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Intergenerational actuarial fairness when longevity increases: Amending the retirement age

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ABSTRACT

Continuous longevity improvements and population ageing have led countries to modify national public pension schemes by increasing standard and early retirement ages in a discretionary, scheduled, or automatic way, and making it harder for people to retire prematurely. To this end, countries have adopted alternative retirement age strategies, but our analyses show that the measures taken are often poorly designed and consequently misaligned with the pension scheme's ultimate goals. This paper discusses how to implement automatic indexation of the retirement age to life expectancy developments while respecting the principles of intergenerational actuarial fairness and neutrality among generations of the respective policy scheme design. With stable demographic conditions, we show in policy designs in which extended working lives translate into additional pension entitlements, the pension age must be automatically updated to keep the period in retirement constant. Alternatively, policy designs that pursue a fixed replacement rate are consistent with retirement age policies targeting a constant balance between active years in the workforce and years in retirement. Under conditions of population ageing, the statutory pension age will have to increase at a faster rate to meet the intergenerational equity criteria. The empirical strategy employed a Bayesian Model Ensemble approach to stochastic mortality modelling to address model risk and generate forecasts of intergenerationally and actuarially fair pension ages for 23 countries from 2000 to 2050. The findings show that the pension age increases needed to accommodate the effect of longevity developments on pay-as-you-go equilibrium and to reinstate equity between generations are sizeable and well beyond those employed and/or legislated in most countries. A new wave of pension reforms may be at the doorsteps.

1. Introduction

With continuously increasing longevity in old age, linking retirement ages and pension benefits to life expectancy has been one of the most common policy measures of countries' efforts to achieve long-term affordability and fiscal sustainability of national universal pension schemes. This reform trend is part of a broader strategy of introducing automatic adjustment or stabilisation mechanisms, i.e., rules that automatically adjust a scheme's parameters (e.g., contribution rate, benefit level, indexation rate, retirement age) to demographic and/or economic developments in a predetermined fashion, instead of relying on usually unpredictable ad hoc political interventions (Godínez-Olivares et al., 2016; Alonso-García et al., 2018; Boado-Penas et al., 2020; Devolder et al., 2021).

In the past more than two decades various strategies have been pursued in OECD countries to increase the age of exit from the labour force of the older population. These include legislation to phase out special pensions and, generally, to phase out paths into early pensions, and increasing the statutory minimum retirement ages. To this end, several alternative retirement age strategies have been adopted (see, e.g., OECD, 2019a). And many countries have passed legislation that will increase the standard and early retirement ages (at least sixteen OECD countries). And some have opted

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to automatically index the standard and early retirement ages to life expectancy. One of the most prominent measures adopted by countries in this context has been to link pensions to life expectancy.

Countries have introduced and adjusted the links to life expectancy in multiple ways, as recognised and discussed in numerous contexts, including our work (Alho et al., 2013; Bovenberg et al., 2015; OECD, 2019a; Holzmann et al., 2020; Ayuso et al., 2021a). The topics of interest in the context of life expectancy indexation of standard retirement fall under a number of categories: (i) Linking newly granted pensions to sustainability factors or life expectancy coefficients (Finland, Portugal), or to old-age dependency ratios (Germany, Japan); (ii) Automatically indexing normal and early retirement ages (Denmark, Estonia, Finland, Greece, Italy, the Netherlands, Norway, Portugal, Slovakia and the United Kingdom); (iii) Transforming public earnings-related plans into nonfinancial defined contribution (NDC) schemes (Italy, Latvia, Norway, Poland, and Sweden), which automatically adjust retirement benefits to life expectancy in the process of annuitisation of individual account balances; (iv) Determining the qualifying conditions for an old-age pension, for instance, by indexing the number of contribution years required for a full pension to life expectancy (France and Italy); (v) Introducing risk-sharing arrangements in public and private individual or employer-sponsored pension plans (Belgium, the Netherlands, and the United States); (vi) Introducing mandatory and voluntary funded defined contribution (DC) schemes to replace or supplement public pension provisions (Australia, Chile, Estonia, Hungary, Israel, Mexico, Poland, Slovakia, Sweden and Switzerland); (vii) Conditioning the annual indexation of pensions in payment to a scheme's solvency position (Japan, and the Netherlands); and (viii) Linking pension penalties (incentives) for early (late) retirement to the contribution length (Portugal).

The adopted retirement age policies differ in several important design aspects (e.g., the triggering event, the frequency of revision, indexation lags, the decision-making process, and the variables used to determine the change). The way in which the statutory pension age is changed can be informed by alternative (explicit or embedded) policy objectives (Stevens, 2016; Bravo and Ayuso, 2021). For instance, some policies target a constant expected retirement period (e.g., Denmark, The Netherlands). Others target a constant balance between time spent in work (contributing) and in retirement or a constant ratio of adult life (or total lifespan) spent in retirement (e.g., the United Kingdom, Czechia, Malta). Others target a stable old-age dependency ratio (see Hyndman et al., 2021). Some have preferred to set an indicative target age for retirement to serve as a benchmark for guiding individual retirement decisions, aiming to replace the long-term well-established implicit target age of 65 (e.g., Sweden). Others have adopted ad hoc rule-based approaches to share the burden of longevity between workers and pensioners (e.g., Portugal). However, recent empirical evidence shows that the use of inappropriate longevity measures and poor policy design results in economic and social policy outcomes that substantially deviate from their initial policy design and/or reform intentions (Ayuso et al., 2021b).

Changes in the statutory pension ages may come about through alternative decision-making processes, including government-sponsored and/or independent commissions on pensions or welfare reforms (e.g., Denmark, France, Germany, Ireland), legislated measures (e.g., Spain, Hungary), automatic or semi-automatic adjustment mechanisms (e.g., The Netherlands, Denmark, Slovakia), negotiations with social partners, impositions on governments by actors in charge of lending money as conditionality programmes (e.g., IMF, the 'Troika' in Portugal and Greece) or considered as an undisputable necessity in the respective national context (e.g., Italy in 2011). Changes in the statutory normal and early pension age are often accompanied by other pension reforms, including special dispositions for workers who started their contribution careers earlier.

Although the objective of introducing automatic stabilisers in pension schemes is primarily cost containment, there are other dimensions of welfare restructuring in the politics and social outcomes of pension reforms. They include recalibration and/or rationalisation, i.e., introducing economic and actuarial rationality for validating the advocated changes, enhancing the credibility of the system, social trust, and the support of the intergenerational contract by averting otherwise public finance crises and major pension entitlement cuts in the future, addressing old age poverty, providing socially adequate benefits, and eluding the political risks of regular negotiations between social partners to approve unpopular reforms that involve retrenchments (Hassel et al., 2019; Carrera and Angelaki, 2020). The rise in old age dependency increases the grey vote, raising discussions about the pro-elderly policy bias and the rise of gerontocracy in pension reform (Tepe and Vanhuysse, 2010).

Reforming pension arrangements under conditions of population ageing satisfying the requirements of fairness between generations has, for example, been advocated by the European Commission from the turn of the century: "Member states should undertake ambitious reforms of pension systems in order to contain pressures on public finances, to place pension systems on a sound financial footing and ensure a fair intergenerational balance" (European Commission, 2003:61). Except for a restricted number of cases, notably the NDC countries, the introduction of automatic stabilisers and other policy measures tends to be poorly aligned with intergenerational equity.

Also noteworthy in the present context is the established practice often employed in the financial management of public nonfinancial defined benefit (NDB) pension schemes of backloading the cost of longer lives and population ageing onto the shoulders of active younger generations, which offers neither stability nor intergenerational fairness. This approach neglects intergenerational fairness and disregards the balance between costs and benefits between generations of taxpayers and pensioners. This can ultimately undermine public support for the intergenerational contract. In fact, this approach runs against fair intergenerational risk sharing, which is one of the key rationales behind government mandates of compulsory pension schemes.

In contrast to employing the practice described in the preceding paragraph, this paper addresses and discusses how to automatically index the retirement age to life expectancy satisfying the requirements of intergenerational actuarial fairness and neutrality among generations. We investigate alternative retirement age policies based on a stylised Bismarckian career average reevaluated earnings-related pension scheme in which a participant's pension granted at retirement is strictly linked to the retiree's entire contribution history. We assume that the scheme begins in financial balance and a steady state, with stable demographic (old-age dependency ratio) and economic (employment rate and wage rate) conditions. We derive an intergenerationally actuarially neutral condition for policy reforms and examine alternative automatic adjustment mechanisms and pension policy rules designed to maintain the long-term financial sustainability of a universal public pension scheme under conditions characterized by the progressive ageing of the pension-age population.

With the same degree of generality, we then focus on a country's normal retirement age as the key policy instrument and investigate how to implement indexation to life expectancy so as not to affect the intergenerational redistribution of the universal public pension scheme, i.e., making it neutral in its effect on the earnings replacement rate of each generation's contributions.

To discuss the alternative policy options, we adopt an intergenerational actuarial fairness and neutrality principle to pension design and reform at the margin. Fairness and neutrality can have different meanings to different people in the context of an earnings-related pension system, so we need to be more precise in their use here. We use the term fairness in the sense of actuarial fairness, i.e., a fair pension scheme requires the present value of lifetime contributions to equal the actuarial present value of lifetime benefits at the time of retirement. Stated differently, an actuarially intragenerational fair pension scheme is one in which every individual's contribution in any given period yields the same expected increment to retirement income given that the distribution of risks, mainly including the risk of life expectancy, is unknown ex-ante. This scheme ex-ante guarantees equality through the random distribution of risk. And intergenerational neutrality requires this to be true over generations.

Tackling the fairness challenge across generations in response to demographic or economic developments depends on the pension scheme's underlying design (DB or DC) and on the way policy interventions are designed and implemented. They ultimately determine how the cost of life expectancy improvements is shared between current and future pensioners. Assuming labour market participation and retirement decisions are not distorted by the policy intervention and that all other pension scheme parameters are kept constant, this paper provides comprehensive empirical results for two possible policy designs. These differ in that they assume the extra contribution years may or may not generate additional pension entitlements, i.e., they convey alternative ways of sharing the longevity risk burden between workers and pensioners.

In the first policy design, the extra contribution period translates into a higher replacement rate by keeping the accrual rate per year constant. Under this constant accrual-rate-per-year (CAR) policy design, we show that if lifetime earnings are revalued at the scheme's discount interest rate and pension benefits are indexed at the same rate, intergenerational actuarial fairness requires standard pension ages to be indexed in line with the development of life expectancy, adjusted by the change in the pension scheme's old-age dependency ratio. With stable demographic conditions, the CAR policy design targets a constant retirement period.

In the second policy design, the additional contribution period is accompanied by a reduction in the accrual rate per year such that the replacement rate remains constant over time. Under the same assumptions as above, we show that to satisfy the requirements of fairness between generations the constant replacement rate (CRR) policy design prescribes that the retirement age must be adjusted such that the expected years in retirement relative to contribution years (or relative to adult life if we assume a constant labour market entry age), adjusted by the rate of increase in the scheme's old-age dependency ratio, must remain constant over time. This is consistent with retirement age policies targeting a constant ratio between years in work (or in adult life) and retirement, while introducing adequacy safeguards and intergenerational fairness by keeping the replacement rate constant across generations. Conceptually and in practice, "mixed interventions" are also feasible within our framework, considering other social, demographic, and/or economic criteria.

Given the above two alternatives, the overall empirical strategy of this paper is to examine mortality outcomes for 23 representative countries, comparing the dynamics of actual and legislated (and projected in the case of countries that have already adopted automatic indexation mechanisms) retirement ages. The aim of the exercise is to evaluate the CAR and CRR approaches for all of the countries in the study in light of what is required to attain schemes that are intergenerationally actuarially fair based on actual and projected data for the period 2000 to 2050. The analysis encompasses medium- and high-income countries with a diversity of actual pension architecture, including both financial and non-financial (DB and DC) schemes.

