models seek to simplify risk estimation by mapping individual asset exposures onto a reduced set of common risk factors. This methodology is commonly adopted to improve scalability and computational efficiency, particularly in portfolios with a large number of positions or complex asset structures.

| Model Class | Model number | Exceedances | Exceedance Rate (%) | p-value (%) |
|---|---|---|---|---|
| Normal | 5 | 90 | 3.46 | 0.00 |
| | 9 | 85 | 3.27 | 0.00 |
| | 14 | 84 | 3.23 | 0.00 |
| | 19 | 82 | 3.15 | 0.00 |
| SGSt | 33 | 67 | 2.58 | 3.71 |
| | 39 | 64 | 2.46 | 0.00 |
| | 44 | 60 | 2.31 | 0.00 |
| | 46 | 58 | 2.33 | 0.00 |
| | 54 | 59 | 2.27 | 0.00 |
| | 59 | 58 | 2.23 | 0.00 |
| Historical | 71 | 72 | 2.77 | 0.00 |
| | 78 | 84 | 3.23 | 0.00 |
| | 93 | 74 | 2.85 | 0.00 |
| | 103 | 81 | 3.12 | 0.00 |
| Quantile Regression | 109 | 56 | 2.15 | 0.00 |
| | 112 | 53 | 2.04 | 0.00 |
| | 117 | 50 | 1.92 | 0.00 |
| | 121 | 55 | 2.12 | 0.00 |
| | 124 | 55 | 2.12 | 0.00 |
| | 131 | 56 | 2.15 | 0.00 |
| | 142 | 58 | 2.23 | 0.00 |
| | 145 | 57 | 2.19 | 0.00 |

**Table 6.3. UC test results over the global period - Systematic VaR.** Models in bold indicate those that pass the UC test at the 5% significance level.

However, as shown in Table 6.3, none of the Systematic VaR models passed the UC test. This outcome reveals a significant discrepancy between the risk captured by the mapped portfolio and the true exposures embedded in the original portfolio. The results suggest that the factor-mapping process introduces distortions that lead to an underestimation of tail risk.

The rejection of all Systematic VaR configurations implies that the simplified portfolio structure fails to preserve essential risk characteristics. In this case, the relatively low level of portfolio diversification may have amplified this divergence, further reducing the reliability of the simplified representation. As a result, Systematic VaR models are excluded from the remainder of the analysis. The study henceforth focuses exclusively on Total VaR models, which are based on the complete, untransformed portfolio data and provide a more accurate and robust framework for risk quantification.

Following the initial screening based on the UC test, models failing to meet the minimum statistical adequacy criteria were removed from further evaluation. The next step involved applying the BCP test to the subset of models that were not rejected by the UC test. The BCP test assesses the independence of exceedance events over time, thereby detecting potential clustering effects that the UC test does not capture. Out of the 22 shortlisted configurations, 12 models passed both tests. The results of the UC and BCP tests for the full set of 152 models are reported in Appendix D, Table D.1.

Table 6.4 presents the BCP test results for the remaining Total VaR models, specifically reporting the lowest p-value obtained across ten different lag structures evaluated for each model. This approach facilitates the identification of models exhibiting temporal dependence in exceedances at any lag within the tested range. Low p-values in this test may signal instability in model calibration or insufficient sensitivity to evolving market conditions. Only those models that perform satisfactorily in both the UC and BCP tests are retained for the subsequent analysis of stability and robustness over shorter time horizons.

| Model Class | Model number | Worst p-value | Lag |
|---|---|---|---|
| SGSt | **44** | **36.29** | **1** |
| | 46 | 2.15 | 2 |
| | **54** | **32.80** | **1** |
| | 59 | 3.08 | 2 |
| Quantile Regression | **109** | **39.86** | **1** |
| | **112** | **26.07** | **1** |
| | **117** | **31.95** | **10** |
| | **121** | **26.07** | **1** |
| | **124** | **12.92** | **3** |
| | **131** | **36.29** | **1** |
| | **142** | **47.12** | **1** |
| | 145 | 3.08 | 2 |

**Table 6.4. BCP test results over the global period.** Models in bold indicate those that pass the BCP test at the 5% significance level.

Out of the 12 models considered, nine passed both the UC and the BCP tests, confirming their statistical adequacy in terms of exceedance frequency and independence. Beyond this restricted set, additional configurations from the broader set of 152 models also met both criteria, bringing the total number of validated models to 26. The complete results of this evaluation are presented in Appendix D, Table D.2.

Table 6.5 presents the UC test results across ten consecutive annual subperiods, allowing for a detailed assessment of exceedance consistency over time. This additional validation step is critical to distinguish models that exhibit genuine stability from those whose performance may result from period-specific calibration or random variation. Based on this analysis, model selection is further refined by identifying the configuration that demonstrates the highest consistency in UC test outcomes across individual years.

| Model class | | SGSt | | Quantile Regression | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Model number | | 44 | 54 | 109 | 112 | 117 | 121 | 124 | 131 | 142 |
| 2022-2023 | Exceedance rate (%) | 0.77 | 0.77 | 0.77 | 0.77 | 1.15 | 0.38 | 0.38 | 0.38 | 0.38 |
| | p-value (%) | 69.67 | 69.67 | 69.67 | 69.67 | 80.77 | 25.44 | 25.44 | 25.44 | 25.44 |
| 2021-2022 | Exceedance rate (%) | 1.15 | 1.15 | 0.77 | 0.77 | 1.15 | 0.77 | 0.38 | 1.15 | 1.15 |
| | p-value (%) | 80.77 | 80.77 | 69.67 | 69.67 | 80.77 | 69.67 | 25.44 | 80.77 | 80.77 |
| 2020-2021 | Exceedance rate (%) | 2.31 | 2.31 | 1.54 | 1.15 | 1.92 | 2.31 | 1.92 | 1.92 | 2.69 |
| | p-value (%) | 7.01 | 7.01 | 41.87 | 80.77 | 18.44 | 7.01 | 18.44 | 18.44 | 2.34 |
| 2019-2020 | Exceedance rate (%) | 1.54 | 1.54 | 1.92 | 1.92 | 1.92 | 1.92 | 1.54 | 1.54 | 1.54 |
| | p-value (%) | 41.87 | 41.87 | 18.44 | 18.44 | 18.44 | 18.44 | 41.87 | 41.87 | 41.87 |
| 2018-2019 | Exceedance rate (%) | 0.38 | 0.38 | 0.77 | 0.77 | 0.77 | 0.38 | 0.38 | 0.38 | 1.15 |
| | p-value (%) | 25.44 | 25.44 | 69.67 | 69.67 | 69.67 | 25.44 | 25.44 | 25.44 | 80.77 |
| 2017-2018 | Exceedance rate (%) | 1.15 | 1.15 | 1.15 | 1.15 | 1.54 | 1.15 | 1.54 | 1.54 | 1.54 |
| | p-value (%) | 80.77 | 80.77 | 80.77 | 80.77 | 41.87 | 80.77 | 41.87 | 41.87 | 41.87 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 2016-2017 | Exceedance rate (%) | 1.15 | 1.15 | 1.15 | 1.15 | 0.77 | 1.15 | 1.15 | 1.15 | 1.15 |
| | p-value (%) | 80.77 | 80.77 | 80.77 | 80.77 | 69.67 | 80.77 | 80.77 | 80.77 | 80.77 |
| 2015-2016 | Exceedance rate (%) | 1.15 | 1.15 | 2.31 | 1.15 | 0.77 | 1.15 | 0.77 | 1.15 | 1.54 |
| | p-value (%) | 80.77 | 80.77 | 7.01 | 80.77 | 69.67 | 80.77 | 69.67 | 80.77 | 41.87 |
| 2014-2015 | Exceedance rate (%) | 1.54 | 1.54 | 1.15 | 1.15 | 1.54 | 1.54 | 1.54 | 1.54 | 1.54 |
| | p-value (%) | 41.87 | 41.87 | 80.77 | 80.77 | 41.87 | 41.87 | 41.87 | 41.87 | 41.87 |
| 2013-2014 | Exceedance rate (%) | 1.54 | 1.15 | 1.54 | 1.54 | 1.54 | 0.77 | 0.77 | 1.92 | 1.15 |
| | p-value (%) | 41.87 | 80.77 | 41.87 | 41.87 | 41.87 | 69.67 | 69.67 | 18.44 | 80.77 |

**Table 6.5. UC test for annual sub-periods.** The table indicates the results for the UC test for the sub-periods of the models that passed the UC and the BCP test for the global period.

Based on the results presented in Table 6.5, Model 124 emerges as the most consistent and reliable among the shortlisted candidates. It records an exceedance rate below 1% in five out of the ten annual subperiods, more than any other model in the comparison set. This pattern highlights the model's temporal stability and its strong alignment with the intended confidence level under a range of market conditions.

In addition, Model 124 presents a global exceedance rate of 1.04%, the closest to the theoretical 1% across all 152 configurations evaluated. This reinforces the model's robustness by demonstrating that it neither systematically underestimates nor overstates tail risk. The combination of stable annual performance and accurate global calibration substantiates its selection as the most suitable model for forward looking VaR management. Furthermore, Model 124 satisfied the UC test across all individual years in the evaluation period, confirming the temporal stability of its risk forecasts.

Accordingly, Model 124 is selected as the optimal VaR specification for this study. The following section applies this model to assess and manage portfolio risk under both unhedged and hedged strategies throughout the one-year out-of-sample period.

# Chapter 7.

## Value-at-Risk Management

As established in the previous chapter, the model selected for the measurement and management of VaR is Model 124. This configuration corresponds to a Quantile Regression approach, incorporating volatility estimates from an EWMA model with a smoothing parameter $\lambda = 0.94$ and a historical sample size of 500 observations. The model is applied under two distinct strategies, both evaluated over a one-year horizon.

The first strategy is passive, allowing the portfolio to evolve without any form of risk mitigation. In contrast, the second strategy enforces a daily VaR constraint, serving as a trigger for active risk control. Whenever the estimated VaR exceeds the predefined limit, a hedging mechanism is activated. This involves reducing exposure to the risk factors that contribute most significantly to total portfolio risk, as identified via Marginal VaR, through the use of futures contracts.

In both scenarios, bond coupon payments are reinvested as they are received throughout the year. Reinvestment is conducted by proportionally adjusting both long and short equity positions in line with their respective portfolio weights on the trading day immediately preceding each coupon payment. This ensures that the portfolio's relative equity exposure remains consistent over time, preserving its intended risk structure. Further details regarding the timing and allocation of these reinvestments are provided in Appendix B.

During the backtesting period, the portfolio's daily VaR typically ranged between €100 000 and €200 000, with pronounced spikes during the COVID-19 crisis (2020–2022). Based on this historical pattern and given that the portfolio value on 27 January 2023 was approximately €9.5 million, the maximum acceptable daily VaR is set at 1.4% of the portfolio value, corresponding to €133 000.

This chapter is organized into three sections. It begins by outlining the methodology for decomposing VaR by risk factor. It then describes the hedging strategy implemented when the VaR threshold is breached. Finally, it presents a comparative analysis of the portfolio's performance under the hedged and unhedged strategies, focusing on their respective impacts on risk-adjusted returns.

### 7.1 VaR decompositions

To support a more effective risk management framework, VaR is computed at the level of individual risk factors. This allows for the decomposition of total portfolio VaR, enabling the identification and monitoring of the most significant sources of risk.

This analysis employs the Marginal Value-at-Risk (Marginal VaR), which quantifies the contribution of a specific subset of risk factor exposures to the portfolio's total VaR. This metric is central to the hedging strategy, as it informs both the selection of risk factors to hedge and the sizing of the corresponding hedge positions.

Formally, let $\Theta$ denote the vector of portfolio exposures to all risk factors, and $\Theta_s$ represent a specific subset (or "slice") of these exposures. The Marginal VaR associated with $\Theta_s$ is defined as:

$$Marginal\ VaR^S = \nabla f(\Theta)^T \Theta^S = \sum_{i=1}^{n} \frac{\partial VaR}{\partial \theta_i} \times \Theta_i^S \tag{36}$$

Here $\nabla f(\Theta)$ denotes the gradient vector of the VaR function with respect to the risk factor loadings, capturing the sensitivity of portfolio VaR to marginal changes in each individual exposure. This gradient is expressed as:

$$\nabla f(\Theta) = \frac{\partial f(\Theta)}{\partial \Theta} = \begin{bmatrix} \frac{\partial VaR}{\partial \theta_1} \\ \vdots \\ \frac{\partial VaR}{\partial \theta_n} \end{bmatrix} \tag{37}$$

From an economic perspective, the Marginal VaR provides insight into how sensitive the portfolio's total risk is to incremental changes in each risk factor exposure. A higher Marginal VaR for a given factor indicates that a slight increase in exposure would result in a disproportionately significant increase in overall portfolio risk.