To generate forecasts of retirement age by age, sex, and year, we estimate period and cohort survival curves from stochastic mortality models. Currently, model selection and model combination are the two competing approaches in mortality modelling and forecasting. The customary procedure is to pursue a winner-take-all approach by which, for each population, a single model is chosen from a set of candidate methods using some criteria (for example, forecasting accuracy). To this end, a growing number of single- and multi-population, discrete- and continuous-time, age-period-cohort stochastic mortality models, principal component methods, smoothing approaches, and statistical machine learning techniques are proposed in the actuarial and demographic literature (e.g., Lee and Carter, 1992; Brouhns et al., 2002; Renshaw and Haberman, 2003, 2006; Currie, 2006, 2016; Cairns et al., 2006, 2009; Hyndman and Ullah, 2007; Plat, 2009; Pascariu et al., 2020; Basellini et al., 2020; Hunt and Blake, 2021; Bravo and Nunes, 2021; Perla et al., 2021; Li et al., 2021; Chen et al., 2022; Ashofteh et al., 2022 and references therein).

The use of different selection procedures, alternative accuracy metrics, different data-coverage periods, misspecification problems, and the presence of structural breaks in the data-generating process can lead to different model choices and time series forecasts. Empirical studies show that no single mortality model outperforms in all countries or subpopulations/cohorts of countries, or across time.

The point of departure for this study is that the current empirical work in actuarial science, economics, finance, and social modelling is subject to substantial conceptual (model specification) uncertainty (Steel, 2020). What is more, this uncertainty has been preventing countries from using cohort life expectancy measures in pension policy. And the use of period instead of cohort life expectancy markers in pension design results in systematic underestimation of the remaining lifetime at retirement (Alho et al., 2013). Recent empirical studies have shown that the gap between period and cohort-based life expectancy projections and actual longevity outcomes at retirement is sizable, persistent, and still increasing in most countries (Bravo et al., 2021). This ultimately translates into an ex-ante unintended financial transfer from future to current generations and intergenerational inequity. The goal of achieving intergenerational equity requires combining the inherent path dependency in pension schemes with a forward-looking approach to dealing with the economic, demographic, and social risks associated with life expectancy projection modelling.

To tackle the model risk problem in stochastic mortality modelling, a recent strand of the literature proposes the use of model combinations (see, e.g., Kontis et al., 2017; Bravo et al., 2021; Barigou et al., 2022). Despite its higher degree of complexity, the composite Bayesian Model Ensemble (BME) approach developed in Bravo et al. (2021) to project life expectancy at retirement is adopted in this paper to provide a sounder basis for statistical inference and policy design. Because populations even in specific countries can nevertheless be heterogeneous over time, the ensemble approach increases the degrees of freedom where the end goal is the successful projection of life expectancy as an input in policy design.

The empirical strategy employed for each country involves the: (i) identification of the model confidence set; (ii) computation of posterior probabilities for each model; (iii) generation of forecasts using the composite model; and (iv) computation of Bayesian prediction intervals for stochastic process, model, and parameter risks using the Model-Averaged Tail Area (MATA) approach proposed by Turek and Fletcher (2012). The model space considered in this paper includes nine heterogeneous stochastic mortality models that encompass principal component methods, two-dimensional smoothing approaches, and the well-known generalised age-period-cohort (GAPC) models. The approach is explained in greater detail in the next section of this paper.

To examine the impact of population ageing (increase in the old-age dependency ratio) on intergenerationally balanced retirement age policies, we generate population forecasts for an illustrative country, Portugal, using the cohort-component method, accounting for the stochastic dynamics of fertility, migration, and mortality. Portugal is one of the countries with the oldest populations in the world and the total population is expected to decline significantly (>20%) in the next decades (European Commission, 2021).

Generally speaking, our empirical results for both the CAR and CRR policy designs show that under stable population ageing scenarios, actual (2000-2021) and legislated (planned) retirement age increases have been and will thus in the future be insufficient to cope with populations' extended survival prospects, if the pension scheme is to preserve the intergenerational fairness and neutrality condition. The difference between the intergenerationally fair and the actual/legislated retirement ages is, as expected, higher under a CAR policy than under a CRR approach, with gaps accumulating over time in both cases.

The results also show that despite the important retirement age increases legislated in many OECD countries in the last two decades, the expected duration of retirement will continue to increase. The adoption of a CRR retirement age policy contributes to reducing the expected period in retirement by 1.91 years in 2050 compared to legislated reforms. Although considerable, this is not enough. The legislated corrections fall short of what is needed to prevent a rise in expected retirement duration and, in many cases, will not be offset by an increase in the relative size of the labour force. Except for Belgium and the Netherlands, the results show that the expected period in retirement relative to the contribution period (and to adult life) is forecast to increase in all countries, peaking at 74.5 percent in France in 2050.

The required pension age corrections are well beyond the scheduled changes planned or ongoing in many countries and could trigger a new wave of pension reforms, including moving away from early retirement rules and further closing routes into early labour market exit. For both policy designs, the results show that population ageing accelerates the retirement age increases required to ensure equal treatment for all generations. However, the required correction raises distributional issues due to the widening gap in life expectancy by socioeconomic group. This concern is addressed in greater depth in Section 4.

The remainder of the paper is organised as follows. Section 2 presents the key concepts and statistical methods used in the paper. These are the principles of intergenerational actuarial fairness and neutrality as employed in pension design. It also recaps the BME approach for mortality modelling and life expectancy computation and describes the data used in fitting the models. Section 3 reports summary forecasts of cohort life expectancy at retirement age by country and provides detailed numerical results for the two alternative retirement age policy designs considered in this study. Section 4 critically discusses the results and concludes. The technical details are relegated to the Appendix.

2. Materials and methods

2.1. Actuarially fair and neutral retirement age policies

Consider a stylised career average re-evaluated earnings-related non-financial defined benefit (NDB) pension scheme with old age entry pension actuarially computed based on the entire contribution effort. Without loss of generality, we adopt an intergenerational actuarial fairness and neutrality principle to pension design and reform at the margin. The actuarial pay-as-you-go aggregate balance constraint[1] in year t equals the revalued contribution effort (notional and real capital) and the pension wealth, i.e.,

$$A_t \cdot c_t \cdot V_t + F_t = L_t \cdot P_{x_r(t)} \cdot a^{\pi,y}_{x_r(t)}, \tag{1}$$

where c_t is the contribution rate; A_t is the number of active workers in the scheme; $V_t \equiv V(x_r, x_e, w_t, \upsilon_t)$ is the lifetime pensionable average salary w_t of all active workers, earned since labour market entry age x_e and revalued using an indexation or valorisation rate υ_t; x_r is the statutory retirement age; F_t represents, if any, the scheme's external sources of funding (e.g., a buffer fund, general or dedicated taxes); L_t is the number of pensioners; $a^{\pi,y}_{x_r(t)}$ is the annuity factor computed at $x_r(t)$ using a cohort approach,

$$a^{\pi,y}_{x_r(t)} := \sum_{\tau=1}^{\omega-x_r} \left(\frac{1+\pi}{1+y}\right)^{\tau} {}_{\tau}p_{x_r(t)}, \tag{2}$$

where π is the uprating rate for pensions, y is the annuity factor discount rate used in the PAYG scheme, and ${}_{\tau}p_x$ denotes the τ-year survival rate of a population cohort aged x at time t:

$${}_{\tau}p_x(t) := \mathbb{E}\left[\exp\left(-\int_0^{\tau} \mu_{x+s}(s)\,ds\right)|\mathcal{G}_t\right], \tag{3}$$

where $\mathcal{G}_t$ describes the information at time t, and $\mu_x(t)$ is a stochastic force of mortality process on a filtered probability space $(\Omega, \mathbb{G}, \mathbb{P})$. For policy analysis, we discard pre-retirement mortality and assume we are equipped with ex ante unbiased cohort-based mortality rate projections. $P_{x_r(t)}$ is the annual pension benefit, calculated as follows:

$$P_{x_r(t)} = \theta_t\left(x_{r(t)} - x_e\right) \cdot \overline{RE}_{x_r(t)} \cdot RF_{x_r(t)} \cdot b_{x_r(t)}, \tag{4}$$

where θ_t is a linear (usually flat) accrual rate for each year of service,[2] $\left(x_{r(t)} - x_e\right)$ is the contribution period, $RF_{x_r(t)}$ is a demographic (often called sustainability) factor introduced in some countries (e.g., Finland, Portugal, Spain) to reduce entry pension benefits as life expectancy increases[3]; $b_{x_r(t)}$ are pension decrements (increments) for early $\left(b_{x_r(t)} < 1\right)$ or postponed $\left(b_{x_r(t)} > 1\right)$ retirement, and $\overline{RE}_{x_r(t)} \equiv \overline{RE}\left(x_{r(t)}, x_e, w_t, \upsilon_t\right)$ is the lifetime average revalued earnings at retirement age

$$\overline{RE}_{x_r(t)} = \frac{RE_{x_r(t)}}{x_r(t) - x_e}, \text{ with}$$
$$RE_{x_r(t)} = \left(w_t^{x_r(t)} + \sum_{x=x_0}^{x_r(t)-1} w^{x_r(t)}_{t-x_r(t)+x} \prod_{j=t-x_r(t)+x+1}^{t} (1+\upsilon_j)\right), \tag{5}$$

[1] Several alternative mutually complementary indicators may be considered to evaluate intergenerational fairness in pension schemes, for instance, the ratio between the present value of lifetime benefits and the accumulation at retirement, the scheme's internal rate of return, the affordability and stability of the social contribution rate across generations, the benefit adequacy or the scheme's balance sheet solvency. Another possibility is to adopt a generational accounting approach that assumes that the government (explicit and implicit) debt reflects taxes paid minus transfers received over the remaining lifetime of both current and future generations.

[2] Accrual rates generally follow a linear flat schedule, although there are exceptions, e.g., Finland which adopted a non-linear accrual schedule until 2017.

[3] The demographic sustainability factor is typically designed as the ratio between the life annuity factor (e.g., Finland) or the total population period life expectancy calculated at some reference age (e.g., age 65 in Portugal) in the base year and the corresponding value in the year the insured reaches the set retirement age. For example, in the Portuguese pension scheme the sustainability factor is defined as $SF_t := \frac{\dot{e}_{65,2000}}{\dot{e}_{65,t-1}}$ (Bravo and Ayuso, 2021; Bravo and Herce, 2022), applying only to those retiring early relative to the statutory age.

where υ_t denotes the valorisation or indexation rate by which past earnings are adjusted to take into account changes in living standards between the time pension rights are accrued and the time they are claimed. In DB pension schemes, the most common practice is to revalue earlier years' pay with the growth of average earnings and/or inflation, with few (e.g., Portugal) considering productivity growth. The uprating of the pension-point value in points schemes and the notional interest rate in and notional-accounts (NDC) systems are the corresponding parameters of the valorisation rate in DB plans.[4]

The actuarial pay-as-you-go aggregate balance in equation (1) is influenced by the size, age structure, and dynamics of the population (fertility, mortality, net migration) and by the labour market conditions. As such, shocks reflecting demographic changes such as population ageing (due to, e.g., low fertility rates, increasing longevity, and/or insufficient net migration flows) or changes in the labour market (e.g., variations in the participation rate, the employment rate, structural unemployment, or in the forms of employment) affecting the size and composition of the working population impact the PAYG equilibrium. Let D_t denote the scheme's old-age dependency ratio (OADR) - the ratio between the number of pensioners L_t and the number of active workers A_t -, $D_t = L_t/A_t$. The aggregate balance constraint (1) can be rewritten as

$$c_t \cdot V_t + \frac{F_t}{A_t} = D_t \cdot P_{x_r(t)} \cdot a^{\pi,y}_{x_r(t)}, \tag{6}$$

From (6) it is clear that if the longevity prospects of the population increase and/or the scheme's old-age dependency ratio changes (e.g., due to population ageing or higher labour market participation rates), the pension scheme parameters (e.g., the early and normal retirement ages, the contribution rate, the sustainability factor coefficient, the accrual rate per year, the indexation rate of pensions) must be updated to ensure that the scheme remains actuarially fair and neutral across generations and does not require additional external sources of funding. This means, for instance, that if the remaining lifetime at retirement age is underestimated by using period-based life expectancy estimates, the scheme will be in deficit and the actuarial balance equation will not hold; i.e., the scheme will not be neutral among generations (Palmer and Zhao de Gosson de Varennes, 2020).