Thus, Marginal VaR serves both as a diagnostic tool for identifying the primary drivers of risk and as a quantitative foundation for targeted hedging. By capturing the marginal impact of each exposure, this decomposition framework enables more precise and effective risk control interventions.

### 7.2. Hedging Strategy

The hedging strategy implemented in this study is governed by a predefined maximum daily VaR threshold, which serves as the trigger for activating risk mitigation measures. This threshold was established based on the distribution of historical daily VaR estimates generated during the backtesting period.

Rather than relying on the overall average or on extreme values, the threshold was determined using the average of the VaR values at the 33rd and 67th percentiles of the ordered VaR observations. This approach mitigates the influence of outliers and non-representative values at both ends of the distribution, producing a more robust and informative threshold. The resulting value, €132 069, was rounded to €133 000 to enhance interpretability and practicality.

This level reflects the typical risk range experienced under standard market conditions and provides a consistent and disciplined criterion for triggering the hedging mechanism.

#### 7.2.1 Hedging Decision Framework

The hedging strategy implemented in this study aims to ensure that the portfolio's daily Value-at-Risk (VaR) remains below the predefined Economic Capital (EC) threshold of €133 000. Whenever the estimated VaR exceeds this limit, a systematic risk mitigation procedure is triggered. The intervention involves opening hedging positions via equity index futures, thereby reducing the portfolio's exposure to the most risk-contributing market factors.

This strategy follows a parsimonious and rule-based framework, designed to promote operational efficiency and empirical robustness. Rather than relying on discretionary judgement or static asset weights, the hedging mechanism is guided by a daily Marginal VaR decomposition of the portfolio. On each day when the VaR estimate breaches the EC threshold, the Marginal VaR of each equity index is computed. These values, representing the partial derivatives of total VaR with respect to each index exposure, quantify the marginal contribution of each index to the portfolio's overall market risk.

To translate these contributions into actionable hedging weights, each positive Marginal VaR is expressed as a proportion of the sum of all positive marginal contributions. These weights are then used to allocate notional exposure across futures contracts, ensuring that the hedge specifically targets the most significant sources of risk. Indices with negative marginal contributions are excluded from the hedge, as neutralizing such exposures would eliminate their natural diversification benefit and could unintentionally increase total portfolio risk.

The hedge is implemented by establishing long or short positions in index futures, in proportion to each index's relative marginal risk contribution. The use of futures contracts ensures liquidity, low transaction costs, and scalability, rendering the approach suitable for dynamic and responsive risk control.

Additionally, the portfolio includes fixed-income instruments that generate periodic coupon payments. To preserve the intended equity allocation and avoid structural distortions in risk exposure over time, all received coupons are reinvested proportionally across existing equity holdings, based on their relative portfolio weights on the trading day immediately prior to each payment. This rule-based reinvestment mechanism ensures that the strategic allocation and overall risk profile of the portfolio remain stable throughout the evaluation period.

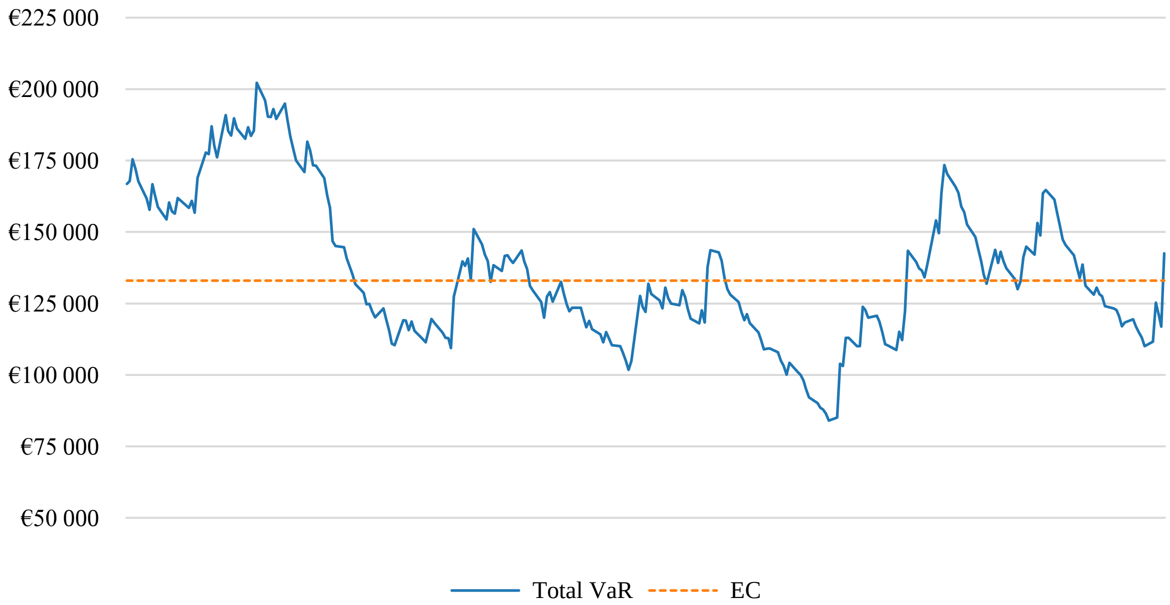


**Figure 7.1. Evolution of the unhedged daily VaR.**

As shown in Figure 7.1, which plots the evolution of the unhedged daily VaR throughout the one-year evaluation period, the portfolio frequently approached or surpassed the predefined EC threshold of €133 000. In total, the hedging strategy was activated 124 times, representing nearly half of the trading days.

The following section provides a detailed breakdown of the Marginal VaR decomposition observed on each intervention day, beginning with the first instance on 30 January 2023.

### 7.2.2 Marginal VaR Decomposition

Throughout the one-year evaluation period, the portfolio's daily VaR was estimated using the selected Quantile Regression model (Model 124). Each VaR estimate was systematically compared to the predefined EC threshold of €133 000. When this threshold was exceeded, the proportional hedging strategy described in Section 7.2.1 was activated.

On 30 January 2023, the portfolio's estimated daily VaR reached €166 827.21, exceeding the EC threshold and triggering the first hedging intervention. In line with the framework, the initial step consisted of decomposing the total unhedged VaR through a Marginal VaR analysis. This process identifies the risk factor components contributing most significantly to total portfolio risk, serving as the empirical basis for calibrating the hedge.

| **Risk Factor Type** | **Equity** | | | | | | **Currency** | | **Interest Rate** | |
|---|---|---|---|---|---|---|---|---|---|---|
| Marginal VaR (EUR) | 93 641.40 | | | | | | 41 332.87 | | 31 852.94 | |
| Marginal VaR (%) | 56.13% | | | | | | 24.77% | | 19.09% | |
| **Risk Factor Group** | **FCHI** | **GSPC** | **IXIC** | **GDAXI** | **DJI** | **HSI** | **USDEUR** | **HKDEUR** | **IR_EUR** | **IR_USD** |
| Marginal VaR (EUR) | 9 824.95 | 31 726.42 | 45 513.66 | -4 091.93 | 10 835.74 | -167.44 | 41 903.81 | - 570.94 | 17 993.70 | 13 859.24 |
| Marginal VaR (%) | 5.89 % | 19.02 % | 27.28 % | - 2.45 % | 6.50 % | -0.10 % | 25.12 % | - 0.34 % | 10.79 % | 8.31 % |

**Table 7.1. Marginal VaR decomposition by risk factor**. Reporting the contribution of each factor to the portfolio's total VaR on 30 January 2023.

The decomposition by risk factor group provides a more granular perspective on the underlying risk architecture of the portfolio. On this specific date, equity exposures represented over half of the total VaR, with the Nasdaq Composite (IXIC) and the S&P 500 (GSPC) jointly accounting for a substantial share. The S&P 500, in particular, contributes not only to equity risk but also introduces currency exposure, as futures positions in this index are denominated in U.S. dollars. Consequently, interventions involving this instrument simultaneously affect the portfolio's sensitivity to exchange rate fluctuations, specifically the USDEUR pair. This interaction underscores the importance of accounting for cross-factor linkages in an integrated risk management process.

The proportional hedging strategy was implemented using the Marginal VaR contributions as a basis for constructing notional weights. On each intervention date, the Marginal VaR of each equity index was normalised by the sum of positive Marginal VaRs, yielding a set of weights used to distribute notional exposure across index futures. Indices with negative marginal contributions were excluded from the hedge, as their exposures functioned as natural diversifiers that reduced the total risk. This selective approach ensured that the hedge targeted only risk-enhancing factors while preserving the integrity of the risk decomposition.

Interventions occurred exclusively in response to material risk deviations, minimising unnecessary trading and maintaining the portfolio’s structural stability during periods of moderate volatility. On activation dates such as 30 January 2023, the Marginal VaR allocation mechanism ensured that the hedge was directed towards the most influential sources of market risk. This dynamic and empirically grounded approach consistently restored the portfolio’s VaR to within the acceptable bound while preserving its responsiveness to evolving risk conditions.

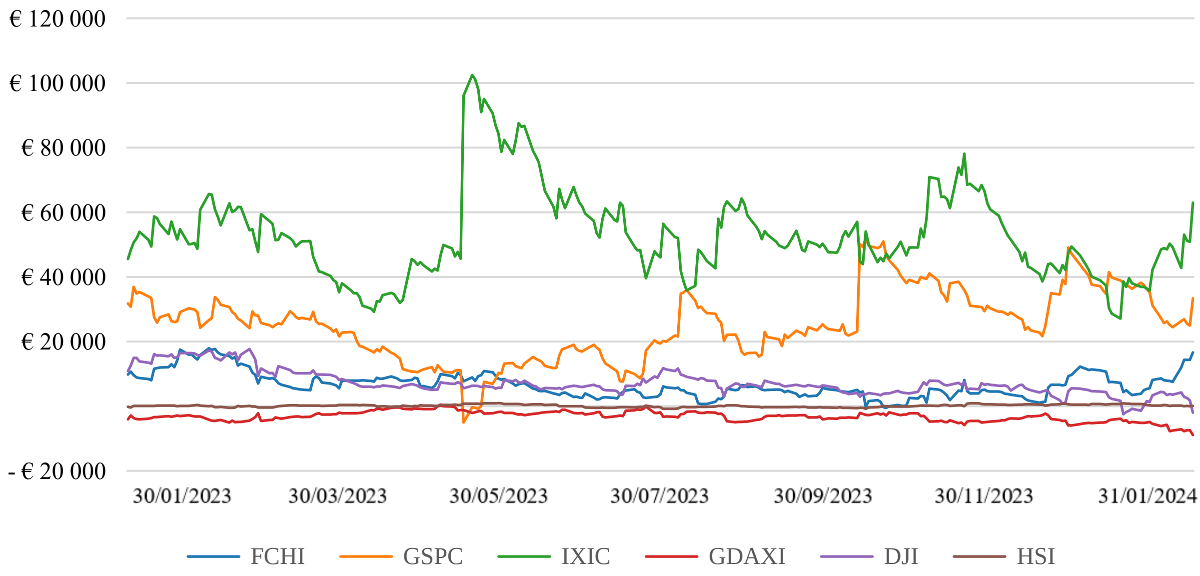


**Figure 7.2. Evolution of daily notional exposures by index.**

The proportional distribution of notional exposures across the six equity indices was derived from the Marginal VaR decomposition of the unhedged portfolio. As shown in Figure 7.2, indices with higher relative contributions to total risk, such as the Nasdaq Composite (IXIC) and the S&P 500 (GSPC), were allocated larger hedge weights. This outcome reflects the strategy's objective of concentrating risk mitigation efforts on the primary sources of systematic market risk. Moreover, the inclusion of futures contracts denominated in foreign currencies, particularly those linked to the U.S. dollar, introduced an additional layer of currency exposure. This interaction highlights the importance of implementing integrated risk management strategies capable of capturing and addressing cross-risk factor dynamics within multi-asset portfolios.

### 7.3. Value-at-Risk Management Results

This subchapter presents a comparative analysis of the portfolio's performance under two distinct risk management approaches. The first corresponds to the original, unmanaged configuration, referred to as the Unhedged Portfolio, while the second incorporates the proportional hedging strategy developed in this study, referred to as the Hedged Portfolio. The assessment focuses on three key dimensions: the evolution of Value-at-Risk, the daily profit and loss, and overall performance indicators. Particular attention is given to the cumulative impact of the hedging interventions, with the aim of evaluating their effectiveness in mitigating downside risk and improving risk-adjusted returns.

Figure 7.3 displays the daily VaR estimates for both portfolio configurations over the one-year out-of-sample evaluation period. The Unhedged Portfolio represents a passive approach without any risk control mechanism, whereas the Hedged Portfolio reflects the implementation of the rule-based strategy derived from the daily Marginal VaR decomposition.

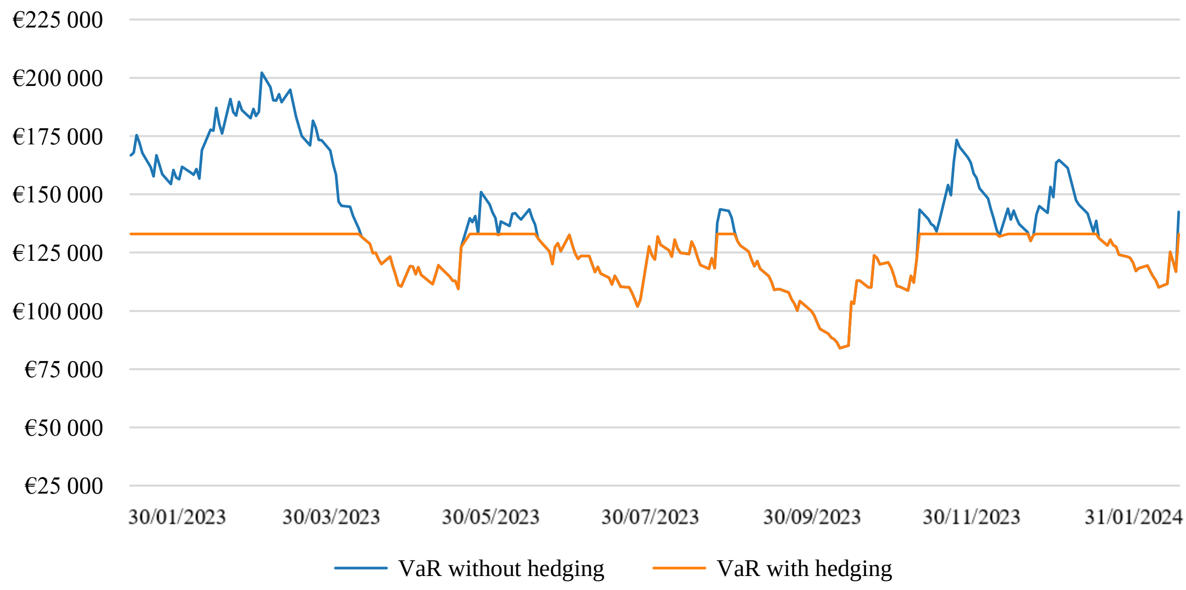


**Figure 7.3. Daily VaR of the portfolio with hedging and without hedging.**

As illustrated, the comparison reveals a clear divergence in risk exposure between the two strategies. In the absence of hedging, the portfolio's VaR exhibits greater volatility and reaches a peak exceeding €202 000 on 17 March 2024. This value corresponds to an increase of approximately 52 percent relative to the predefined EC threshold of €133 000, highlighting the potential for significant risk accumulation when no mitigation mechanism is applied.

Throughout the evaluation period, the hedging strategy was activated on 124 out of 265 trading days, demonstrating its responsiveness to shifts in market risk conditions. The consistent gap between the two VaR trajectories confirms the framework's ability to contain risk within acceptable boundaries. These findings support the effectiveness of the proposed methodology and reinforce its practical relevance for real-time portfolio risk management under varying market conditions.

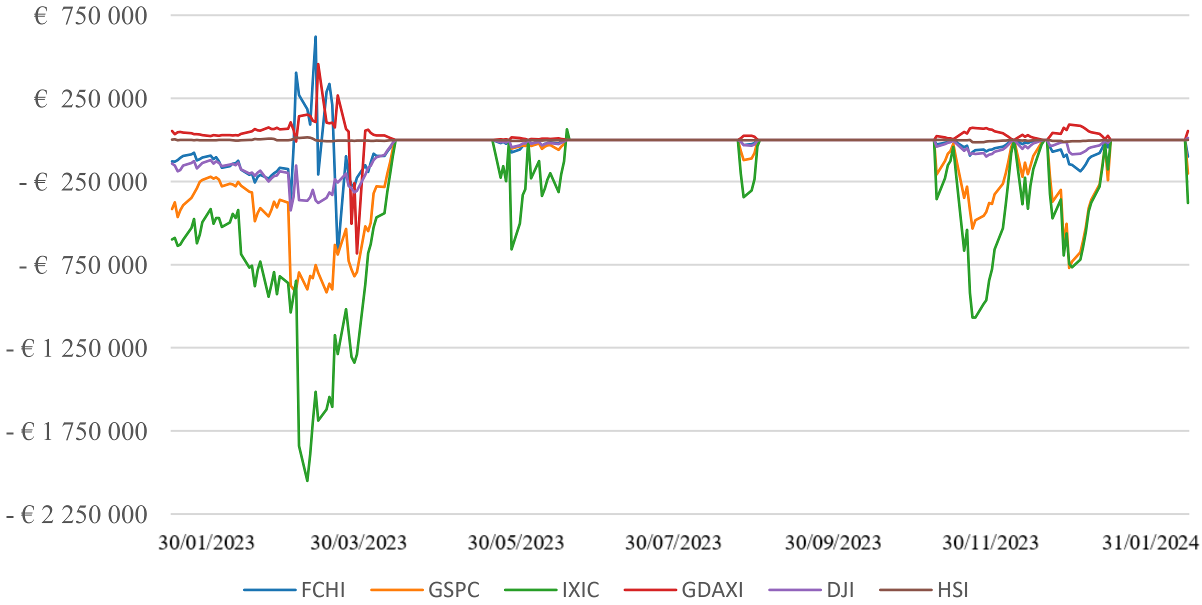


**Figure 7.4. Daily notional values of hedging positions.** The exposures reflect the proportional allocation derived from the Marginal VaR decomposition, ensuring daily alignment between hedge structure and the portfolio's evolving risk profile.

As illustrated in Figure 7.4, the daily notional values of the hedging positions established throughout the evaluation period reflect the application of the proportional allocation methodology introduced in Section 7.2.2. This approach, grounded in the Marginal VaR decomposition of the unhedged portfolio, ensures that the hedge dynamically targets the most significant sources of market risk. The structure of these hedging interventions, shown in detail in Figure 7.2, directly informs the distribution of notional exposure over time. Accordingly, Figure 7.4 provides a visual confirmation of the consistent and risk-sensitive implementation of the strategy, highlighting the alignment between the theoretical framework and its practical execution.

In addition to its role in risk reduction, the effectiveness of the hedging strategy must also be evaluated through its influence on portfolio returns. While the Value-at-Risk metric captures potential downside risk, it does not convey the realised financial impact of the strategy. For this reason, the analysis now shifts to the portfolio's daily profit and loss, which complements the risk-based assessment by reflecting actual market performance. The P&L series incorporates both market-driven fluctuations and the effects of the hedging decisions made throughout the evaluation period. This approach provides a more complete understanding of how the strategy contributed to the portfolio's return dynamics and offers further insight into its practical value under real-world conditions.

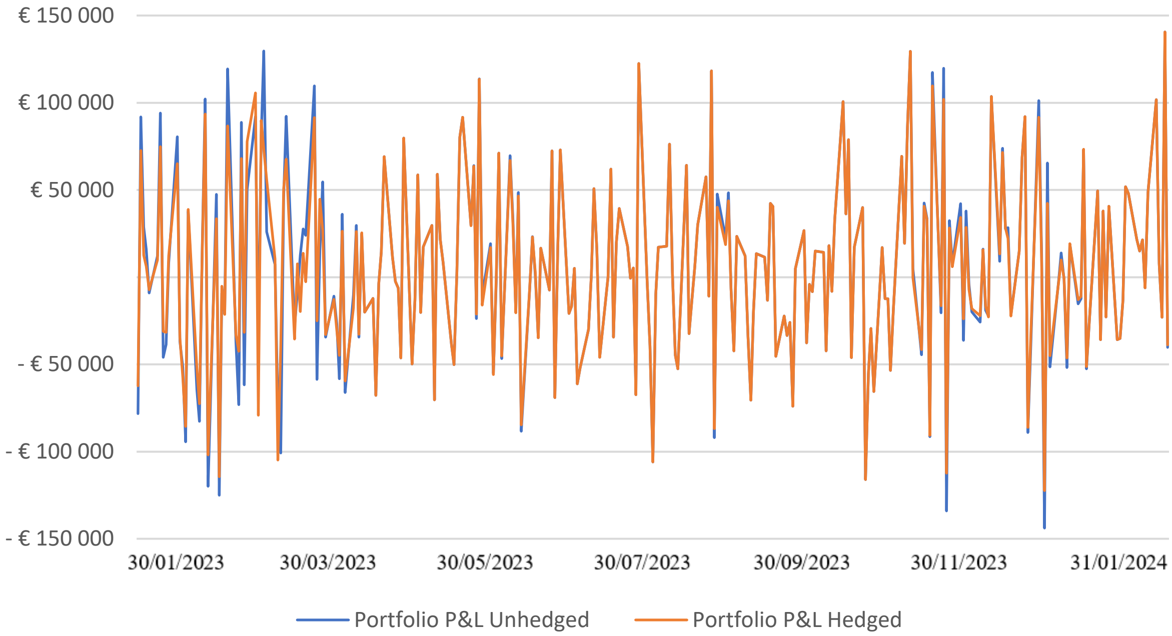


**Figure 7.5. Daily P&L of the portfolio with and without hedging.**

As shown in Figure 7.5, the impact of the hedging strategy on the portfolio's daily P&L exhibits considerable variation throughout the one-year period. On certain days, the implementation of the hedge improves performance relative to the unhedged configuration, while on others it results in a marginal reduction in returns. This asymmetrical effect aligns with the fundamental objective of hedging, which is primarily to reduce downside risk, even if it occasionally limits upside potential.

Specifically, on days when the portfolio registers negative P&L, the hedge typically acts as a buffer, reducing the magnitude of losses. Conversely, on days characterised by favourable market movements, the hedge may constrain performance, as the protective positions limit the full capture of positive returns. This trade-off between risk mitigation and return optimisation is inherent to the structure of the strategy and highlights the need for a balanced and context-aware approach to risk management.

This asymmetry is substantiated by the results in Table 7.2, which summarises the distribution of daily P&L differences between the hedged and unhedged portfolios. The hedge contributed positively in 73.8% of the days when the unhedged portfolio recorded losses, while only 23.8% of days with positive unhedged P&L showed a performance gain with hedging.

These figures confirm that the strategy effectively dampens adverse outcomes at the cost of limited upside capture.

| Statistics | P&L hedged – P&L unhedged | | |
|---|---|---|---|
| | Total | When P&L > 0 | When P&L < 0 |
| Number of days | 124 | 63 | 61 |
| Positive Difference (%) | 48.40% | 23.80% | 73.80% |
| Average (EUR) | - 451.11 | - 5 941.86 | 5 219.67 |
| Median (EUR) | - 98.42 | - 3 471.66 | 2 583.69 |
| Maximum (EUR) | 61 463.04 | 29 846.14 | 61 463.04 |
| Minimum (EUR) | - 56 363.71 | - 56 363.71 | - 36 905.01 |

**Table 7.2. P&L differences between hedged and unhedged portfolios**

To further assess the net impact of the hedging strategy on portfolio performance, the next step involves analysing the difference in daily P&L between the hedged and unhedged configurations. This differential provides a direct measure of the incremental effect of the risk mitigation process, highlighting the days on which the hedge contributed positively or negatively to the portfolio's outcome.