Assume that the parameters that are not policy instruments are kept constant. To ensure the scheme remains fair and neutral across the members of the initial (labelled 0) and the current (labelled t) generations, the contribution to benefit ratio must be kept constant, i.e.,

$$\frac{c_t \cdot V_t + \frac{F_t}{A_t}}{D_t \cdot P_{x_r(t)} \cdot a^{\pi,y}_{x_r(t)}} = \frac{c_0 \cdot V_0 + \frac{F_0}{A_0}}{D_0 \cdot P_{x_r(0)} \cdot a^{\pi,y}_{x_r(0)}}, \tag{7}$$

or, equivalently,[5]

$$\frac{c_t \cdot V_t + \frac{F_t}{A_t}}{c_0 \cdot V_0 + \frac{F_0}{A_0}} = \frac{D_t}{D_0} \cdot \frac{\theta_t\left(x_{r(t)} - x_e\right)}{\theta_0\left(x_{r(0)} - x_e\right)} \cdot \frac{\overline{RE}_{x_r(t)}}{\overline{RE}_{x_r(0)}} \cdot \frac{RF_{x_r(t)}}{RF_{x_r(0)}} \cdot \frac{b_{x_r(t)}}{b_{x_r(0)}} \cdot \frac{a^{\pi,y}_{x_r(t)}}{a^{\pi,y}_{x_r(0)}}. \tag{8}$$

Assume now that individuals of both cohorts retire at the full old-age pension age (i.e., $b_{x_r(t)} = b_{x_r(0)} = 1$), that the sustainability factor coefficient is constant over time (i.e., $RF_{x_r(t)}/RF_{x_r(0)} = 1$), and that pension scheme receives no external funding (i.e., $F_t = F_0 = 0$). The intergenerational fairness condition (8) simplifies to:

$$\frac{c_t}{c_0} \cdot \frac{V_t}{V_0} = \frac{D_t}{D_0} \cdot \frac{\theta_t\left(x_{r(t)} - x_e\right)}{\theta_0\left(x_{r(0)} - x_e\right)} \cdot \frac{\overline{RE}_{x_r(t)}}{\overline{RE}_{x_r(0)}} \cdot \frac{a^{\pi,y}_{x_r(t)}}{a^{\pi,y}_{x_r(0)}}. \tag{9}$$

Equations (8) and (9) offer a full menu of automatic adjustment mechanisms and pension policy rules to absorb the impact of economic and demographic shocks and preserve actuarial fairness and neutrality across generations. It frames a credible social contract between different generations, explicitly integrating intra- and intergenerational equity concerns. However, some of the policy options are (politically and socially) difficult to implement and sustain in practice. Moreover, depending on the pension scheme's overall architecture (a combination of state, occupational, and private components), and the technical design (DB, DC) of individual schemes in the system, as well as the way the interventions are devised and adopted, they may have important implications for the way the cost of providing for pensions is shared among generations as life expectancy increases.

As previously mentioned, our starting position is a pension scheme with no ex ante redistributive objectives in which proposed interventions aim to eliminate the wealth redistribution effects and the distortions on individual labour supply and savings decisions created by the life expectancy developments. In DC (DB) schemes a zero ex ante distortion takes place if account balances (accumulated rights) at the time of retirement are converted into an annuity based on cohort survival probabilities estimated using an unbiased projection method. The size of the unfunded pension liabilities or, equivalently, of the intergenerational tax/subsidy created before and after the policy intervention, is suggested as a performance measure.

Conceptually, the policy interventions can take place in the accumulation, annuitisation, and decumulation phases or can encompass mixed interventions that combine elements of all three stages (Ayuso et al., 2021a). Given the nature of the distortions addressed in this paper, we believe that redesign is best implemented in the latter two phases. This can be done, for instance, by reducing the initial pension through an actuarially designed reduction factor in response to a longer period of benefits or by linking pension benefits or pension indexation to survival developments (Bravo et al., 2021). We note, however, that in a pure NDB scheme, the natural adjustment would come through an update in the contribution rate to achieve fiscal balance, redistributing risk from pensioners to contributors.[6] In contrast, by generic construction, an NDC system's contribution rate should be constant across generations.

This paper instead focuses on the retirement age adjustments required to restore actuarial fairness in response to life expectancy

[4] It can be shown that if the rate used to revalue past earnings in a DB scheme is the same as the notional interest rate in NDC schemes (Sweden, Italy) and that of the valuation procedure of pension-point schemes (e.g., France, Germany), the initial benefit structure can be constructed to be similar to the NDC structure (see, e.g., Queisser and Whitehouse, 2006) - but only if one disregards the notional interest component composed of the rate of change in the "contribution-based" labour force (see, e.g., Palmer, 2013).

[5] A similar but narrower condition can be found in Meneu et al. (2016), discarding the scheme's old-age dependency ratio, the reduction factor, and the pension decrement/increment correction.

[6] From (9), keeping the other pension parameters constant, in response to the population's higher survival prospects, the new contribution rate necessary to restore the global equilibrium of the PAYG scheme would be determined such that the following condition holds:

$$c_t = c_0 \cdot \frac{a^{\pi,y}_{x_r(t)}}{a^{\pi,y}_{x_r(0)}} \cdot \frac{D_t}{D_0}. \tag{10}$$

Note, however, that an increase in the contribution rate creates a negative impact on labour costs affecting labour demand, wages, labour market equilibrium, and the pension scheme's long-term sustainability.

developments and population ageing. Assuming incentives for individuals are neutral – i.e., assuming that labour market entry and exit (retirement) ages are not distorted – pension age increases come with an equivalent increase in the contribution period and the effective retirement age. As noted before, at least two possible designs are possible depending on whether the added contribution period generates additional pension entitlements: (i) the extra contribution period translates into a higher replacement rate by keeping the accrual rate per year constant; or (ii) the increase in contribution years is accompanied by a reduction in the accrual rate per year such that the replacement rate remains constant. Mixed interventions sharing the longevity risk burden between different generations are also possible considering other social and economic criteria.[7]

2.1.1. Constant accrual-rate-per-year policy

Under a CAR policy, the required retirement age and the contribution period adjustments are accompanied by an increase in the replacement rate since the accrual rate per year is kept constant (i.e., $\theta_t = \theta_0$), while keeping all other pension system parameters unchanged. In a scenario of positive longevity developments, the contribution period will have to increase to restore actuarial fairness, generating higher replacement rates $\theta_t\left(x_{r(t)} - x_e\right) > \theta_0\left(x_{r(0)} - x_e\right)$; i.e., higher pensions and an enlarged pension scheme. Depending on the way the corrections are made, the shorter pension payment period may counterbalance the higher benefit levels. At an aggregate level, if the increased survival prospects negatively impact the old-age dependency ratio, the scheme's PAYG equilibrium deteriorates. From equation (9), the new equilibrium retirement age is the result of the following updating rule

$$a^{\pi,y}_{x_r(t)} = \frac{D_0}{D_t} \cdot \frac{V_t/V_0}{RE_{x_r(t)}/RE_{x_r(0)}} \cdot a^{\pi,y}_{x_r(0)}. \tag{11}$$

If lifetime earnings are revalued at the scheme's discount rate[8] (i.e., if $\upsilon_t = y_t \ \forall t$), the adjustment rule (11) reduces to:

$$a^{\pi,y}_{x_r(t)} = \frac{D_0}{D_t} \cdot a^{\pi,y}_{x_r(0)}. \tag{12}$$

By further assuming the uprating rate for pensions matches the scheme's discount rate (i.e., $\pi_t = y_t \ \forall t$), equation (12) reduces to:

$$\dot{e}^{C}_{x_r(t)} = \frac{D_0}{D_t} \cdot \dot{e}^{C}_{x_r(0)}. \tag{13}$$

The simplifying assumptions regarding lifetime earnings revaluation and pension indexation lead to an implicit equation for estimating the retirement age in which the pension age adjustments influence and are influenced by the dynamics of life expectancy at the retirement age and the scheme's old-age dependency ratio. Equations (12) and (13) suggest that to cope with populations' survival prospects and population ageing while keeping the accrual rate per year constant, the pension age must be updated so that the actuarial present value (or the cohort life expectancy), adjusted by the change in the dependency ratio, remains constant over time. In other words, the simple rule of adjusting pension age by the same magnitude of the life expectancy increase, targeting a constant retirement period (the Netherlands, Denmark), would only be considered actuarially fair and neutral across generations if accompanied by a properly calibrated CAR policy and stable demographic conditions (a constant old-age dependency ratio). In this scenario, the added contribution years generate additional pension entitlements fully covered by additional contributions.[9] In a scenario of population ageing (increase in the scheme's old-age dependency ratio), equation (13) shows that retirement age increases fully in line with increases in cohort life expectancy at retirement are not sufficient to restore the PAYG equilibrium and equity constraint. This means that the retirement period would have to be reduced, and younger generations would have to further adjust their retirement decisions to sustain the PAYG conditional pension promise.

If lifetime earnings are revalued below the scheme discount rate (i.e., if $y_t > \upsilon_t \ \forall t$), it is clear from equation (11) that the required retirement age adjustments would have to be smaller than in the baseline case since the lifetime revalued earnings would not completely reflect the additional contribution period. If lifetime earnings are revalued above the scheme's discount rate (i.e., $y_t < \upsilon_t \ \forall t$), the opposite occurs. If pensions are adjusted below the scheme's discount rate (i.e., if $\pi_t < y_t \ \forall t$), the required pension age adjustments would naturally be smaller; the opposite occurs if pensions are revalued every year above y_t. Finally, if the pension scheme DB pension formula includes a demographic (sustainability) factor linked to longevity developments reducing entry pension benefits if life expectancy increases or if additional external sources of funding (e.g., buffer fund, general or dedicated taxes) are used to finance a fraction of the total expenditures, the required pension age adjustments require to restore equilibrium would be smaller compared to the baseline case.

2.1.2. Constant replacement rate policy

Under a CRR policy, the required adjustment in the retirement age and contribution period is accompanied by a reduction in the accrual rate per year, such that the replacement rate (global accrued rate) remains constant across generations; i.e., $\theta_t\left(x_{r(t)} - x_e\right) = \theta_0\left(x_{r(0)} - x_e\right)$ or, equivalently:

$$\theta_t = \theta_0 \cdot \frac{\left(x_{r(0)} - x_e\right)}{\left(x_{r(t)} - x_e\right)}, \tag{14}$$

with $\theta_t < \theta_0$ since $\left(x_{r(0)} - x_e\right) < \left(x_{r(t)} - x_e\right)$. In this scenario, the impact of a longer contribution period on pension entitlements would be mitigated since it would come only because of the impact of extra work years on average lifetime revalued earnings. This effect is expected to be small since contrary to "best years" DB formulas, full contribution period DB pension formulas smooth the effect of abnormally low or high labour income years on initial benefits.

From (9), the new equilibrium retirement age would be the result of the following updating rule:

$$a^{\pi,y}_{x_r(t)} = \frac{D_0}{D_t} \cdot \frac{\left(x_{r(t)} - x_e\right)}{\left(x_{r(0)} - x_e\right)} \cdot \frac{V_t/V_0}{RE_{x_{r(t)}}/RE_{x_{r(0)}}} \cdot a^{\pi,y}_{x_r(0)}, \tag{15}$$

which, if lifetime earnings are revalued at the scheme's discount rate, reduces to:

$$a^{\pi,y}_{x_r(t)} = \frac{D_0}{D_t} \cdot \frac{\left(x_{r(t)} - x_e\right)}{\left(x_{r(0)} - x_e\right)} \cdot a^{\pi,y}_{x_r(0)}. \tag{16}$$

By further assuming the uprating rate for pensions matches the discount rate, the fairness condition (16) reduces to

7 For the pension scheme as a whole to be revenue neutral, the actuarial adjustments should reflect as closely as possible the group-specific life expectancy and benefit amount.

8 This is particularly the case in NDC schemes in which the notional pension wealth and the benefit computation incorporate the internal (implicit) rate of return from a PAYG system and the expected remaining lifetime at retirement.