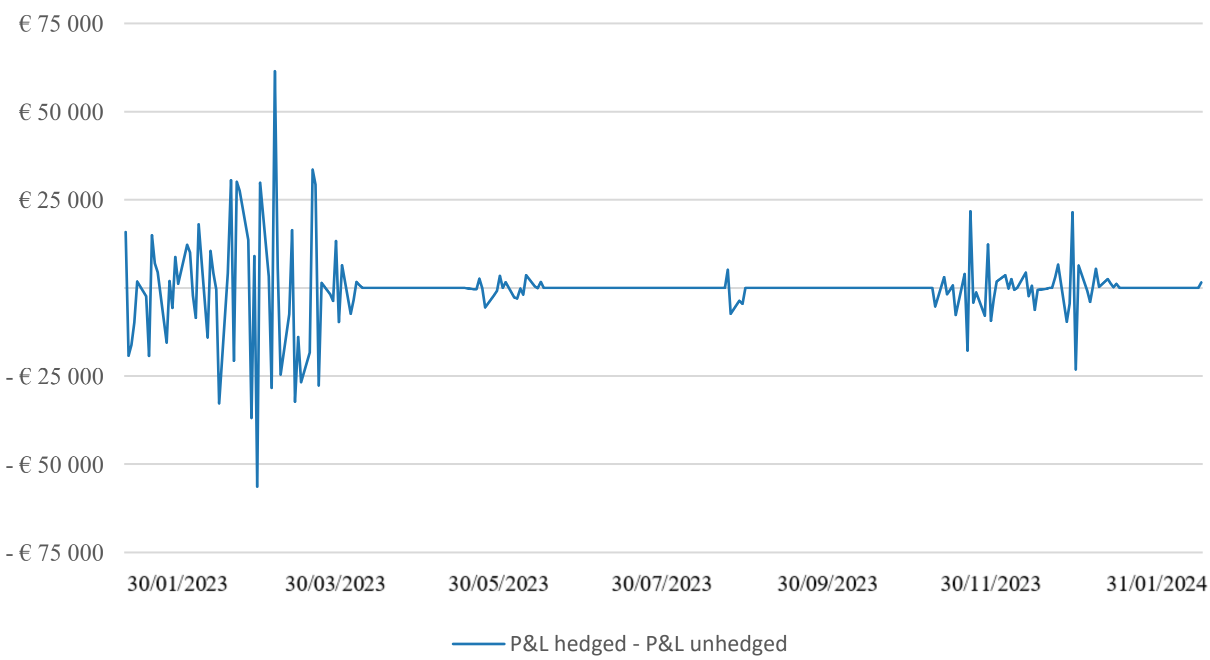


**Figure 7.6. Daily differences in P&L between the hedged and unhedged portfolios.** Positive values indicate days when the hedging strategy reduced losses or enhanced gains, while negative values reflect a reduction in returns due to risk mitigation.

The most substantial positive contribution of the hedging strategy occurred on 22 March 2023, when it improved the portfolio's daily P&L by approximately €61 000 relative to the unhedged configuration. On that day, the unhedged portfolio recorded a loss of €101 000, whereas the hedged portfolio registered a profit of €39 000. This reversal illustrates the effectiveness of the hedging mechanism in absorbing severe market shocks and transforming potential losses into gains. This outcome highlights the strategy's capacity to mitigate adverse market movements and preserve portfolio value during periods of heightened volatility. Conversely, the most pronounced negative effect occurred on 16 March 2023, when the hedged portfolio underperformed the unhedged configuration by nearly €56 000. On that day, the unhedged portfolio achieved a profit of €129 732, while the hedged portfolio returned €73 368. This divergence illustrates the inherent trade-off of risk reduction strategies, which may constrain upside capture in pursuit of greater stability during downturns.

Focusing on the 124 trading days in which the hedging strategy was actively deployed, the average daily difference in P&L between the hedged and unhedged portfolios was - €451.11, while the median stood at - €98.42. These figures suggest that, although the hedge was effective in containing extreme losses, it was associated with a modest reduction in daily profitability. This result is consistent with the primary objective of the strategy, which prioritises risk containment rather than return maximisation. The observed asymmetry reinforces the notion that effective risk management often requires a deliberate trade-off between potential gains and increased stability under adverse market conditions.

Building upon the P&L comparison, the next step in evaluating the hedging strategy involves analysing the behaviour of exceedances. These are instances in which the actual daily loss surpasses the VaR estimate. This metric provides a direct assessment of model performance and the effectiveness of risk control. By comparing the frequency and timing of exceedances under the hedged and unhedged configurations, it becomes possible to quantify the extent to which the strategy succeeded in keeping portfolio losses within the predicted bounds. This analysis complements the preceding performance evaluation by shifting the focus from return outcomes to risk containment.

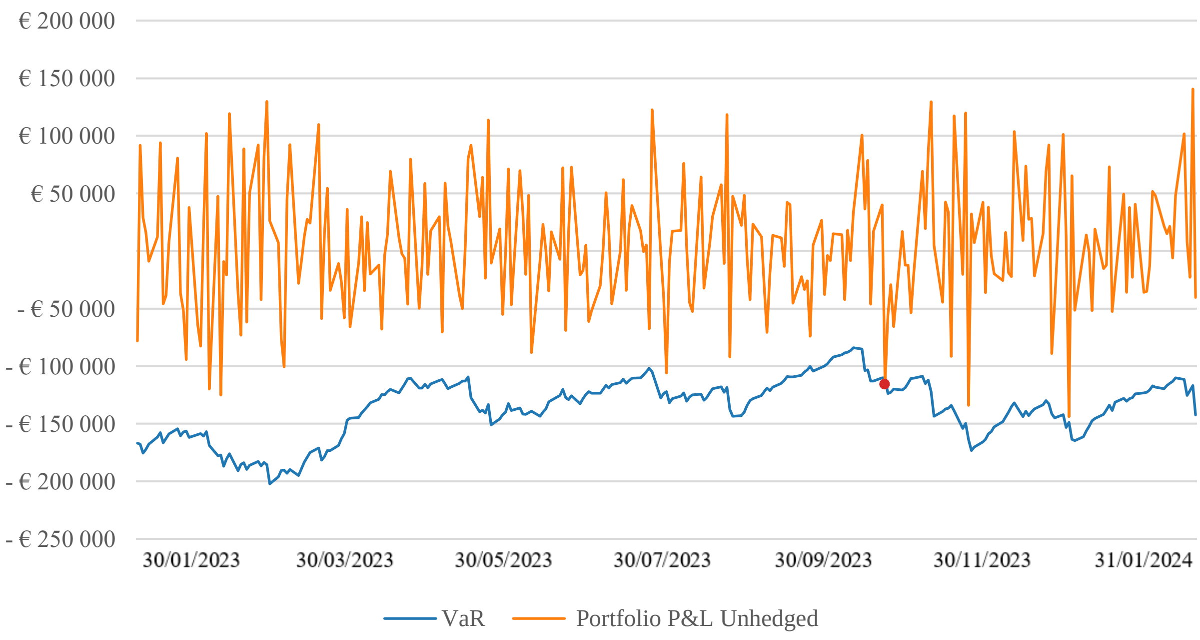


**Figure 7.7. Daily exceedances of the unhedged portfolio**. Observations below the VaR line indicate instances where actual losses exceeded predicted risk.

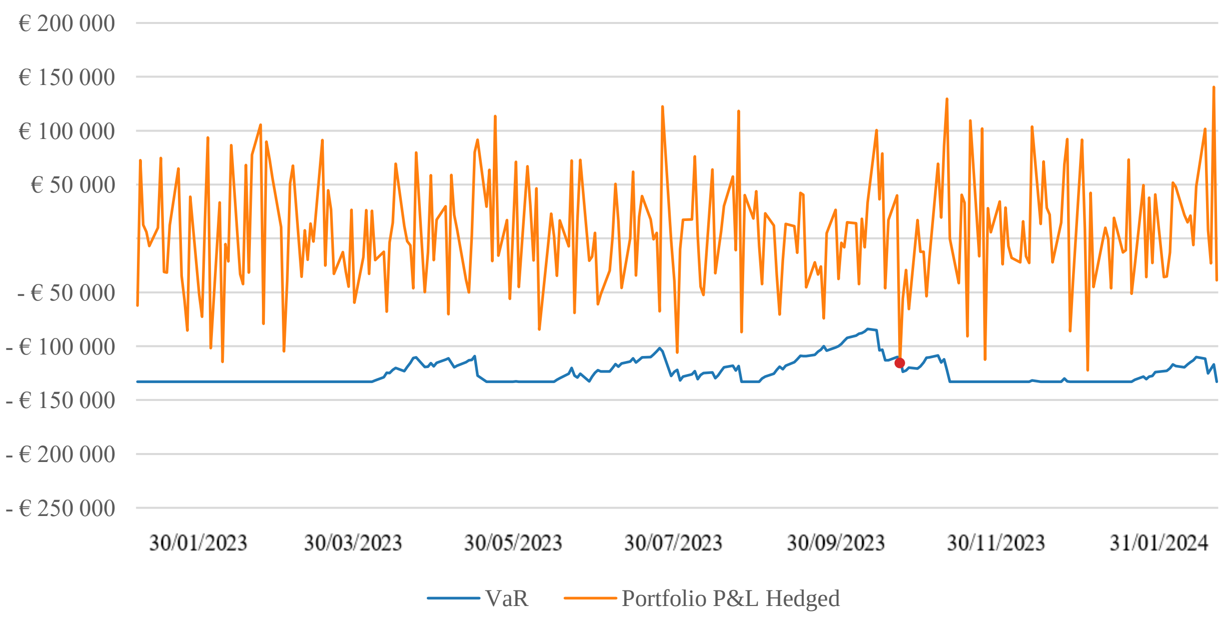


**Figure 7.8. Daily exceedances of the hedged portfolio.** Observations below the VaR line indicate instances where actual losses exceeded predicted risk.

Upon examining the exceedance profiles of both portfolio configurations, it is observed that a single exceedance occurred in each case on 17 October 2023. On that day, the actual loss exceeded the estimated VaR, which stood at €110 105.27 for both the hedged and unhedged portfolios. Notably, this exceedance did not trigger a hedging intervention, as the portfolio's VaR remained below the predefined Economic Capital threshold of €133 000. This outcome reinforces the design of the framework, which activates hedging strictly in response to material risk levels. It also highlights the importance of distinguishing between statistical model deviations and economically significant risk events when evaluating the effectiveness of risk control mechanisms.

To deepen the evaluation of the hedging strategy, the following analysis integrates both VaR and P&L metrics for the unhedged and hedged portfolio configurations. This joint perspective allows for a more nuanced assessment of the strategy's ability not only to mitigate losses but also to stabilise returns in varying market conditions.

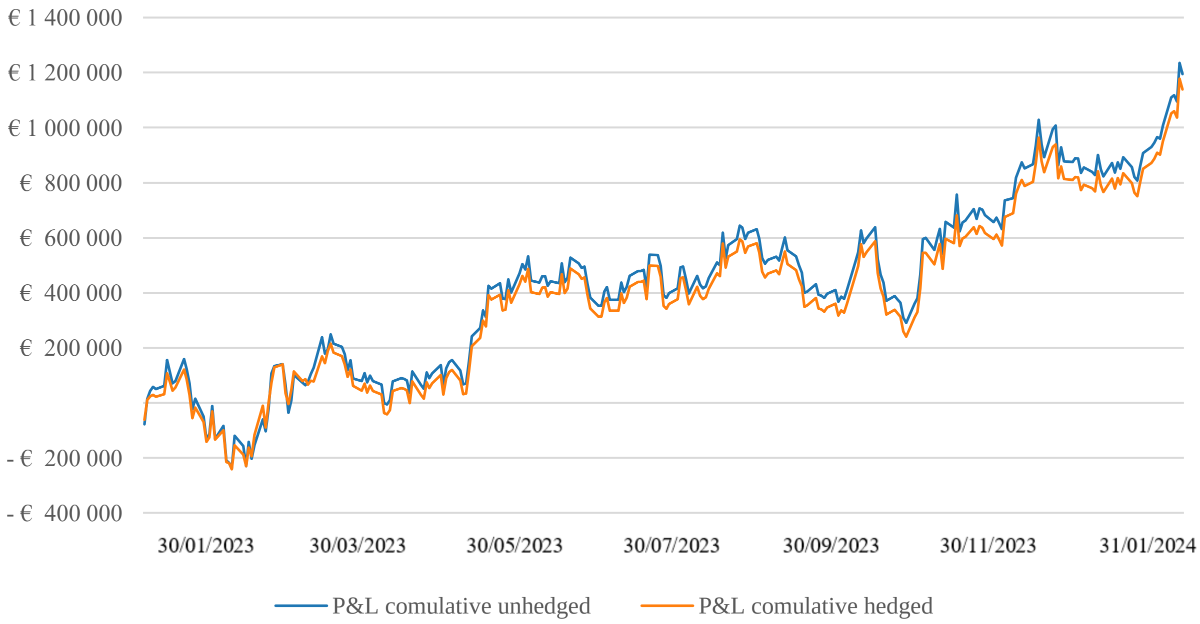


**Figure 7.9. Cumulative P&L of Hedged vs. Unhedged Portfolio**

A comparative analysis of the cumulative P&L trajectories indicates that, although both portfolios concluded the one-year evaluation period with positive returns, the hedged portfolio generated a lower total profit relative to the unhedged configuration. This outcome exemplifies the inherent trade-off in risk mitigation strategies: while hedging serves to attenuate extreme losses, it can also constrain upside potential by dampening exposure to favourable market movements.

Nevertheless, this performance gap must be interpreted in the broader context of portfolio volatility and downside risk. Throughout the year, the hedged portfolio exhibited lower return volatility, fewer pronounced drawdowns, and closer alignment with VaR estimates. These features underscore the strategy's effectiveness in achieving its primary objective of enhancing risk control and promoting return stability. In this sense, the results reinforce the strategic role of hedging in containing tail risk and protecting portfolio value during periods of heightened uncertainty.

To complement this analysis, the Return on Risk-Adjusted Capital (RORAC) is adopted as the key metric for evaluating performance efficiency. RORAC captures the relationship between the return generated by a portfolio and the risk capital deployed to support it, offering an integrated view of profitability relative to risk exposure. By accounting for both return generation and downside potential, this measure enables a robust comparison between the hedged and unhedged strategies, particularly under conditions of financial uncertainty.

Formally, RORAC is computed as the ratio between the portfolio's daily profit and loss (P&L) and the Value-at-Risk (VaR) estimated for the same day:

$$RORAC = \frac{P\&L}{VaR} \tag{38}$$

where P&L denotes the portfolio's daily profit and loss, and VaR corresponds to the Value-at-Risk estimated for the same day. The analysis begins on 30 January 2023 and spans the full one-year out-of-sample period.

To ensure a comprehensive evaluation, two complementary methodologies are used. The first method calculates the average daily RORAC from Equation (38), capturing the consistency of risk-adjusted performance throughout the evaluation period. The second method derives the ratio between the average daily P&L and the average daily EC, providing a high-level aggregate indicator of return efficiency. Together, these perspectives allow for a robust comparison between the hedged and unhedged strategies in terms of their economic performance under uncertainty.

$$Average\ RORAC^{(1)} = \frac{Average\ P\&L}{Average\ EC} \tag{39}$$

$$Average\ RORAC^{(2)} = \frac{1}{n}\sum_{t=1}^{n}\frac{P\&L_t}{EC_t} \tag{40}$$

These two expressions represent the alternative approaches used to quantify average risk-adjusted return. Equation (39) relies on aggregated means, whereas Equation (40) accounts for daily fluctuations in both return and risk.

Table 7.3 summarises the risk-adjusted performance results for both the hedged and unhedged portfolios, using the two RORAC methodologies described previously. The table reports the total profit and EC over the one-year period, as well as their respective daily averages.

| | Unhedged | Hedged |
|---|---|---|
| P&L (€) | 1 194 786.56 | 1 138 849.38 |
| Average P&L | 4 508.63 | 4 297.54 |
| EC (€) | 36 051 784.97 | 32 970 711.74 |
| Average EC | 136 044.47 | 124 417.78 |
| Average RORAC (1) | 3.31% | 3.45% |
| Average RORAC (2) | 3.48% | 3.76% |

**Table 7.3. Summary of performance and risk-adjusted efficiency.** RORAC (1) corresponds to the ratio of total P&L to total EC, while RORAC (2) reflects the average of daily RORAC values over the evaluation period.

Based on the results presented in Table 7.3, it becomes evident that the hedged portfolio outperforms the unhedged configuration in terms of capital efficiency across both risk-adjusted performance metrics. Under the first methodology, RORAC (1), defined as the ratio between total profit and total Economic Capital (EC), the hedged portfolio achieves a value of 3.45 per cent, exceeding the 3.31 per cent recorded for the unhedged counterpart. This indicates that, over the full investment horizon, the hedged strategy generated more return per unit of capital committed to absorbing risk.

A similar conclusion is reached under the second metric, RORAC (2), which reflects the average of daily risk-adjusted returns throughout the year. In this case, the hedged portfolio attains a value of 3.76 per cent, clearly outperforming the 3.48 per cent achieved by the unhedged version. This result highlights not only greater capital efficiency at the aggregate level but also improved consistency in daily risk-adjusted performance.

It is important to note that the hedging framework intentionally excluded indices with negative marginal contributions to Value-at-Risk. By doing so, it preserved natural diversification effects and avoided unnecessary offsetting of risk-reducing exposures. This conceptual refinement contributed to the robustness of the strategy without compromising performance.

Taken together, these findings support the view that dynamic hedging based on Marginal VaR decomposition can enhance portfolio performance by improving the efficiency with which risk capital is allocated. In investment contexts where downside protection and capital preservation are prioritised, such outcomes justify the integration of proactive risk control mechanisms.

# Chapter 8.

# Conclusion

This dissertation set out to assess and manage the Value-at-Risk (VaR) of a diversified portfolio, ensuring that daily risk exposure remained within a predefined Economic Capital (EC) threshold of €133 000. The portfolio included both equity and fixed-income instruments, representing a broad cross-section of major developed markets. The methodology employed throughout the study prioritised empirical rigour and practical applicability, focusing on selecting, validating, and operationalising VaR models capable of supporting dynamic risk control interventions.

A comprehensive evaluation of 152 model configurations across four major classes (Normal, SGSt, Historical Simulation and Quantile Regression) highlighted the inadequacy of assuming normality in financial return distributions. Parametric models based on Gaussian assumptions systematically failed standard backtesting procedures, while approaches involving strict factor mappings introduced distortions that reduced forecast accuracy. Through a multi-stage process of statistical validation using the Unconditional Coverage (UC) and Berkowitz, Christoffersen, and Pelletier (BCP) tests, a Quantile Regression model with EWMA volatility (Model 124) emerged as the most robust configuration. This model delivered the most consistent exceedance rates across both global and annual subperiods, exhibiting strong alignment with the theoretical confidence level.

The selected model was subsequently embedded within a dynamic, proportional hedging strategy based on Marginal VaR decomposition. This approach enabled the daily identification of risk concentrations and guided the adjustment of hedging positions through equity index futures. Importantly, the strategy was only activated when the estimated VaR exceeded the EC threshold, ensuring operational efficiency and minimising trading frequency. Furthermore, the hedge was applied exclusively to indices with positive marginal contributions to portfolio VaR, thereby preserving the diversification benefits of negatively contributing exposures. A rule-based coupon reinvestment mechanism complemented this framework, maintaining a stable asset allocation and consistent risk profile throughout the evaluation period.

Empirical evidence showed that the unhedged portfolio frequently breached the VaR limit, with the most severe exceedance surpassing €225 000. In contrast, the hedged portfolio consistently maintained risk levels within the prescribed boundaries. While this came at the cost of a slight reduction in total nominal profit, the trade-off proved beneficial when viewed through the lens of risk-adjusted efficiency.

Analysis of cumulative P&L trajectories revealed that the hedged portfolio generated lower absolute returns but with reduced volatility and fewer drawdowns. Most importantly, the Return on Risk-Adjusted Capital (RORAC) results provided clear evidence of improved capital efficiency. Using the first method, defined as total profit divided by total EC, the hedged portfolio achieved a RORAC of 3.45 per cent, compared to 3.31 per cent for the unhedged configuration. Under the second approach, based on the average of daily RORAC values, the hedged strategy again outperformed, with a result of 3.76 per cent versus 3.48 per cent. These results demonstrate not only enhanced annual capital productivity but also more consistent daily performance.

In conclusion, this research demonstrates the practical effectiveness of a model-driven, dynamically executed risk management strategy. By combining rigorous model selection with a rule-based hedging mechanism grounded in Marginal VaR decomposition, the proposed framework stabilised portfolio risk, improved resilience to market shocks and enhanced the efficiency with which economic capital was deployed. These findings support the broader application of quantitative risk control methodologies in institutional portfolio management, particularly in environments characterised by uncertainty and multi-asset complexity.

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# Appendices

## Appendix A: Detailed Statistics

Appendix A. Summary Statistics of Portfolio P&L and Returns

| P&L | Mean | Median | Maximum | Minimum | Standard Deviation | Skewness | Kurtosis |
|---|---|---|---|---|---|---|---|
| 2013-2023 | 4 387.32 € | 5 838.73 € | 391 134.68 € | -526 714.99 € | 59 554.02 € | -0.269 | 6.445 |
| 2023 | 153.13 € | -8 695.81 € | 193 644.78 € | -117 390.19 € | 70 406.91 € | 0.984 | 2.028 |
| 2022 | -2 301.27 € | -2 490.23 € | 391 134.68 € | -273 484.85 € | 83 487.76 € | 0.241 | 1.432 |
| 2021 | 5 473.03 € | 5 109.42 € | 165 025.78 € | -166 931.91 € | 50 557.46 € | -0.145 | 1.107 |
| 2020 | 5 285.87 € | 11 271.25 € | 377 837.98 € | -526 714.99 € | 94 532.97 € | -0.528 | 6.657 |
| 2019 | 6 788.27 € | 5 696.49 € | 106 999.24 € | -115 490.84 € | 40 333.00 € | -0.384 | 0.415 |
| 2018 | 2 478.23 € | 5 785.36 € | 182 459.18 € | -180 954.69 € | 59 856.36 € | -0.193 | 0.766 |
| 2017 | 4 183.29 € | 7 987.38 € | 190 091.63 € | -123 846.45 € | 38 805.01 € | 0.097 | 2.127 |
| 2016 | 5 313.87 € | 9 383.73 € | 154 147.00 € | -169 874.51 € | 47 056.55 € | -0.423 | 0.833 |
| 2015 | 4 097.73 € | 2 404.77 € | 215 825.06 € | -231 444.20 € | 65 198.27 € | -0.175 | 1.440 |
| 2014 | 8 236.92 € | 9 356.81 € | 146 838.48 € | -131 772.15 € | 40 770.92 € | -0.235 | 1.542 |
| 2013 | 4 426.30 € | 1 599.01 € | 137 203.42 € | -109 051.80 € | 44 624.48 € | -0.01 | 0.082 |

**Appendix A. Descriptive Statistics of the P&L and return of the portfolio, during the global test period, from 11 February 2013 to 27 January 2023.**

## Appendix B: Portfolio Data

Appendix B. Portfolio value and asset allocation over time with coupon reinvestment

| Date | Bond ISIN | Coupon Reinvested (€) | Reinvestment Description | Equity Allocation | Bond Allocation | Portfolio value (€) |
|---|---|---|---|---|---|---|
| 27-01-2023 | - | - | Initial setup | 42 % | 58 % | 9 499 993.85 |
| 06-03-2023 | LU2591860569 | 19 800. 00 | Equity Reallocation | 43 % | 57 % | 9 394 280.76 |
| 17-05-2023 | US91282CFV81<br>US91282CJJ18 | 15 743.26<br>19 347.14 | Equity Reallocation | 43 % | 57 % | 9 619 910.67 |
| 06-07-2023 | DE0001135226 | 47 025.00 | Equity Reallocation | 47 % | 53 % | 10 068 759.37 |
| 21-10-2023 | DE000BU25018 | 12 000.00 | Equity Reallocation | 48 % | 52 % | 9 996 483.62 |
| 17-11-2023 | US91282CFV81<br>US91282CJJ18 | 15 736.58<br>19 338.93 | Equity Reallocation | 49 % | 51 % | 10 294 699.16 |
| 15-01-2024 | NL0000102317 | 60 500.00 | Equity Reallocation | 50 % | 50 % | 10 558 967.85 |

**Appendix B. Coupon reinvestment allocation over time**. Presents the evolution of the portfolio's value and asset class allocation during the out-of-sample period. Bond coupon proceeds were reinvested proportionally across existing equity positions to preserve the portfolio's risk structure, with fixed-income allocation adjusted accordingly. This rule-based approach ensured consistency in asset mix and supported the stability of the overall risk profile.