9 This scenario is referred to as the "100% shift scenario" in Schwan and Sail (2013).

$$\dot{e}^C_{x_r(t)} = \frac{D_0}{D_t} \cdot \frac{\left(x_{r(t)} - x_e\right)}{\left(x_{r(0)} - x_e\right)} \cdot \dot{e}^C_{x_r(0)}, \tag{17}$$

or, equivalently,

$$\frac{\dot{e}^C_{x_r(t)}}{\left(x_{r(t)} - x_e\right)} = \frac{\dot{e}^C_{x_r(0)}}{\left(x_{r(0)} - x_e\right)} \cdot \frac{D_0}{D_t}. \tag{18}$$

Equation (18) provides an interesting and important retirement age policy result. It shows that in an actuarially fair and neutral pension scheme, to deal with populations' extended survival prospects while keeping the replacement rate (global accrual rate) constant over time, the retirement age must be updated such that expected years in retirement relative to years of work and the contribution period, adjusted by the rate of increase in the scheme's old-age dependency ratio, must remain constant over time. This means that the extra lifetime must be divided proportionally over the working and retirement periods; i.e., the working population and retirees share the burden of life expectancy improvements. Moreover, for a constant labour market entry age and a stable old-age dependency ratio, pursuing the retirement age policy expressed in equation (18) is equivalent to a policy design targeting the expected years in retirement as a fixed share of adult life. Stated differently, in an actuarially – and thus intergenerationally – fair and neutral pension scheme, a retirement age policy targeting a constant balance (ratio) between time spent in work (or in adult life) and retirement (see, for example, the reform proposals in the UK) is consistent with a constant replacement rate (adequacy) across generations, as long as the scheme's old-age dependency ratio does not deteriorate. In a scenario of population ageing, lower participation rate, or higher structural unemployment levels, the ratio $\frac{D_0}{D_t}$ declines. To keep the scheme fair across generations, this will require future pensioners to enjoy a shorter fraction of their lives in retirement compared to previous generations.

From (17), the pension age increase required to keep constant the time spent in work (contributing) and in retirement, $\Delta x_{r(t)} = x_{r(t)} - x_{r(0)}$, is given by the initial contribution career multiplied by the percentage increase in life expectancy adjusted by the change in the scheme's old-age dependency ratio:

$$\Delta x_{r(t)} = \left(x_{r(0)} - x_e\right) \cdot \left(\frac{\dot{e}^C_{x_r(t)}}{\dot{e}^C_{x_r(0)}} \cdot \frac{D_t}{D_0} - 1\right). \tag{19}$$

Under this policy design, the extra period in retirement that is consistent with the intergenerational actuarial fairness condition is a fraction of the additional contribution years. From (18), we also conclude that targeting a constant balance between time spent in work and retirement requires updating the contribution period by a factor equal to the percentage increase in cohort life expectancy at the retirement age, adjusted by the OADR variation. Of course, society may decide to depart from the intergenerational fairness condition and adopt alternative longevity risk-sharing mechanisms between current and future pensioners combining actuarial fairness, financial sustainability, and social adequacy. One possible strategy is to pursue the following updating scheme:

$$\left(x_{r(t)} - x_e\right) = \left(x_{r(0)} - x_e\right) \cdot \left(\frac{\dot{e}^C_{x_r(t)}}{\dot{e}^C_{x_r(0)}} \cdot \frac{D_t}{D_0}\right)^{\phi}, \tag{20}$$

where ϕ is a risk-sharing coefficient with $\phi = 1$ corresponding to the retirement age policy set by equations (17) and (18). For values of ϕ in the range $]0, 1[$ the retirement age updates would only partially reflect life expectancy developments, whereas for $\phi = 0$ the policy option would be to keep the contribution period constant over time.

Compared to the CAR policy, a retirement age policy targeting a CRR requires smaller pension age increases to cope with life expectancy developments. This is because of the reduced impact of additional contribution years on pension entitlements as a consequence of the smaller accrual per contribution year. Compared with (11), the pension age adjustment prescribed by (15) no longer translates into a higher replacement rate at retirement. Once again, if lifetime earnings are revalued below (above) the scheme's discount rate, the required pension age adjustments would have to be comparatively smaller (higher) than in the baseline case. As in the previous case, if pensions are updated below (above) the scheme's discount rate, the required pension age correction would be smaller (higher). Finally, in the empirical part of the paper we follow the OECD baseline full-career simulation model and assume a labour market entry at the age of 22, common for all countries and subpopulations.

2.2. Forecasting the survival function

2.2.1. Bayesian model ensemble or averaging

This section draws heavily and resumes the stochastic mortality modelling and forecasting approach developed by Bravo et al. (2021) and applied here to produce life expectancy forecasts. The rationale behind the BME is that instead of producing best-estimate projections based on a single model presumed to be the true one, identified based on user-specified criteria (for example, Bayesian Information Criterion, forecasting accuracy measure, cross-validation), the projection model is determined combining (averaging) a set or subset (model confidence set) of models. The BME model combination aims at finding a composite model that best approximates the actual data generation process (known historical data) and its multiple sources of risk. The BME composite model design should by definition be superior to individual candidate models because, first, it explicitly addresses model uncertainty. Second, each model's shortcomings are ideally compensated within a statistically (data) driven optimal combination. Third, conditioning the statistical inference on a set of statistical models minimises the individual model-based biases and produces more realistic confidence intervals. This in turn improves the out-of-sample forecasting precision and provides a more accurate representation of forecast uncertainty for decision making.

Let each candidate model (learner) be denoted by M_l, $l = 1, \ldots, K$. This encompasses the set of probability distributions comprising the likelihood function $\mathcal{L}\left(y|\xi_l, M_l\right)$ of the observed data y in terms of model specific parameters ξ_l, and $\pi\left(\xi_l, M_l\right)$ the prior density of ξ_l under M_l. Consider a quantity of interest Δ present in all models, for instance, the predictive quantity of y. The marginal posterior distribution across all models is given by

$$\pi\left(\Delta|\mathbf{y}\right) = \sum_{k=1}^{K} \pi\left(\Delta|\mathbf{y}, M_k\right) \pi\left(M_k|\mathbf{y}\right), \tag{21}$$

where $\pi\left(\Delta|\mathbf{y}, M_k\right)$ denotes the forecast probability density function (PDF) based on model M_k alone, and $\pi\left(M_k|\mathbf{y}\right)$ is the posterior probability of model M_k given the observed data. The weight assigned to each model M_k is given by its posterior probability

$$\pi\left(M_k|\mathbf{y}\right) = \frac{\pi\left(\mathbf{y}|M_k\right)\pi\left(M_k\right)}{\sum_{l=1}^{K}\pi\left(\mathbf{y}|M_l\right)\pi\left(M_l\right)}, \tag{22}$$

with $\sum_{k=1}^{K} \pi\left(M_k|\mathbf{y}\right) = 1$. The BME PDF is a weighted average of the PDFs of the individual candidate models, weighted by their posterior model probabilities (Raftery et al., 2005).

The model combination approach requires (i) the identification (selection) of the model space (model confidence set), (ii) choosing a specific ensemble learning strategy (e.g., BME, Bagging, Stacking, Boosting). In the first stage, we identify the model space by ranking individual learners according to out-of-sample forecasting precision.[10] We split the data into a training and test set and implement a backtesting procedure considering five-year holding periods for all models and populations. The predictive performance was measured by the symmetric mean absolute percentage error (SMAPE), defined as

$$SMAPE := \frac{1}{n_{x,t}} \sum_{x=x_{\min}}^{x_{\max}} \sum_{t=t_{\min}}^{t_{\max}} \frac{\left|\dot{\mu}_{x,t} - \mu_{x,t}\right|}{0.5 \times \left(\dot{\mu}_{x,t} + \mu_{x,t}\right)}, \tag{23}$$

where $\dot{\mu}_{x,t}$ and $\mu_{x,t}$ denote the point forecast and observed mortality rates, respectively, and $n_{x,t} = (x_{\max} - x_{\min} + 1)(t_{\max} - t_{\min} + 1)$.

Secondly, we compute the posterior probability for each model using the normalised exponential (Softmax) function using:

$$\pi\left(M_k|\mathbf{y}\right) = \frac{\exp\left(-\left|\mathcal{S}_k\right|\right)}{\sum_{l=1}^{K}\exp\left(-\left|\mathcal{S}_l\right|\right)},\ k = 1, ..., K, \tag{24}$$

with $\mathcal{S}_k = \psi_k / \max\{\psi_l\}_{l=1,...,K}$ and $\psi_k = SMAPE$ for model k. The Softmax function is derived from the logistic function, commonly adopted in forecasting, regression, and classification exercises considering traditional or statistical learning (for example, machine learning, deep learning) methods as a combiner or an activation function. The function possesses a desirable characteristic in that it assigns larger weights to models with smaller out-of-sample forecasting errors, with weights following an exponential distribution.[11]

Model-averaged Bayesian credible intervals are derived using the MATA construction. The method consists of estimating confidence limits such that the weighted sum of error rates, computed using the BME posterior probability $\pi\left(M_k|\mathbf{y}\right)$, produces the required overall error rate.

2.2.2. Candidate stochastic mortality models

The set of individual single population heterogeneous stochastic mortality models considered in this study comprises a selection of well-known and commonly used GAPC parametric models, principal component methods, and smoothing approaches. The set of individual learners used adds ensemble diversity, a desirable property for a good model combination (Albuquerque et al., 2022). Table 1 recapitulates the analytical structure of the nine individual candidate models considered in this study; additional technical details are provided in Appendix A for completeness. The set comprises: (i) Six single-population GAPC models (LC, APC, RH, CBD, M7, Plat); (ii) A univariate functional demographic time-series model: the weighted Hyndman and Ullah (2007) Functional Demographic Model considering geometrically decaying weights (HUw); (iii) A bivariate functional data model: the Regularized Singular Value Decomposition (RSVD) model (Huang et al., 2009; Zhang et al., 2013); (iv) A two-dimensional smooth constrained P-splines model (CPspl), which imposes smoothness in mortality rates across years and ages (Camarda, 2019).

The first six models are well-known GAPC models: [LC] is the age-period Lee-Carter model under a Poisson setting for the number of deaths (Brouhns et al., 2002; Renshaw and Haberman, 2003); [APC] is the age-period-cohort model (Currie, 2006); [RH] is the Lee-Carter model extended to include cohort effects and particular substructure obtained by setting $\beta_x^{(0)} = 1$ and an additional approximate identifiability constraint (Renshaw and Haberman, 2006; Haberman and Renshaw, 2011); [CBD] is the Cairns-Blake-Dowd model considering a predictor structure with two age-period terms, prespecified age-modulating parameters $\beta_x^{(1)} = 1$ and $\beta_x^{(2)} = (x - \bar{x})$, with $\bar{x}$ the average age in the data, and no cohort effects (Cairns et al., 2006); [M7] is the CBD model with cohort effects and a quadratic age effect (Cairns et al., 2009); [Plat] is the Plat (2009) model with particular substructure obtained by setting $\kappa_t^{(3)} = 0$ to focus only on older ages.

Table 1
Analytical structure of the stochastic mortality models used in this study.

Model	Model structure
LC	$\eta_{x,t} = \alpha_x + \beta_x^{(1)}\kappa_t^{(1)}$
APC	$\eta_{x,t} = \alpha_x + \kappa_t^{(1)} + \gamma_{t-x}$
RH	$\eta_{x,t} = \alpha_x + \beta_x^{(1)}\kappa_t^{(1)} + \beta_x^{(0)}\gamma_{t-x}$
CBD	$\eta_{x,t} = \kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)}$
M7	$\eta_{x,t} = \kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)} + \left((x - \bar{x})^2 - \sigma_x^2\right)\kappa_t^{(3)} + \gamma_{t-x}$
Plat	$\eta_{x,t} = \alpha_x + \kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)} + (\bar{x} - x)^+\kappa_t^{(3)} + \gamma_{t-x}$
HUw	$y_t(x_i) = f_t(x_i) + \sigma_t(x_i)\epsilon_{t,i}$
CPspl	$\eta = \mathbb{B}\alpha$
RSVD	$m(x,t) = \sum_{j=1}^{q} d_j U_j(t) V_j(x) + \epsilon(x,t)$

Note: $\eta_{x,t}$ denotes the linear predictor; α_x and $\beta_x^{(i)}$ denote age-specific terms; $\kappa_t^{(i)}$ and γ_{t-x} are period and cohort indices; σ_x^2 is the mean of $(x-\bar{x})^2$; $y_t(x_i) = log(m_{x_i,t})$; $f_t(x_i)$ is a continuous and smooth function; $\sigma_t(x_i)$ is a volatility term; $\epsilon_{t,i}$ and $\epsilon(x,t)$ are error terms; $\mathbb{B}$ are B-spline bases with a roughness penalty; α is a vector of parameters.

Since some of the GAPC models described in Table 1 are particular cases of larger models, trimming models and determining a model confidence set may lead to better estimates of each model's posterior probabilities in the BME forecast (Hansen et al., 2011). For instance, model LC is nested within model RH, with $\beta_x^{(0)} = 0$ for all x, and $\gamma_{t-x} = 0$ for all c, being a special case of APC with $\beta_x^{(1)} = 1$ for all x and no cohort effects. Model APC is a special case of RH with $\beta_x^{(1)} = \beta_x^{(0)} = 1$ for all x. The CBD model is a restricted version of M7 with $\kappa_t^{(3)} = 0$ for all t and $\gamma_{t-x} = 0$ for all c. To address model diversity concerns and the existence of nested models, we follow Samuels and Sekkel (2017) and use a fixed-rule trimming scheme prior to averaging in which three out of the six GAPC candidates are discarded. The set of best models is determined based on the forecasting precision in the validation (test) period. Individual models are first calibrated using total population data from 1960 to the most recent year available. Since our focus is to discuss the implications of life expectancy improvements on retirement age policies, models are calibrated using an age range of $60 - 95$. Prediction intervals for age-specific mortality rates considering both stochastic process and parameter risk are derived using a bootstrap approach with 5000 bootstrap samples (Brouhns et al., 2005; Koissi et al., 2006). The Denuit and Goderniaux (2005) life table closing method with ultimate age set at $\omega = 125$ is assumed for all years, countries, and populations to ensure comparable and comprehensive cross-country results, with extrapolation starting at age 96.[12] The model fitting, forecasting, and simulation procedures were implemented using an R statistical software routine.

2.2.3. Life expectancy measures

Equipped with forecasts of age-specific mortality rates by year and sex for each population g, $m_{x,g}(t)$, the (complete) cohort and

[10] For details on the use of the model confidence set approach in longevity modelling see, e.g., Shang and Haberman (2018).

[11] Alternative choices for the posterior probability allocation include the normalized C-probability, the natural odds-based probability, the extreme C-probability, the normalized extreme C-probability, and the Sigmoid function (Bravo, 2022).

[12] We conducted a preliminary investigation on the impact of using alternative life table closure method testing alternative approaches (e.g., the Kannisto and Coale-Kisker methods) and concluded that the impact on the empirical results is negligible.

Table 2
Selected HMD countries and available data period used.

Available data	Countries and Regions
1960 – 2016	Australia (AUS), Canada (CAN), Denmark (DNK), Iceland (ISL), Netherlands (NDL), Poland (POL), Spain (ESP), England and Wales (ENW),
1960 – 2017	Austria (AUT), France (FRA), Ireland (IRL), Japan (JPN), Slovakia (SVK), Sweden (SWE), Switzerland (CHE), U.S.A. (USA)
1960 – 2018	Belgium (BEL), Finland (FIN), Norway (NOR)
1992 – 2008	Chile (CHL)
1990 – 2017	Germany (DEU)
1983 – 2016	Israel (ISR)
1960 – 2015	Portugal (PRT)

period life expectancy measures for an x-year old individual in year t are given, respectively, by:

$$\dot{e}^{C}_{x,g}(t) := \frac{1}{2} + \sum_{k=1}^{\omega-x} \exp\left(-\sum_{j=0}^{k-1} m_{x+j,g}(t+j)\right), \tag{25}$$

and by

$$\dot{e}^{P}_{x,g}(t) := \frac{1}{2} + \sum_{k=1}^{\omega-x} \exp\left(-\sum_{j=0}^{k-1} m_{x+j,g}(t)\right), \tag{26}$$

with ω denoting the highest attainable age, from which the concept of life expectancy gap at age x in year t, $\dot{e}^{Gap}_{x,g}(t)$, defined as the systematic difference between period and cohort life expectancy measures (Ayuso et al., 2021a) can be easily computed as $\dot{e}^{Gap}_{x,g}(t) := \dot{e}^{C}_{x,g}(t) - \dot{e}^{P}_{x,g}(t)$.

2.3. Mortality and pension age data

The datasets used in this study comprise mortality data and full pension age data. Mortality data are obtained from the Human Mortality Database (2021) and consist of observed death counts, $D_{x,t}$, and exposure-to-risk, $E_{x,t}$, classified by age at death ($x = 60, \ldots, 95$), year of death ($t = 1960, \ldots., 2018$) and sex for 23 homogeneous national populations (countries or areas) in different regions of the world. Table 2 lists the countries considered in this study together with details about data availability in the defined historical "lookback window", set from 1960 (or the most distant year available) to 2018 (or the most recent year available).

The pension age data include actual and forecasted standard retirement age by sex from 2000 to 2050 for 23 countries. The sample used in this study is representative of the diversity of retirement age policies adopted worldwide in the last two decades. The full (or normal) pension age considered in this paper is the age at which a worker can take his or her public pension without any decrement for early retirement.[13] For countries where a gender gap in standard retirement ages still exists, the male pension age is used as the benchmark. As of 2021, significant differences persist in the male pension age between countries and, in some cases, between genders, with retirement age ranging between 62 years (France) and 67 years (Norway, Iceland, Israel). In those countries in which the pension age is different for men and women, women have a lower retirement age. Our approach to gender differences in pension age is consistent with current trends toward harmonisation of legislated normal pension ages between genders.

In EU and OECD Member States, the most general normal pension age is still 65 years. Since 2000, 13 of the 23 countries studied in this paper have increased their full normal pension age, either by (i) introducing automatic indexation to life expectancy (Denmark, Estonia, Greece, Italy, the Netherlands, Portugal, Slovakia, Finland, Cyprus) with diverse policy goals, or (ii) adopting scheduled or ad hoc interventions. Some reform reversals occurred, for instance, in Canada, Poland, and Slovakia. Canada planned to increase the age for basic and means-tested pensions to 67 but finally decided against it. Poland reversed its planned increase to 67, dropping retirement ages back to previous levels (65 for men and 60 for women). The largest progression of the normal retirement age over the period 2008–2060 is projected in Denmark and the Netherlands, but a significant dispersion of pension ages is projected to persist in the long run (Carone et al., 2016; Ayuso et al., 2021b).

All countries have early retirement pathways (for example, in conjunction with very long contribution careers, long-term unemployment, or sickness insurance schemes for older workers), usually causing a reduction in pension benefits. In some countries (Sweden, Norway, Finland) people can retire flexibly; i.e., they can take out a full or partial old-age pension within a certain age range (for example, currently between 62 and 68 years in Sweden). However, access to resource-tested schemes (for example, minimum or guaranteed pensions) is restricted to those of a certain minimum age (65 in Sweden, rising to 66 in 2023). Following OECD guidelines, this age is used as the pensionable age herein. Variations in the pension age are observed between and within countries. For instance, in some countries (e.g., Australia), differences arise between the minimum public pension (age pension) and the retirement age of mandatory private schemes (superannuation), and different early retirement schemes may coexist.

3. Empirical results

3.1. Forecasts of the retirement age

Fig. 1 exhibits the BME point forecast of the cohort life expectancy at age 65 for the total population from 1960 to 2050 by country, along with the 95 percent MATA prediction intervals accounting for both (i) the uncertainty arising from the error in the forecast of the individual stochastic mortality model parameters, and (ii) the parameter uncertainty resulting from model fitting. We forecast for all countries a continuation of the long-term positive trend in cohort life expectancy, with Japan, France, and Switzerland leading the list in 2050 with 28.28, 26.90, and 26.34 years of expected remaining lifetime at age 65, respectively. We forecast that the total population cohort life expectancy at age 65 will increase by 47 percent in Japan, 44 percent in England and Wales, 42 percent in Finland, 38 percent in Australia, and 29 percent in the United States. If the full pension age is selected as the policy instrument to correct the distortion introduced by developments in life expectancy in intergenerational fairness, the retirement age must increase to restore the equilibrium condition.

Fig. 2 plots the actual and legislated full pension ages by country from 2000 to 2050, together with the point forecasts of the retirement age under both the CAR and CRR policy options in the baseline scenario; the baseline assumes that the lifetime earnings indexing rate, the scheme's discount rate, and the pension annual indexation rate are all equal (i.e., $\upsilon_t = y_t = \pi_t \ \forall t$) and that the

[13] In this paper we use the terms "pension age" and "retirement age" interchangeably meaning the statutory eligibility age for full old-age pension.

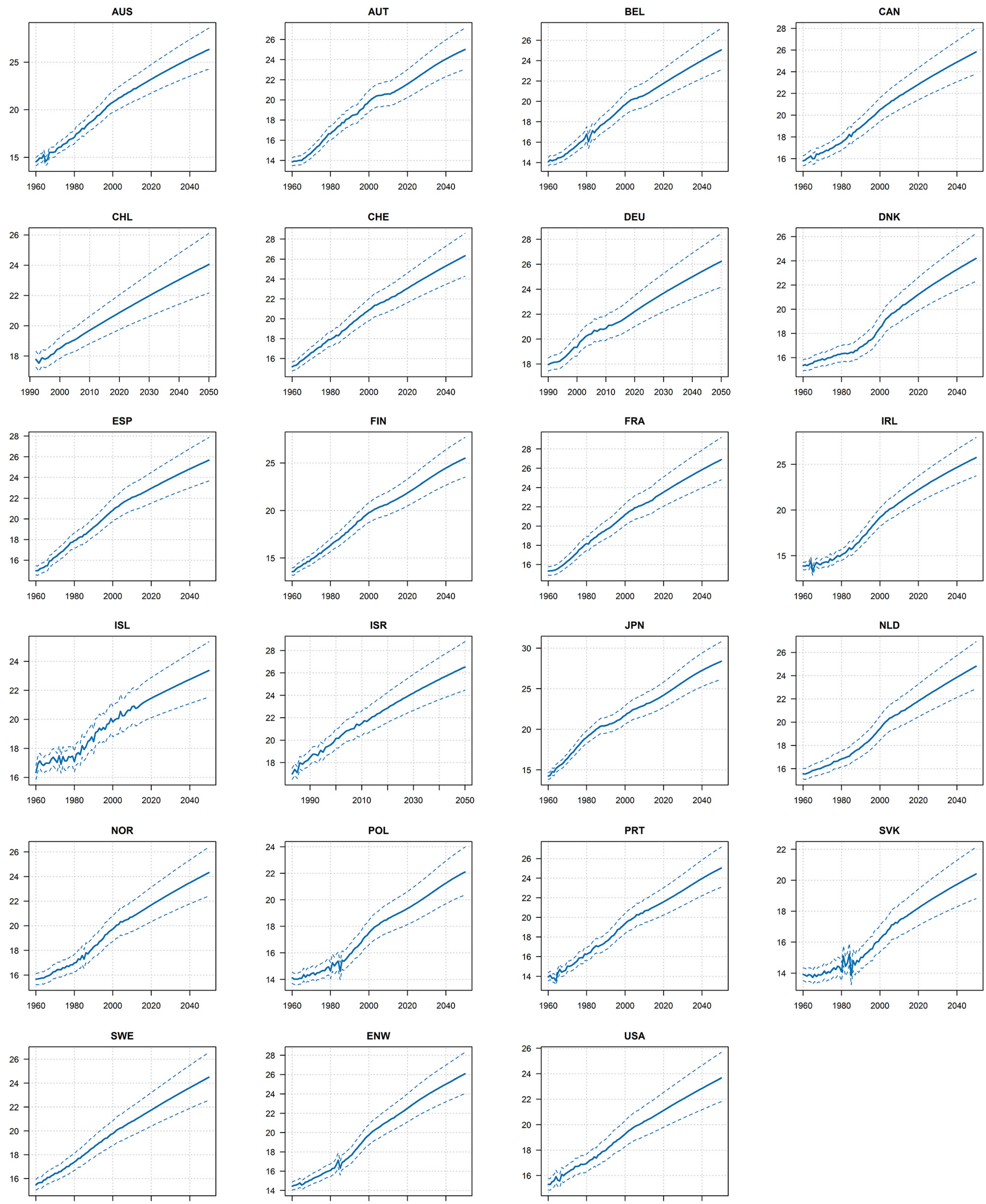

Fig. 1. Forecasts of the total population cohort life expectancy at age 65, along with 95% prediction intervals. *Note*: We note that, throughout the paper, the figures necessarily use different ordinate scales for each country.

scheme's old-age dependency ratio remains stable over time.[14] The year 2000 is selected as the starting point for our analysis since it marks the beginning of the most recent wave of pension reforms addressing the impact of population ageing and life expectancy increases in OECD countries after nearly a half-century of constant pension ages. Forecasts of the legislated pension age in countries following an automatic indexation mechanism to period life expectancy were derived using forecasts of the period life expectancy at the reference age and the formula stated in each country's national pension law.[15]

For all countries except Belgium (and partially Germany and Slovakia), which started from comparatively (much) lower retirement ages in 2000, the results for both the CAR and CRR retirement age policies show that the actual (2000–2021) and legislated retirement age increases have been and will be insufficient to cope with populations' extended survival prospects and to preserve the intergenerational fairness and neutrality conditions. The difference between the intergenerationally fair retirement ages and the actual/legislated retirement ones is, as expected, higher under a CAR policy option than under a CRR policy alternative, with gaps accumulating over time in both cases (Table 3). For instance, in 2020 the cross-country average difference between actual pension ages and those required to deal with cohort life expectancy improvements at labour market exit ages observed since 2000 is 1.59 years; the highest gaps are in Finland (3.55 years), Denmark (3.16 years), Chile (2.77 years), and Japan (2.63 years). Under a CAR policy option, these gaps are forecasted to increase to a cross-country average difference of 3.92 years in 2050; the highest corrections will be required in Japan (6.63 years), Finland (6.03 years), and Chile (5.99 years). The lowest values (discarding Belgium and Slovakia) are in the Netherlands (0.69 years), Denmark (2.30 years), and Portugal (2.54 years), countries that introduced automatic indexation of retirement ages but pursued alternative retirement age approaches.