## Appendix C: VaR Models Details

Appendix C.1. Normal VaR Model configurations

| Model number | Model Class | EWMA Smoothing Factor |
|---|---|---|
| 1 | Normal | 0.9 |
| 2 | Normal | 0.905 |
| 3 | Normal | 0.91 |
| 4 | Normal | 0.915 |
| 5 | Normal | 0.92 |
| 6 | Normal | 0.925 |
| 7 | Normal | 0.93 |
| 8 | Normal | 0.935 |
| 9 | Normal | 0.94 |
| 10 | Normal | 0.945 |
| 11 | Normal | 0.95 |
| 12 | Normal | 0.955 |
| 13 | Normal | 0.96 |
| 14 | Normal | 0.965 |
| 15 | Normal | 0.97 |
| 16 | Normal | 0.975 |
| 17 | Normal | 0.98 |
| 18 | Normal | 0.985 |
| 19 | Normal | 0.99 |
| 20 | Normal | 0.995 |

**Appendix C.1. Overview of Normal VaR Models evaluated**.

Appendix C.2. SGSt VaR Model configurations

| Model number | Model Class | EWMA Smoothing Factor | Sample Size |
|---|---|---|---|
| 21 | SGSt | 0.92 | 250 |
| 22 | SGSt | 0.925 | 250 |
| 23 | SGSt | 0.93 | 250 |
| 24 | SGSt | 0.935 | 250 |
| 25 | SGSt | 0.94 | 250 |
| 26 | SGSt | 0.945 | 250 |
| 27 | SGSt | 0.95 | 250 |
| 28 | SGSt | 0.955 | 250 |
| 29 | SGSt | 0.96 | 250 |
| 30 | SGSt | 0.965 | 250 |
| 31 | SGSt | 0.97 | 250 |
| 32 | SGSt | 0.92 | 500 |
| 33 | SGSt | 0.925 | 500 |
| 34 | SGSt | 0.93 | 500 |
| 35 | SGSt | 0.935 | 500 |
| 36 | SGSt | 0.94 | 500 |
| 37 | SGSt | 0.945 | 500 |
| 38 | SGSt | 0.95 | 500 |
| 39 | SGSt | 0.955 | 500 |
| 40 | SGSt | 0.96 | 500 |
| 41 | SGSt | 0.965 | 500 |
| 42 | SGSt | 0.97 | 500 |
| 43 | SGSt | 0.92 | 750 |
| 44 | SGSt | 0.925 | 750 |
| 45 | SGSt | 0.93 | 750 |
| 46 | SGSt | 0.935 | 750 |
| 47 | SGSt | 0.94 | 750 |
| 48 | SGSt | 0.945 | 750 |
| 49 | SGSt | 0.95 | 750 |
| 50 | SGSt | 0.955 | 750 |
| 51 | SGSt | 0.96 | 750 |
| 52 | SGSt | 0.965 | 750 |
| 53 | SGSt | 0.97 | 750 |
| 54 | SGSt | 0.92 | 1 000 |
| 55 | SGSt | 0.925 | 1 000 |
| 56 | SGSt | 0.93 | 1 000 |
| 57 | SGSt | 0.935 | 1 000 |
| 58 | SGSt | 0.94 | 1 000 |
| 59 | SGSt | 0.945 | 1 000 |
| 60 | SGSt | 0.95 | 1 000 |
| 61 | SGSt | 0.955 | 1 000 |
| 62 | SGSt | 0.96 | 1 000 |
| 63 | SGSt | 0.965 | 1 000 |
| 64 | SGSt | 0.97 | 1 000 |

**Appendix C.2. Overview of SGSt VaR Models evaluated**.

Appendix C.3. Historical VaR Models configurations

| Model number | Model Class | EWMA Smoothing Factor | Sample Size |
|---|---|---|---|
| 65 | Historical | 0.92 | 250 |
| 66 | Historical | 0.925 | 250 |
| 67 | Historical | 0.93 | 250 |
| 68 | Historical | 0.935 | 250 |
| 69 | Historical | 0.94 | 250 |
| 70 | Historical | 0.945 | 250 |
| 71 | Historical | 0.95 | 250 |
| 72 | Historical | 0.955 | 250 |
| 73 | Historical | 0.96 | 250 |
| 74 | Historical | 0.965 | 250 |
| 75 | Historical | 0.97 | 250 |
| 76 | Historical | 0.92 | 500 |
| 77 | Historical | 0.925 | 500 |
| 78 | Historical | 0.93 | 500 |
| 79 | Historical | 0.935 | 500 |
| 80 | Historical | 0.94 | 500 |
| 81 | Historical | 0.945 | 500 |
| 82 | Historical | 0.95 | 500 |
| 83 | Historical | 0.955 | 500 |
| 84 | Historical | 0.96 | 500 |
| 85 | Historical | 0.965 | 500 |
| 86 | Historical | 0.97 | 500 |
| 87 | Historical | 0.92 | 750 |
| 88 | Historical | 0.925 | 750 |
| 89 | Historical | 0.93 | 750 |
| 90 | Historical | 0.935 | 750 |
| 91 | Historical | 0.94 | 750 |
| 92 | Historical | 0.945 | 750 |
| 93 | Historical | 0.95 | 750 |
| 94 | Historical | 0.955 | 750 |
| 95 | Historical | 0.96 | 750 |
| 96 | Historical | 0.965 | 750 |
| 97 | Historical | 0.97 | 750 |
| 98 | Historical | 0.92 | 1 000 |
| 99 | Historical | 0.925 | 1 000 |
| 100 | Historical | 0.93 | 1 000 |
| 101 | Historical | 0.935 | 1 000 |
| 102 | Historical | 0.94 | 1 000 |
| 103 | Historical | 0.945 | 1 000 |
| 104 | Historical | 0.95 | 1 000 |
| 105 | Historical | 0.955 | 1 000 |
| 106 | Historical | 0.96 | 1 000 |
| 107 | Historical | 0.965 | 1 000 |
| 108 | Historical | 0.97 | 1 000 |

**Appendix C.3. Overview of Historical VaR Models evaluated.**

Appendix C.4. Quantile Regression VaR Models configurations

| Model number | Model Class | EWMA Smoothing Factor | Sample Size |
|---|---|---|---|
| 109 | Quantile Regression | 0.92 | 250 |
| 110 | Quantile Regression | 0.925 | 250 |
| 111 | Quantile Regression | 0.93 | 250 |
| 112 | Quantile Regression | 0.935 | 250 |
| 113 | Quantile Regression | 0.94 | 250 |
| 114 | Quantile Regression | 0.945 | 250 |
| 115 | Quantile Regression | 0.95 | 250 |
| 116 | Quantile Regression | 0.955 | 250 |
| 117 | Quantile Regression | 0.96 | 250 |
| 118 | Quantile Regression | 0.965 | 250 |
| 119 | Quantile Regression | 0.97 | 250 |
| 120 | Quantile Regression | 0.92 | 500 |
| 121 | Quantile Regression | 0.925 | 500 |
| 122 | Quantile Regression | 0.93 | 500 |
| 123 | Quantile Regression | 0.935 | 500 |
| 124 | Quantile Regression | 0.94 | 500 |
| 125 | Quantile Regression | 0.945 | 500 |
| 126 | Quantile Regression | 0.95 | 500 |
| 127 | Quantile Regression | 0.955 | 500 |
| 128 | Quantile Regression | 0.96 | 500 |
| 129 | Quantile Regression | 0.965 | 500 |
| 130 | Quantile Regression | 0.97 | 500 |
| 131 | Quantile Regression | 0.92 | 750 |
| 132 | Quantile Regression | 0.925 | 750 |
| 133 | Quantile Regression | 0.93 | 750 |
| 134 | Quantile Regression | 0.935 | 750 |
| 135 | Quantile Regression | 0.94 | 750 |
| 136 | Quantile Regression | 0.945 | 750 |
| 137 | Quantile Regression | 0.95 | 750 |
| 138 | Quantile Regression | 0.955 | 750 |
| 139 | Quantile Regression | 0.96 | 750 |
| 140 | Quantile Regression | 0.965 | 750 |
| 141 | Quantile Regression | 0.97 | 750 |
| 142 | Quantile Regression | 0.92 | 1 000 |
| 143 | Quantile Regression | 0.925 | 1 000 |
| 144 | Quantile Regression | 0.93 | 1 000 |
| 145 | Quantile Regression | 0.935 | 1 000 |
| 146 | Quantile Regression | 0.94 | 1 000 |
| 147 | Quantile Regression | 0.945 | 1 000 |
| 148 | Quantile Regression | 0.95 | 1 000 |
| 149 | Quantile Regression | 0.955 | 1 000 |
| 150 | Quantile Regression | 0.96 | 1 000 |
| 151 | Quantile Regression | 0.965 | 1 000 |
| 152 | Quantile Regression | 0.97 | 1 000 |

**Appendix C.4. Overview of Quantile Regression VaR Models evaluated.**

## Appendix D: Backtesting Details

Appendix D.1. UC test results of the Total VaR

| Model number | Model Class | Exceedances | Exceedance rate (%) | p-value (%) |
|---|---|---|---|---|
| 1 | Normal | 64 | 2.46% | 0.00% |
| 2 | Normal | 65 | 2.50% | 0.00% |
| 3 | Normal | 65 | 2.50% | 0.00% |
| 4 | Normal | 65 | 2.50% | 0.00% |
| 5 | Normal | 63 | 2.42% | 0.00% |
| 6 | Normal | 61 | 2.35% | 0.00% |
| 7 | Normal | 58 | 2.23% | 0.00% |
| 8 | Normal | 58 | 2.23% | 0.00% |
| 9 | Normal | 59 | 2.27% | 0.00% |
| 10 | Normal | 56 | 2.15% | 0.00% |
| 11 | Normal | 55 | 2.12% | 0.00% |
| 12 | Normal | 54 | 2.08% | 0.00% |
| 13 | Normal | 52 | 2.00% | 0.00% |
| 14 | Normal | 50 | 1.92% | 0.00% |
| 15 | Normal | 49 | 1.88% | 0.01% |
| 16 | Normal | 49 | 1.88% | 0.01% |
| 17 | Normal | 51 | 1.96% | 0.00% |
| 18 | Normal | 51 | 1.96% | 0.00% |
| 19 | Normal | 47 | 1.81% | 0.02% |
| 20 | Normal | 52 | 2.00% | 0.00% |
| 21 | SGSt | 78 | 3.00% | 0.00% |
| 22 | SGSt | 75 | 2.88% | 0.00% |
| 23 | SGSt | 76 | 2.92% | 0.00% |
| 24 | SGSt | 76 | 2.92% | 0.00% |
| 25 | SGSt | 75 | 2.88% | 0.00% |
| 26 | SGSt | 75 | 2.88% | 0.00% |
| 27 | SGSt | 75 | 2.88% | 0.00% |
| 28 | SGSt | 76 | 2.92% | 0.00% |
| 29 | SGSt | 74 | 2.85% | 0.00% |
| 30 | SGSt | 68 | 2.62% | 0.00% |
| 31 | SGSt | 73 | 2.81% | 0.00% |
| 32 | SGSt | 42 | 1.62% | 0.38% |
| 33 | SGSt | 39 | 1.50% | 1.70% |
| 34 | SGSt | 39 | 1.50% | 1.70% |
| 35 | SGSt | 40 | 1.54% | 1.06% |
| 36 | SGSt | 41 | 1.58% | 0.64% |
| 37 | SGSt | 40 | 1.54% | 1.06% |
| 38 | SGSt | 41 | 1.58% | 0.64% |
| 39 | SGSt | 37 | 1.42% | 4.15% |
| 40 | SGSt | 37 | 1.42% | 4.15% |
| 41 | SGSt | 37 | 1.42% | 4.15% |
| 42 | SGSt | 37 | 1.42% | 4.15% |
| **43** | **SGSt** | **32** | **1.23%** | **25.37%** |
| **44** | **SGSt** | **33** | **1.27%** | **18.54%** |
| **45** | **SGSt** | **35** | **1.35%** | **9.20%** |
| **46** | **SGSt** | **32** | **1.23%** | **25.37%** |
| **47** | **SGSt** | **34** | **1.31%** | **13.22%** |
| **48** | **SGSt** | **34** | **1.31%** | **13.22%** |
| **49** | **SGSt** | **35** | **1.35%** | **9.20%** |
| **50** | **SGSt** | **35** | **1.35%** | **9.20%** |
| **51** | **SGSt** | **36** | **1.38%** | **6.25%** |