The results obtained for the Netherlands and Denmark are particularly interesting to analyse, since both countries introduced automatic indexation of pension ages by adopting a retirement age policy that explicitly targets a constant period in retirement, an outcome demonstrated in section 2.1.1 to be consistent with the CAR policy option. Our results show, however, that in both countries the actual/legislated pension age increases are well below what will be required to preserve intergenerational fairness, particularly in Denmark. In both countries, this is explained essentially by poor policy design, particularly (i) the use of an incorrect (period) life expectancy measure instead of a cohort estimate (the life expectancy gap) in the indexation formula, and (ii) the existence of additional provisions capping the maximum increase in the pension age per period, indexation lags, and other design features that affect the final policy outcome (see Ayuso et al., 2021b for details). The results for Germany show that the scheduled pension increases follow roughly a CAR retirement age policy until 2029 when the ongoing updating path ceases. The results for the United States and Spain roughly approximate a CRR retirement age policy up to 2026, but further corrections will be required from that year on to cope with forecasted longevity improvements.

The difference between the corrections dictated by the CRR policy to match the intergenerational actuarial balance constraint and those implemented is smaller but still significant. For instance, in 2050 the average cross-country difference between actual/legislated pension ages and those required to deal with cohort life expectancy improvements and to keep up with intergenerational fairness is 2.01 years; the highest gaps are in Japan (4.38 years), Finland (4.08 years), and Chile (4.06 years), with 10 countries requiring an increase in the retirement age of at least 3 years. By 2050, the average cross-country difference between the retirement age corrections required by the CAR and CRR policy options is 1.91 years, with values ranging between 1.14 and 2.50 years.

3.2. Expected duration of retirement

Fig. 3 summarises the forecasts of the expected duration of retirement – the cohort life expectancy at the pensionable age – dictated by the CAR and CRR policies from 2000 to 2050, along with the expected years in retirement under the current/legislated retirement age path pursued by each of the 23 countries analysed in this study. Recall that, by construction, the expected years in retirement dictated by the CAR retirement age policy are constant and equal to those observed in the initial year, set to 2000 for all countries.

Our empirical results show that, first, despite the important increases in retirement age legislated in many OECD countries in the last two decades, the expected duration of retirement is forecast to increase in all countries analysed in this study, except in Belgium for the reasons mentioned above. In 2000, the average expected duration of retirement in the 23 countries analysed was 20.08 years, with values ranging between 16.71 years in Denmark and 25.82 years in France. In 2020, despite the major pension reforms adopted in 15 of the 23 countries, the average expected duration of retirement increased to 21.50 years, with France again leading the cohort life expectancy at the pensionable age (26.34 years for the total population). We forecast that the positive trend in the average duration of retirement will continue in the future, reaching 23.75 years in 2050, with a maximum of 29.80 years in France and 28.14 years in Japan (Table 4).

In relative terms, the largest increases in the expected duration of retirement are forecast for Chile (+29.8 percent or 5.51 years), Japan (+29.7 percent or +6.45 years), and Finland (+28.6 percent, or +5.67 years). Fig. 3 also shows that the only country in which the expected duration of retirement is forecast to roughly stabilise around 20 years is the Netherlands, above the 18.26 targeted by the legislated retirement age policy linking full pension age to life expectancy.

Second, we conclude that the adoption of a CRR retirement age policy would contribute to reducing the expected period of retirement by 1.91 years in 2050 when compared with legislated reforms. The results also show, however, that the increase in pension ages dictated by the CRR policy falls short of what will be needed to prevent a rise in the expected retirement duration and, in many cases, will not prevent the decline in the relative size of the labour force.

Fig. 4 summarises for all countries the expected duration of retirement relative to contribution years under the actual/legislated CAR and CRR retirement age policies. Recall that by construction, the CRR retirement age policy sets the pension age such that the ratio between expected years in retirement and contribution years is kept constant over time and equal, for each country, to the percentage observed in 2000. Assuming a fixed labour market entry age, set at age 22 in this study, a similar graph can be derived for

[14] The empirical results for other parameter combinations confirm the discussion in the section 2.1 and are available upon request.

[15] For instance, the formula stated in the Dutch pension law can be rewritten as:

$$x_r^{NLD}(t) = 65 + \left[\dot{e}_{65}^{P}(t) - 18.26\right],$$

whereas in Denmark it can be expressed as

$$x_r^{DNK}(t) = 60 + \left[\dot{e}_{60}^{P}(t-15) - 14.5\right]$$

with both countries targeting a constant period in retirement (see Ayuso et al. (2021b) for details). The Dutch government plans to adjust the life expectancy link from a one-to-one matching to two thirds link similar to the Portuguese formula in 2025.

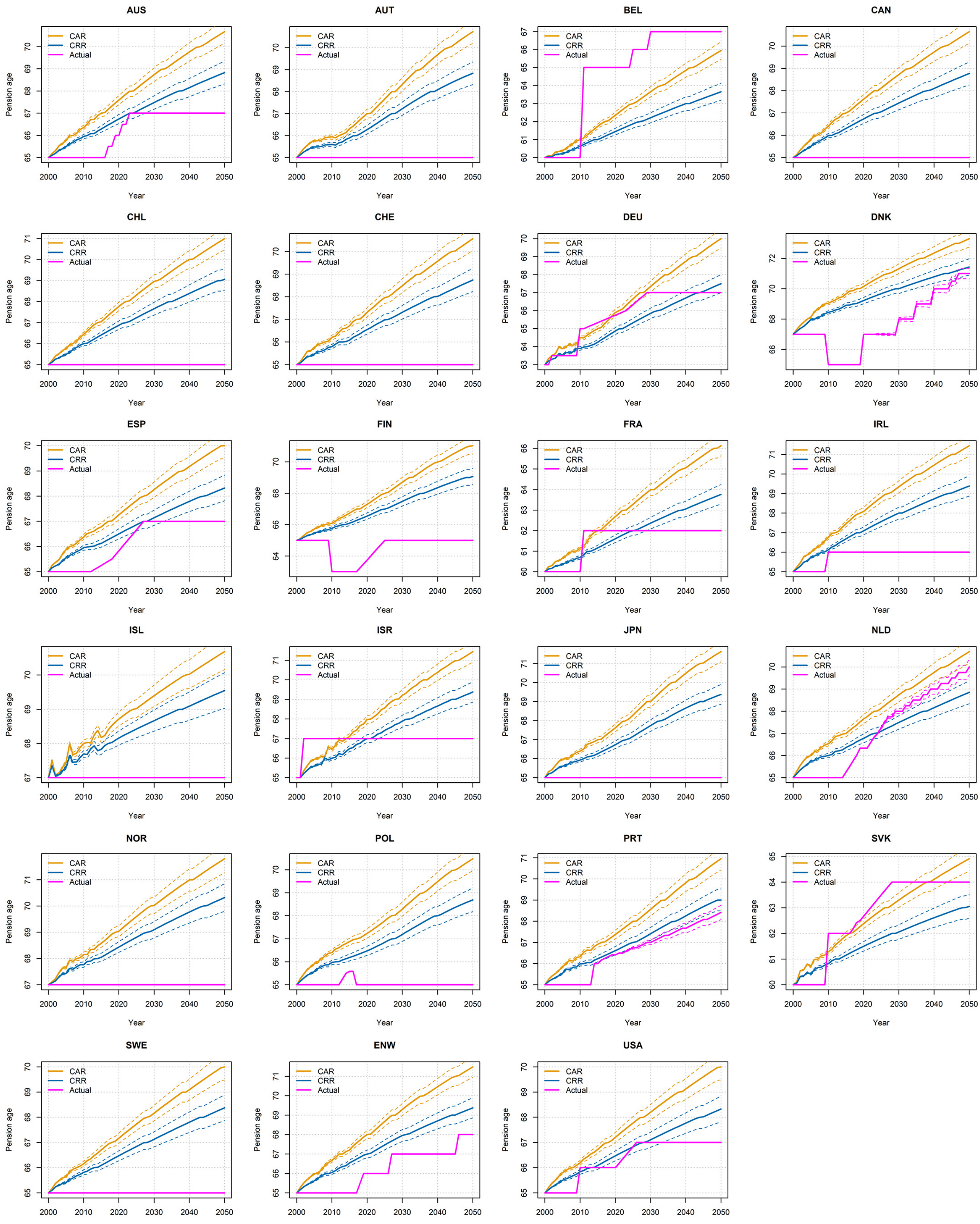

Fig. 2. Forecasts of the retirement age prescribed by the CAR and CRR policy designs, along with 95% prediction intervals.

Table 3
Difference between actual and CAR/CRR policy retirement ages.

Country	2010		2020		2030		2040		2050	
	CAR	CRR	CAR	CRR	CAR	CRR	CAR	CRR	CAR	CRR
AUS	1.36	0.95	1.59	0.74	1.74	0.48	2.76	1.15	3.68	1.83
AUT	0.95	0.60	2.00	1.27	3.33	2.22	4.68	3.09	5.73	3.84
BEL	1.00	0.64	−2.65	−3.55	−3.35	−4.79	−2.15	−4.00	−1.05	−3.34
CAN	1.41	0.99	2.62	1.73	3.73	2.45	4.74	3.10	5.66	3.77
CHL	1.45	1.00	2.77	1.88	3.96	2.66	5.00	3.39	5.99	4.06
CHE	1.21	0.79	2.32	1.56	3.49	2.32	4.58	3.02	5.57	3.74
DEU	−0.50	−1.11	0.15	−0.88	0.34	−1.15	1.75	−0.29	3.00	0.50
DNK	4.05	3.44	3.16	2.22	3.33	2.01	2.36	0.79	2.30	0.44
ESP	1.38	0.95	1.42	0.67	1.26	0.14	2.17	0.79	3.00	1.32
FIN	3.10	2.77	3.55	2.80	3.71	2.49	5.00	3.36	6.03	4.08
FRA	1.16	0.70	0.69	−0.39	2.00	0.37	3.06	1.06	4.14	1.77
IRL	0.76	0.13	2.09	1.09	3.35	2.00	4.45	2.73	5.45	3.39
ISL	1.00	0.69	1.72	1.16	2.40	1.67	3.02	2.09	3.69	2.55
ISR	−0.55	−1.00	0.98	0.00	2.21	0.89	3.38	1.67	4.45	2.38
JPN	1.40	0.93	2.63	1.74	4.00	2.74	5.49	3.66	6.63	4.38
NLD	1.53	1.00	1.32	0.44	0.74	-0.50	0.75	−0.84	0.69	−1.15
NOR	1.15	0.77	2.04	1.44	3.02	2.10	3.99	2.77	4.80	3.32
POL	1.40	0.97	2.23	1.48	3.29	2.19	4.49	3.00	5.48	3.69
PRT	1.33	0.99	0.96	0.20	1.56	0.41	2.16	0.59	2.54	0.59
SVK	−0.71	−1.18	−0.29	−1.17	−0.70	−1.94	0.11	−1.39	0.91	−0.94
SWE	1.15	0.82	2.17	1.47	3.17	2.13	4.10	2.80	5.00	3.38
ENW	1.67	1.05	2.00	1.00	2.29	0.95	3.47	1.71	3.49	1.38
USA	0.19	−0.16	1.21	0.47	1.20	0.11	2.13	0.77	3.00	1.33
Max	4.05	3.44	3.55	2.80	4.00	2.74	5.49	3.66	6.63	4.38
Min	−0.71	−1.18	−2.65	−3.55	−3.35	−4.79	−2.15	−4.00	−1.05	−3.34
Average	1.17	0.73	1.59	0.76	2.18	0.95	3.11	1.52	3.92	2.01

Notes: Difference in years between the forecasted pension age under both a constant accrual-rate-per-year (CAR) and constant replacement rate (CRR) policy options for selected years from 2010 to 2050. Positive (negative) values mean the CAR and/or CRR fair retirement ages are higher (lower) than those implemented and/or legislated.

Table 4
Expected duration of retirement under the legislated and CRR retirement age policies.

Country	2000	2020		2030		2040		2050	
	Legis	Legis	CRR	Legis	CRR	Legis	CRR	Legis	CRR
AUS	20.79	22.23	21.61	22.37	21.96	23.39	22.28	24.33	22.60
AUT	19.84	21.53	20.42	22.88	20.84	24.06	21.23	25.00	21.57
BEL	24.18	21.75	25.08	21.07	25.53	22.14	25.99	23.15	26.39
CAN	20.51	22.82	21.32	23.88	21.65	24.87	21.95	25.80	22.27
CHL	18.50	20.85	19.28	21.96	19.61	23.02	19.92	24.01	20.20
CHE	20.89	23.02	21.62	24.17	21.98	25.25	22.32	26.27	22.66
DEU	21.47	21.58	22.42	21.73	22.92	23.00	23.36	24.18	23.77
DNK	16.71	19.41	17.51	19.55	17.80	18.70	18.08	18.65	18.31
ESP	20.83	22.19	21.54	21.96	21.84	22.82	22.15	23.61	22.40
FIN	19.78	23.47	20.48	23.16	20.89	24.42	21.29	25.44	21.61
FRA	25.82	26.34	26.79	27.57	27.27	28.72	27.71	29.80	28.16
IRL	19.19	21.10	20.10	22.27	20.50	23.33	20.81	24.31	21.10
ISL	18.18	19.64	18.63	20.28	18.83	20.89	19.00	21.47	19.18
ISR	20.16	20.95	21.07	22.23	21.48	23.40	21.84	24.48	22.16
JPN	21.70	24.13	22.55	25.69	23.05	27.07	23.50	28.14	23.85
NLD	19.44	20.57	20.23	20.04	20.54	20.06	20.84	20.02	21.14
NOR	18.05	19.84	18.61	20.74	18.87	21.58	19.13	22.37	19.35
POL	17.50	19.18	18.09	20.12	18.37	21.12	18.66	21.91	18.96
PRT	19.36	20.23	20.07	20.75	20.42	21.40	20.79	21.72	21.12
SVK	20.21	20.06	21.01	20.82	21.29	21.51	21.55	22.14	21.77
SWE	19.77	21.74	20.43	22.69	20.73	23.61	21.03	24.48	21.29
ENW	19.75	21.50	20.70	21.98	21.07	23.06	21.41	23.09	21.71
USA	19.27	20.26	19.91	20.27	20.19	21.08	20.48	21.86	20.72
Max	25.82	26.34	26.79	27.57	27.27	28.72	27.71	29.80	28.16
Min	16.71	19.18	17.51	19.55	17.80	18.70	18.08	18.65	18.31
Average	20.08	21.50	20.85	22.10	21.20	22.98	21.54	23.75	21.84

Notes: By construction, the expected years in retirement dictated by the CAR policy are constant and equal to those observed in 2000.

the relationship between the expected duration of retirement and adult life.

Except for Belgium and the Netherlands, the results show that the expected period in retirement relative to the contribution period is expected to increase in all countries despite recent and legislated increases in standard pension ages. Substantial variations arise in the ratio between retirement and contribution periods among the countries analysed in this study. In 2000, the average cross-country ratio was 47.5 percent, with national values ranging between 37.1 percent in Denmark and 67.9 percent in France. The average cross-country ratio between retirement and contribution periods is forecast to increase to 53.9 percent in 2050, with France

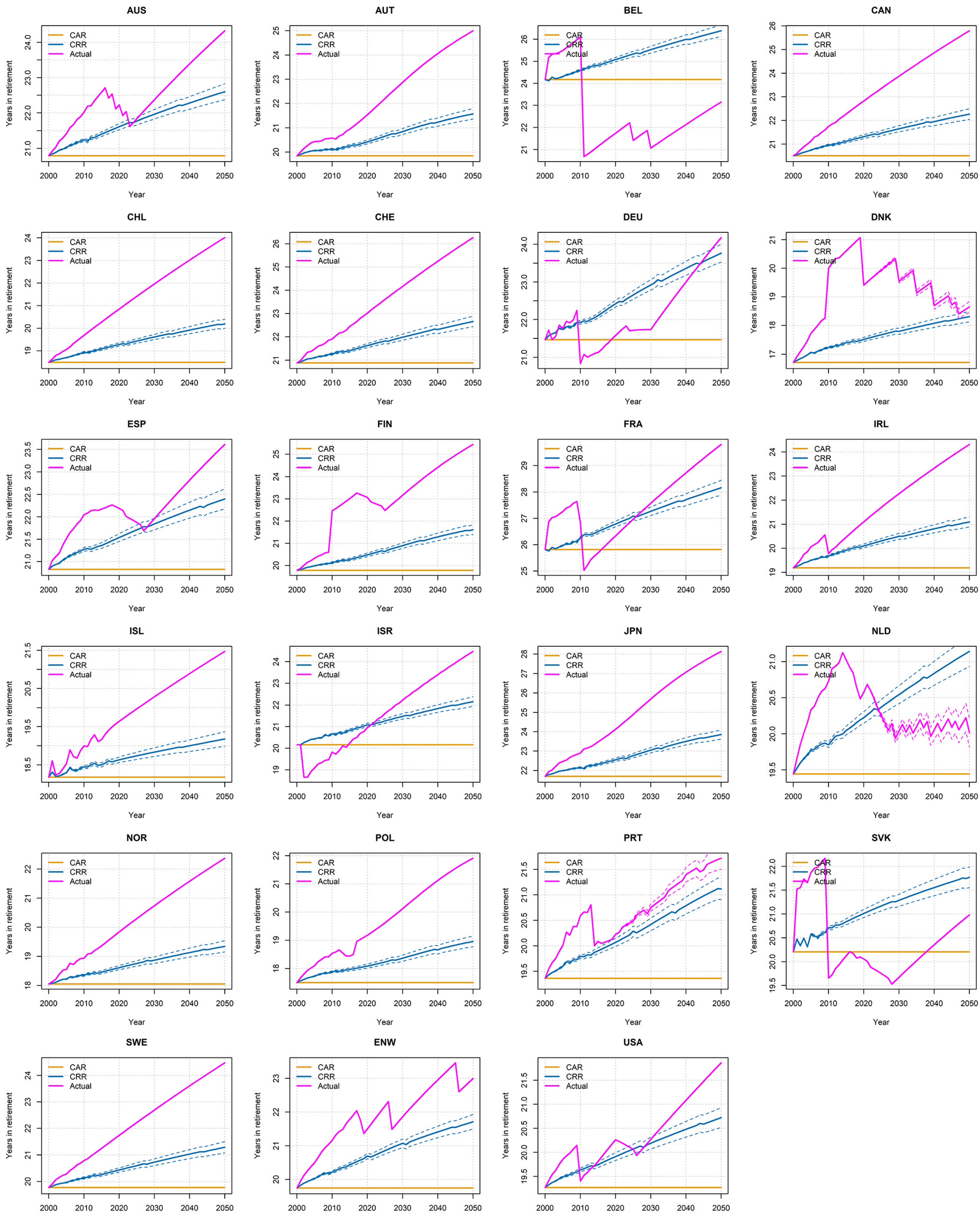

Fig. 3. Forecast of the expected retirement duration dictated by the actual, CAR and CRR retirement age policy designs, along with 95% prediction intervals.

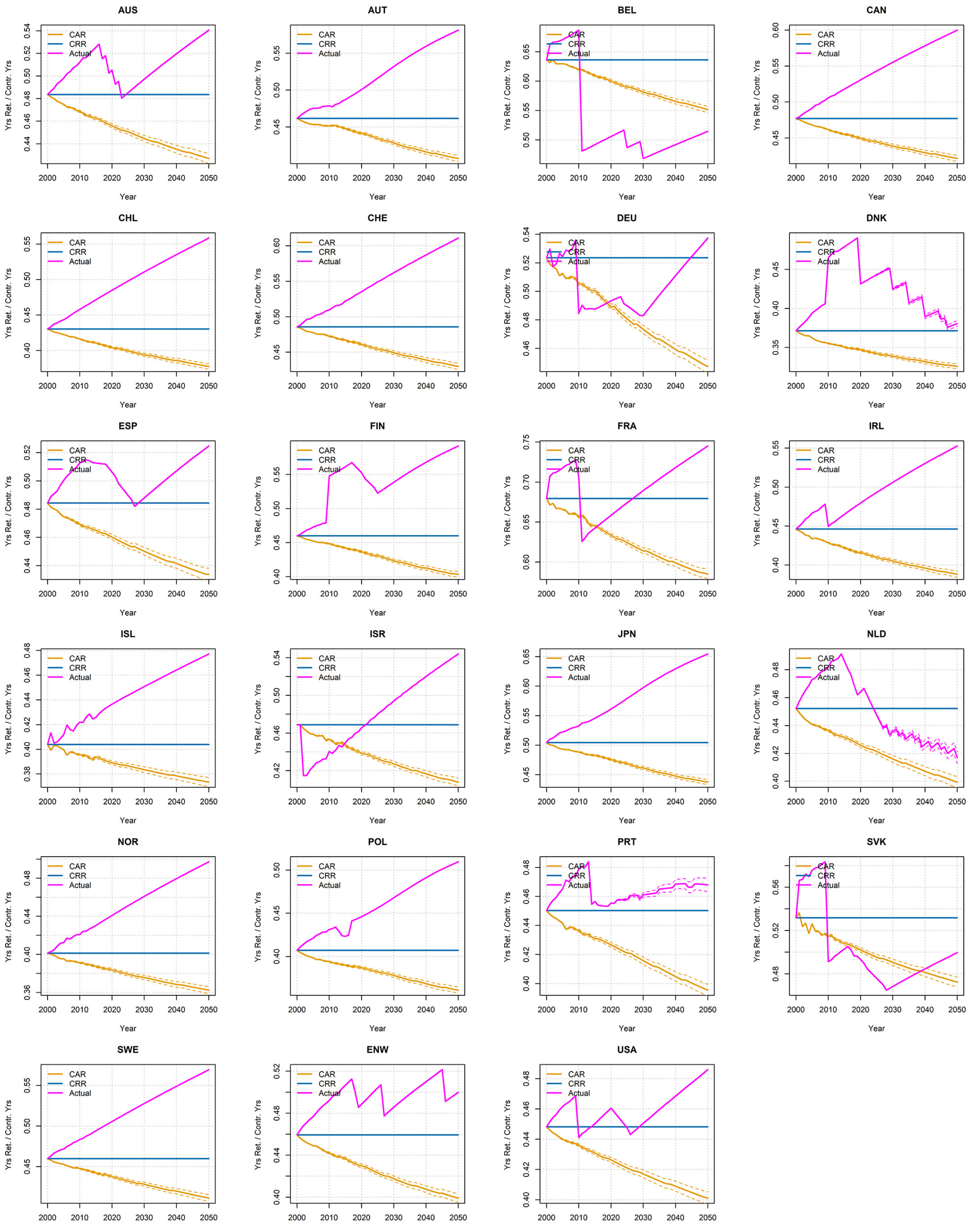

Fig. 4. Forecasts of the expected period in retirement relative to contribution years under the actual, CAR and CRR retirement age policy designs, along with 95% prediction intervals.