| | | | | |
|---|---|---|---|---|
| **52** | **SGSt** | **36** | **1.38%** | **6.25%** |
| **53** | **SGSt** | **32** | **1.23%** | **25.37%** |
| **54** | **SGSt** | **32** | **1.23%** | **25.37%** |
| **55** | **SGSt** | **33** | **1.27%** | **18.54%** |
| **56** | **SGSt** | **34** | **1.31%** | **13.22%** |
| **57** | **SGSt** | **35** | **1.35%** | **9.20%** |
| **58** | **SGSt** | **33** | **1.27%** | **18.54%** |
| **59** | **SGSt** | **33** | **1.27%** | **18.54%** |
| **60** | **SGSt** | **34** | **1.31%** | **13.22%** |
| **61** | **SGSt** | **35** | **1.35%** | **9.20%** |
| **62** | **SGSt** | **36** | **1.38%** | **6.25%** |
| **63** | **SGSt** | **34** | **1.31%** | **13.22%** |
| **64** | **SGSt** | **32** | **1.23%** | **25.37%** |
| 65 | Historical | 66 | 2.54% | 0.00% |
| 66 | Historical | 66 | 2.54% | 0.00% |
| 67 | Historical | 63 | 2.42% | 0.00% |
| 68 | Historical | 60 | 2.31% | 0.00% |
| 69 | Historical | 58 | 2.23% | 0.00% |
| 70 | Historical | 54 | 2.08% | 0.00% |
| 71 | Historical | 46 | 1.77% | 0.04% |
| 72 | Historical | 44 | 1.69% | 0.12% |
| 73 | Historical | 46 | 1.77% | 0.04% |
| 74 | Historical | 44 | 1.69% | 0.12% |
| 75 | Historical | 42 | 1.62% | 0.38% |
| 76 | Historical | 67 | 2.58% | 0.00% |
| 77 | Historical | 66 | 2.54% | 0.00% |
| 78 | Historical | 63 | 2.42% | 0.00% |
| 79 | Historical | 58 | 2.23% | 0.00% |
| 80 | Historical | 53 | 2.04% | 0.00% |
| 81 | Historical | 49 | 1.88% | 0.01% |
| 82 | Historical | 43 | 1.65% | 0.22% |
| 83 | Historical | 40 | 1.54% | 1.06% |
| 84 | Historical | 39 | 1.50% | 1.70% |
| 85 | Historical | 39 | 1.50% | 1.70% |
| 86 | Historical | 38 | 1.46% | 2.69% |
| 87 | Historical | 70 | 2.69% | 0.00% |
| 88 | Historical | 70 | 2.69% | 0.00% |
| 89 | Historical | 66 | 2.54% | 0.00% |
| 90 | Historical | 61 | 2.35% | 0.00% |
| 91 | Historical | 58 | 2.23% | 0.00% |
| 92 | Historical | 56 | 2.15% | 0.00% |
| 93 | Historical | 49 | 1.88% | 0.01% |
| 94 | Historical | 44 | 1.69% | 0.12% |
| 95 | Historical | 41 | 1.58% | 0.64% |
| 96 | Historical | 40 | 1.54% | 1.06% |
| 97 | Historical | 38 | 1.46% | 2.69% |
| 98 | Historical | 67 | 2.58% | 0.00% |
| 99 | Historical | 66 | 2.54% | 0.00% |
| 100 | Historical | 62 | 2.38% | 0.00% |
| 101 | Historical | 57 | 2.19% | 0.00% |
| 102 | Historical | 54 | 2.08% | 0.00% |
| 103 | Historical | 52 | 2.00% | 0.00% |
| 104 | Historical | 48 | 1.85% | 0.01% |
| 105 | Historical | 43 | 1.65% | 0.22% |
| 106 | Historical | 40 | 1.54% | 1.06% |
| 107 | Historical | 38 | 1.46% | 2.69% |
| **108** | **Historical** | **35** | **1.35%** | **9.20%** |
| **109** | **QR** | **34** | **1.31%** | **13.22%** |
| **110** | **QR** | **31** | **1.19%** | **33.88%** |
| **111** | **QR** | **31** | **1.19%** | **33.88%** |

| | | | | |
|---|---|---|---|---|
| **112** | **QR** | **30** | **1.15%** | **44.15%** |
| **113** | **QR** | **30** | **1.15%** | **44.15%** |
| **114** | **QR** | **30** | **1.15%** | **44.15%** |
| **115** | **QR** | **32** | **1.23%** | **25.37%** |
| **116** | **QR** | **33** | **1.27%** | **18.54%** |
| **117** | **QR** | **34** | **1.31%** | **13.22%** |
| **118** | **QR** | **34** | **1.31%** | **13.22%** |
| **119** | **QR** | **36** | **1.38%** | **6.25%** |
| **120** | **QR** | **31** | **1.19%** | **33.88%** |
| **121** | **QR** | **30** | **1.15%** | **44.15%** |
| **122** | **QR** | **31** | **1.19%** | **33.88%** |
| **123** | **QR** | **27** | **1.04%** | **84.47%** |
| **124** | **QR** | **27** | **1.04%** | **84.47%** |
| **125** | **QR** | **27** | **1.04%** | **84.47%** |
| **126** | **QR** | **29** | **1.12%** | **56.15%** |
| **127** | **QR** | **31** | **1.19%** | **33.88%** |
| **128** | **QR** | **32** | **1.23%** | **25.37%** |
| **129** | **QR** | **33** | **1.27%** | **18.54%** |
| **130** | **QR** | **33** | **1.27%** | **18.54%** |
| **131** | **QR** | **33** | **1.27%** | **18.54%** |
| **132** | **QR** | **34** | **1.31%** | **13.22%** |
| **133** | **QR** | **33** | **1.27%** | **18.54%** |
| **134** | **QR** | **33** | **1.27%** | **18.54%** |
| **135** | **QR** | **33** | **1.27%** | **18.54%** |
| **136** | **QR** | **30** | **1.15%** | **44.15%** |
| **137** | **QR** | **29** | **1.12%** | **56.15%** |
| **138** | **QR** | **29** | **1.12%** | **56.15%** |
| **139** | **QR** | **30** | **1.15%** | **44.15%** |
| **140** | **QR** | **31** | **1.19%** | **33.88%** |
| **141** | **QR** | **32** | **1.23%** | **25.37%** |
| **142** | **QR** | **36** | **1.38%** | **6.25%** |
| **143** | **QR** | **34** | **1.31%** | **13.22%** |
| **144** | **QR** | **34** | **1.31%** | **13.22%** |
| **145** | **QR** | **33** | **1.27%** | **18.54%** |
| **146** | **QR** | **30** | **1.15%** | **44.15%** |
| **147** | **QR** | **29** | **1.12%** | **56.15%** |
| **148** | **QR** | **29** | **1.12%** | **56.15%** |
| **149** | **QR** | **30** | **1.15%** | **44.15%** |
| **150** | **QR** | **29** | **1.12%** | **56.15%** |
| **151** | **QR** | **30** | **1.15%** | **44.15%** |
| **152** | **QR** | **31** | **1.19%** | **33.88%** |

**Appendix D.1. UC test results for the global period, for the Total VaR.** Reports the results of the UC test applied to Total VaR estimates over the global evaluation period. Models highlighted in bold correspond to those that pass the test at the 5% significance level.

Appendix D.2. BCP test results of the Total VaR for the models that pass the UC test.

| Model number | Model Class | Worst p-value | Lag |
|---|---|---|---|
| 43 | SGSt | 0.00% | 10 |
| **44** | **SGSt** | **36.29%** | **1** |
| **45** | **SGSt** | **5.75%** | **2** |
| 46 | SGSt | 2.15% | 2 |
| 47 | SGSt | 4.27% | 2 |
| 48 | SGSt | 4.27% | 2 |
| **49** | **SGSt** | **5.75%** | **2** |
| **50** | **SGSt** | **5.75%** | **2** |
| 51 | SGSt | 0.00% | 2 |
| 52 | SGSt | 0.00% | 3 |
| 53 | SGSt | 0.00% | 10 |
| **54** | **SGSt** | **32.80%** | **1** |
| **55** | **SGSt** | **36.29%** | **1** |
| 56 | SGSt | 4.27% | 2 |
| **57** | **SGSt** | **5.75%** | **2** |
| 58 | SGSt | 3.08% | 2 |
| 59 | SGSt | 3.08% | 2 |
| 60 | SGSt | 4.27% | 2 |
| **61** | **SGSt** | **5.75%** | **2** |
| 62 | SGSt | 0.00% | 2 |
| 63 | SGSt | 0.00% | 3 |
| 64 | SGSt | 0.00% | 10 |
| 108 | Historical | 0.00% | 10 |
| **109** | **QR** | **39.86%** | **1** |
| **110** | **QR** | **53.80%** | **1** |
| **111** | **QR** | **53.80%** | **1** |
| **112** | **QR** | **26.07%** | **1** |
| **113** | **QR** | **26.07%** | **1** |
| **114** | **QR** | **26.07%** | **1** |
| **115** | **QR** | **32.80%** | **1** |
| **116** | **QR** | **36.29%** | **1** |
| **117** | **QR** | **31.95%** | **10** |
| 118 | QR | 0.04% | 2 |
| 119 | QR | 0.02% | 2 |
| **120** | **QR** | **29.38%** | **1** |
| **121** | **QR** | **26.07%** | **1** |
| **122** | **QR** | **29.38%** | **1** |
| **123** | **QR** | **12.92%** | **3** |
| **124** | **QR** | **12.92%** | **3** |
| 125 | QR | 0.18% | 2 |
| 126 | QR | 0.57% | 2 |
| 127 | QR | 1.44% | 2 |
| 128 | QR | 2.15% | 2 |
| 129 | QR | 0.00% | 2 |
| 130 | QR | 0.00% | 2 |
| **131** | **QR** | **36.29%** | **1** |
| **132** | **QR** | **39.86%** | **1** |
| **133** | **QR** | **36.29%** | **1** |
| 134 | QR | 3.08% | 2 |
| 135 | QR | 3.08% | 2 |
| 136 | QR | 0.87% | 10 |
| 137 | QR | 0.37% | 10 |
| 138 | QR | 0.37% | 10 |
| 139 | QR | 0.00% | 2 |
| 140 | QR | 0.00% | 2 |
| 141 | QR | 0.00% | 2 |

| | | | |
|---|---|---|---|
| **142** | **QR** | **47.12%** | **1** |
| 143 | QR | 4.27% | 2 |
| 144 | QR | 4.27% | 2 |
| 145 | QR | 3.08% | 2 |
| 146 | QR | 0.87% | 10 |
| 147 | QR | 0.00% | 2 |
| 148 | QR | 0.00% | 2 |
| 149 | QR | 0.00% | 2 |
| 150 | QR | 0.00% | 2 |
| 151 | QR | 0.00% | 2 |
| 152 | QR | 0.00% | 10 |

**Appendix D.2. BCP test results for the global period, for the Total VaR.** Reports the results of the BCP test applied to Total VaR estimates over the global evaluation period. Models highlighted in bold correspond to those that pass the test at the 5% significance level.

Appendix D.3.1 UC test results across annual subperiods for Total VaR – SGSt model

| **Model class** | | **SGSt** | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Model number** | | **44** | **45** | **49** | **50** | **54** | **55** | **57** | **61** |
| 2022-2023 | Exceedance rate (%) | **0.77** | 0.77 | 1.15 | 1.15 | **0.77** | 0.77 | 0.77 | 1.15 |
| | p-value (%) | **69.67** | 69.67 | 80.77 | 80.77 | **69.67** | 69.67 | 69.67 | 80.77 |
| 2021-2022 | Exceedance rate (%) | **1.15** | 1.15 | 1.15 | 1.15 | **1.15** | 1.15 | 1.15 | 1.15 |
| | p-value (%) | **80.77** | 80.77 | 80.77 | 80.77 | **80.77** | 80.77 | 80.77 | 80.77 |
| 2020-2021 | Exceedance rate (%) | **2.31** | 2.69 | 2.69 | 2.69 | **2.31** | 2.69 | 2.69 | 2.69 |
| | p-value (%) | **7.01** | 2.34 | 2.34 | 2.34 | **7.01** | 2.34 | 2.34 | 2.34 |
| 2019-2020 | Exceedance rate (%) | **1.54** | 1.54 | 1.15 | 1.15 | **1.54** | 1.54 | 1.54 | 1.15 |
| | p-value (%) | **41.87** | 41.87 | 80.77 | 80.77 | **41.87** | 41.87 | 41.87 | 80.77 |
| 2018-2019 | Exceedance rate (%) | **0.38** | 0.38 | 0.77 | 0.77 | **0.38** | 0.38 | 0.38 | 0.77 |
| | p-value (%) | **25.44** | 25.44 | 69.67 | 69.67 | **25.44** | 25.44 | 25.44 | 69.67 |
| 2017-2018 | Exceedance rate (%) | **1.15** | 1.15 | 1.54 | 1.54 | **1.15** | 1.15 | 1.54 | 1.54 |
| | p-value (%) | **80.77** | 80.77 | 41.87 | 41.87 | **80.77** | 80.77 | 41.87 | 41.87 |
| 2016-2017 | Exceedance rate (%) | **1.15** | 1.15 | 1.15 | 1.15 | **1.15** | 1.15 | 1.15 | 1.15 |
| | p-value (%) | **80.77** | 80.77 | 80.77 | 80.77 | **80.77** | 80.77 | 80.77 | 80.77 |
| 2015-2016 | Exceedance rate (%) | **1.15** | 1.15 | 1.15 | 1.15 | **1.15** | 1.15 | 1.54 | 1.15 |
| | p-value (%) | **80.77** | 80.77 | 80.77 | 80.77 | **80.77** | 80.77 | 41.87 | 80.77 |
| 2014-2015 | Exceedance rate (%) | **1.54** | 1.54 | 1.54 | 1.54 | **1.54** | 1.54 | 1.54 | 1.54 |
| | p-value (%) | **41.87** | 41.87 | 41.87 | 41.87 | **41.87** | 41.87 | 41.87 | 41.87 |
| 2013-2014 | Exceedance rate (%) | **1.54** | 1.15 | 1.15 | 1.15 | **1.15** | 1.15 | 1.15 | 1.15 |
| | p-value (%) | **41.87** | 80.77 | 80.77 | 80.77 | **80.77** | 80.77 | 80.77 | 80.77 |

**Appendix D.3.1 UC test results across annual subperiods for Total VaR - SGSt model.** Reports the UC test applied to Total VaR model specifications over ten consecutive annual subperiods. Models highlighted in bold correspond to those that pass the test at the 5% significance level.

Appendix D.3.2 UC test results across annual subperiods for Total VaR – Quantile Regression model

| Model class | | Quantile Regression | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Model number | | 109 | 110 | 111 | 112 | 113 | 114 | 115 | 116 | 117 |
| 2022-2023 | Exceedance rate (%) | **0.77** | **0.77** | **0.77** | **0.77** | **0.77** | **0.77** | **1.15** | **1.15** | **1.15** |
| | p-value (%) | **69.67** | **69.67** | **69.67** | **69.67** | **69.67** | **69.67** | **80.77** | **80.77** | **80.77** |
| 2021-2022 | Exceedance rate (%) | **0.77** | **0.77** | **0.77** | **0.77** | **0.38** | **0.77** | **0.77** | **1.15** | **1.15** |
| | p-value (%) | **69.67** | **69.67** | **69.67** | **69.67** | **25.44** | **69.67** | **69.67** | **80.77** | **80.77** |
| 2020-2021 | Exceedance rate (%) | **1.54** | **1.15** | **1.15** | **1.15** | **1.54** | **1.15** | **1.15** | **1.15** | **1.92** |
| | p-value (%) | **41.87** | **80.77** | **80.77** | **80.77** | **41.87** | **80.77** | **80.77** | **80.77** | **18.44** |
| 2019-2020 | Exceedance rate (%) | **1.92** | **1.92** | **1.92** | **1.92** | **1.92** | **1.92** | **1.92** | **1.92** | **1.92** |
| | p-value (%) | **18.44** | **18.44** | **18.44** | **18.44** | **18.44** | **18.44** | **18.44** | **18.44** | **18.44** |
| 2018-2019 | Exceedance rate (%) | **0.77** | **0.77** | **0.77** | **0.77** | **0.77** | **0.77** | **0.77** | **0.77** | **0.77** |
| | p-value (%) | **69.67** | **69.67** | **69.67** | **69.67** | **69.67** | **69.67** | **69.67** | **69.67** | **69.67** |
| 2017-2018 | Exceedance rate (%) | **1.15** | **1.15** | **1.15** | **1.15** | **1.15** | **1.54** | **1.54** | **1.54** | **1.54** |
| | p-value (%) | **80.77** | **80.77** | **80.77** | **80.77** | **80.77** | **41.87** | **41.87** | **41.87** | **41.87** |
| 2016-2017 | Exceedance rate (%) | **1.15** | **1.15** | **1.15** | **1.15** | **1.15** | **1.15** | **1.15** | **1.15** | **0.77** |
| | p-value (%) | **80.77** | **80.77** | **80.77** | **80.77** | **80.77** | **80.77** | **80.77** | **80.77** | **69.67** |
| 2015-2016 | Exceedance rate (%) | **2.31** | **1.54** | **1.54** | **1.54** | **1.15** | **0.77** | **0.77** | **0.77** | **0.77** |
| | p-value (%) | **7.01** | **41.87** | **41.87** | **80.77** | **80.77** | **69.67** | **69.67** | **69.67** | **69.67** |
| 2014-2015 | Exceedance rate (%) | **1.15** | **1.15** | **1.15** | **1.15** | **1.15** | **1.15** | **1.54** | **1.54** | **1.54** |
| | p-value (%) | **80.77** | **80.77** | **80.77** | **80.77** | **80.77** | **80.77** | **41.87** | **41.87** | **41.87** |
| 2013-2014 | Exceedance rate (%) | **1.54** | **1.54** | **1.54** | **1.54** | **1.54** | **1.54** | **1.54** | **1.54** | **1.54** |
| | p-value (%) | **41.87** | **41.87** | **41.87** | **41.87** | **41.87** | **41.87** | **41.87** | **41.87** | **41.87** |

| Model class | | Quantile Regression | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Model number | | 120 | 121 | 122 | 123 | 124 | 131 | 132 | 133 | 142 |
| 2022-2023 | Exceedance rate (%) | **0.38** | **0.38** | **0.38** | **0.38** | **0.38** | **0.38** | **0.38** | **0.38** | 0.38 |
| | p-value (%) | **25.44%** | **25.44%** | **25.44%** | **25.44%** | **25.44%** | **25.44%** | **25.44%** | **25.44%** | 25.44% |
| 2021-2022 | Exceedance rate (%) | **0.77** | **0.77** | **0.77** | **0.38** | **0.38** | **1.15** | **1.15** | **0.77** | 1.15 |
| | p-value (%) | **69.67%** | **69.67%** | **69.67%** | **25.44%** | **25.44%** | **80.77%** | **80.77%** | **69.67%** | 80.77% |
| 2020-2021 | Exceedance rate (%) | **1.54** | **2.31** | **2.31** | **1.92** | **1.92** | **1.92** | **1.92** | **2.31** | 2.69 |
| | p-value (%) | **41.87%** | **7.01%** | **7.01%** | **18.44%** | **18.44%** | **18.44%** | **7.01%** | **7.01%** | 2.34% |
| 2019-2020 | Exceedance rate (%) | **2.31** | **1.92** | **1.92** | **1.54** | **1.54** | **1.54** | **1.54** | **1.54** | 1.54 |
| | p-value (%) | **7.01%** | **18.44%** | **18.44%** | **41.87%** | **41.87%** | **41.87%** | **41.87%** | **41.87%** | 41.87% |
| 2018-2019 | Exceedance rate (%) | **0.38** | **0.38** | **0.38** | **0.38** | **0.38** | **0.38** | **0.38** | **0.38** | 1.15 |
| | p-value (%) | **25.44%** | **25.44%** | **25.44%** | **25.44%** | **25.44%** | **25.44%** | **25.44%** | **25.44%** | 80.77% |
| 2017-2018 | Exceedance rate (%) | **1.54** | **1.15** | **1.54** | **1.54** | **1.54** | **1.54** | **1.54** | **1.54** | 1.54 |
| | p-value (%) | **41.87%** | **80.77%** | **41.87%** | **41.87%** | **41.87%** | **41.87%** | **41.87%** | **41.87%** | 41.87% |
| 2016-2017 | Exceedance rate (%) | **1.15** | **1.15** | **1.15** | **1.15** | **1.15** | **1.15** | **1.15** | **1.15** | 1.15 |
| | p-value (%) | **80.77%** | **80.77%** | **80.77%** | **80.77%** | **80.77%** | **80.77%** | **80.77%** | **80.77%** | 80.77% |
| 2015-2016 | Exceedance rate (%) | **1.15** | **1.15** | **1.15** | **0.77** | **0.77** | **1.15** | **1.15** | **1.15** | 1.54 |
| | p-value (%) | **80.77%** | **80.77%** | **80.77%** | **69.67%** | **69.67%** | **80.77%** | **80.77%** | **80.77%** | 41.87% |
| 2014-2015 | Exceedance rate (%) | **1.54** | **1.54** | **1.54** | **1.54** | **1.54** | **1.54** | **1.54** | **1.54** | 1.54 |
| | p-value (%) | **41.87%** | **41.87%** | **41.87%** | **41.87%** | **41.87%** | **41.87%** | **41.87%** | **41.87%** | 41.87% |
| 2013-2014 | Exceedance rate (%) | **1.15** | **0.77** | **0.77** | **0.77** | **0.77** | **1.92** | **1.92** | **1.92** | 1.15 |
| | p-value (%) | **80.77%** | **69.67%** | **69.67%** | **69.67%** | **69.67%** | **18.44%** | **18.44%** | **18.44%** | 80.77% |

**Appendix D.3.2 UC test results across annual subperiods for Total VaR – Quantile Regression model.** Reports the UC test applied to Total VaR model specifications over ten consecutive annual subperiods. Models highlighted in bold correspond to those that pass the test at the 5% significance level.

Appendix D.3.3. Exceedance rates for Total VaR during the global evaluation period

| **Model number** | **Model Class** | **Exceedance rate (%)** |
|---|---|---|
| 44 | SGSt | 1.27 |
| 54 | SGSt | 1.23 |
| 109 | Quantile Regression | 1.31 |
| 110 | Quantile Regression | 1.19 |
| 111 | Quantile Regression | 1.19 |
| 112 | Quantile Regression | 1.15 |
| 113 | Quantile Regression | 1.15 |
| 114 | Quantile Regression | 1.15 |
| 115 | Quantile Regression | 1.23 |
| 116 | Quantile Regression | 1.27 |
| 117 | Quantile Regression | 1.31 |
| 120 | Quantile Regression | 1.19 |
| 121 | Quantile Regression | 1.15 |
| 122 | Quantile Regression | 1.19 |
| 123 | Quantile Regression | 1.04 |
| 124 | Quantile Regression | 1.04 |
| 131 | Quantile Regression | 1.27 |
| 132 | Quantile Regression | 1.31 |
| 133 | Quantile Regression | 1.27 |

**Appendix D.3.3. Exceedance rates for Total VaR during the global evaluation period.** Reports the exceedance rate for the global evaluation period. Models 123 and 124 exhibited the closest alignment with the theoretical exceedance rate of 1%. Model 124 was ultimately selected as the preferred specification.