Table 5
Expected duration of retirement relative to contribution years under legislated and CAR retirement age policies (in %).

Country	2000	2020		2030		2040		2050	
	Legis	Legis	CAR	Legis	CAR	Legis	CAR	Legis	CAR
AUS	0.484	0.505	0.456	0.497	0.445	0.520	0.435	0.541	0.427
AUT	0.461	0.501	0.441	0.532	0.428	0.560	0.416	0.581	0.407
BEL	0.636	0.506	0.600	0.468	0.582	0.492	0.566	0.514	0.552
CAN	0.477	0.531	0.450	0.555	0.439	0.578	0.430	0.600	0.422
CHL	0.430	0.485	0.404	0.511	0.394	0.535	0.386	0.558	0.378
CHE	0.486	0.535	0.461	0.562	0.449	0.587	0.439	0.611	0.430
DEU	0.524	0.493	0.489	0.483	0.473	0.511	0.459	0.537	0.447
DNK	0.371	0.431	0.347	0.425	0.339	0.389	0.332	0.381	0.326
ESP	0.484	0.506	0.460	0.488	0.450	0.507	0.442	0.525	0.434
FIN	0.460	0.569	0.437	0.539	0.423	0.568	0.413	0.592	0.403
FRA	0.679	0.658	0.634	0.689	0.614	0.718	0.599	0.745	0.585
IRL	0.446	0.480	0.416	0.506	0.405	0.530	0.396	0.553	0.388
ISL	0.404	0.437	0.389	0.451	0.384	0.464	0.378	0.477	0.373
ISR	0.469	0.466	0.438	0.494	0.427	0.520	0.417	0.544	0.408
JPN	0.505	0.561	0.476	0.597	0.462	0.630	0.447	0.655	0.437
NLD	0.452	0.464	0.426	0.436	0.416	0.427	0.407	0.417	0.399
NOR	0.401	0.441	0.384	0.461	0.376	0.480	0.368	0.497	0.362
POL	0.407	0.446	0.387	0.468	0.378	0.491	0.369	0.510	0.361
PRT	0.450	0.455	0.427	0.461	0.416	0.469	0.405	0.468	0.395
SVK	0.532	0.493	0.502	0.512	0.491	0.529	0.481	0.544	0.472
SWE	0.460	0.506	0.438	0.528	0.428	0.549	0.420	0.569	0.411
ENW	0.459	0.489	0.430	0.488	0.418	0.512	0.407	0.502	0.399
USA	0.448	0.461	0.426	0.450	0.417	0.469	0.409	0.486	0.401
Max	0.679	0.658	0.634	0.689	0.614	0.718	0.599	0.745	0.585
Min	0.371	0.431	0.347	0.425	0.339	0.389	0.332	0.381	0.326
Average	0.475	0.496	0.449	0.504	0.437	0.523	0.427	0.539	0.418

Notes: Values in percentage.

peaking at 74.5 percent (Table 5). In Japan, the ratio is expected to increase 15 percentage points from 50.5 percent in 2000 to 65.5 percent in 2050, the largest percentage increase among the countries analysed. We highlight in particular the impact of pension reform reversals on the expected length of the retirement period in Poland and Slovakia, stopping and inverting earlier declines that had been phased in or legislated.

The empirical results also show that the adoption of a CAR retirement age policy to address intergenerational fairness and to cope with life expectancy developments significantly contributes to reducing the proportion of the expected period in retirement relative to contribution years (minus 5.7 percentage points), from an average cross-country ratio of 47.5 percent in 2000 to 41.8 percent in 2050. The reduction is much higher compared to the 2050 projected ratio for 2050 under legislated reforms (41.8 percent in 2050 versus 53.9 percent). For instance, keeping all other pension parameters constant, the adoption of a CAR retirement age policy in France would be sufficient to bring down the fraction of contribution years relative to years in retirement by 9.5 percentage points.

3.3. The impact of population ageing

Over the next three decades, old-age dependency ratios are projected to increase in all 23 countries analysed. Portugal is one of the countries with the oldest populations in the world. By 2050, the old age dependency ratio is projected to reach a peak of more than 65% in the country, the highest value in the European Union (2020). The sociodemographic ageing process in Portugal is driven by historically low fertility rates, significant increases in life expectancy at all ages, and negative natural and migration balances, particularly among working-age individuals. Portugal's population is projected to decline significantly (>20%) in the next decades.

The Portuguese pension system comprises three pillars. The dominant mandatory earnings-related DB public scheme (first pillar) comprises two separate but convergent schemes: (i) a private-sector workers scheme (general social security scheme—RGSS) and (ii) a civil service pension scheme (CGA) covering public servants enrolled before December 2005. Occupational pension schemes and accident insurance form the second pillar. The third pillar, personal pension provision, is voluntary and consists of various private personal funded schemes. There is a common time-dependent statutory retirement age for both men and women, which, from 2015 onwards, is automatically indexed every year by two-thirds of the cumulative period life expectancy improvements computed at the age of 65 (Bravo and Herce, 2022).

We forecast the size and age and sex composition of the Portuguese population using the standard cohort-component method stochastically modelling the components of demographic change. Forecasts of age-specific fertility and net migration rates are generated using the functional demographic data modelling approach (Hyndman and Booth, 2008), calibrated to data provided by Statistics Portugal from 1960 to 2019. Net migration is estimated using the demographic growth-balance equation. Forecasts of labour market participation rates and employment rates of men and women are taken from the 2021 Ageing Working Group (AWG) Report (European Commission, 2021). The retirement timing assumptions are taken from Bravo et al. (2015).

After reaching its peak population in 2008 of 10.6 million people, Portugal's population has been gradually declining. As of the 2021 census, Portugal's population is 10.344 million people. By 2050, we forecast the population to be 9.15 million people and by 2080 the population is expected to be down to 8.3 million people. This is a consequence of insufficient (below replacement level) fertility levels, a decline in reproductive potential, and negative net migration flows. The natural balance turned negative in 2009 and we forecast that it will remain so in the future, driven by the ageing of the population and a low birth rate. The number of elderly people (65 years and older) will increase from 2.4 to 3.1 million in 2080. The ageing rate will almost double, from 159 to 300 elderly people per 100 young people in 2080, due to the decrease in the young population (<15) and the increase in the elderly population (65+). The working-age population (15 to 64 years old) will decrease from 6.5 to 5.2 million people in 2050.

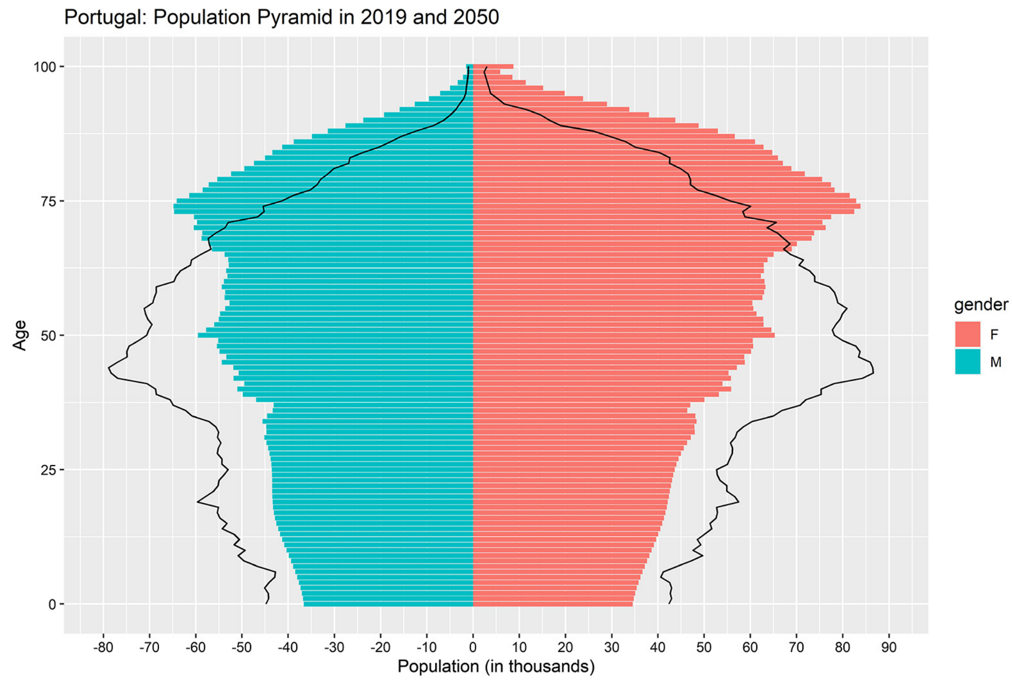

Fig. 5. Portugal - Age structure of the population in 2019 and 2050 by gender.

Fig. 5 represents the distribution of ages across the Portuguese population in 2019 (solid black line) by gender and the 2050 point forecast produced by the cohort-component method. The evolution of the population pyramid confirms that the Portuguese population is experiencing a triple demographic ageing process: (i) the long-term rise in average life expectancy is revealed in the widening of the top of the pyramid; (ii) the very low fertility levels contribute to narrowing the pyramid's base; (iii) the reduction in the reproductive potential as women get older contributes to reducing the number of births, further narrowing the pyramid's base. Portugal has one of the lowest birth rates in the EU, standing at 7.73 in 2021. The only countries below Portugal are Italy (7.1%) and Greece (7.1%).

The demographic dynamics have led to a deterioration in the demographic and pension scheme's OADR, despite an increase in labour market participation rates and employment rates, in particular for older workers and women, partially encouraged by recent pension reforms heavily penalizing early retirement and reducing the pension entitlements of future pensioners, with increasing risk of poverty in retirement (Fig. 6). In 2020, the demographic OADR (65+/15-64) was 34.7% of the working age population, compared to 24.0% in 2000. By 2050, our results show that the point forecast is 66.8%, i.e., only 1.5 working age individuals per individual aged 65 and over. This essentially reflects the negative demographic driver, with forecasts showing a reduction in working age population and total labour supply.

Fig. 7 shows, for Portugal, the actual and forecast values of pension age, years in retirement, and years in retirement relative to contribution years dictated by the CAR and CRR policy designs with (CAR-OADR, CRR-OADR) and without (CAR, CRR) considering population ageing, as measured by increases in the pension scheme's old-age dependency ratio. The results express both the demographic dynamics but also the forecasts of the labour market participation rates, employment rates, and labour market exit trends.

The results show, for both policy designs, that under conditions of population ageing, the statutory pension age will have to increase at a faster pace to satisfy the requirements of fairness between generations and financial balance. The reduction in the number of contributors per pensioner demands workers to stay longer in the labour market to meet the benefit obligations of retired generations. The alternative would be to substantially increase contribution rates as shown in Bravo et al. (2015) and/or further reduce the benefit generosity of future pensions, compromising pension adequacy. For instance, under a CAR policy design the statutory pension age would have to increase to 69.89 years by 2025 (against the legislated/predicted 66.75 years and the predicted 68 years in a steady state demographic scenario) and to 74.97 years by 2050 (against the legislated 68.42 years and the predicted 70.96 years in a constant pension scheme's OADR scenario).

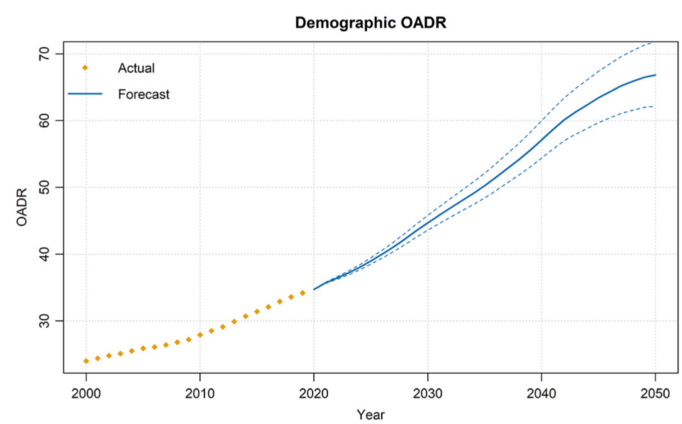

Fig. 6. Portugal - Actual and forecast values of the demographic old-age dependency ratio (65+/15-64), with 95% confidence limits.

Under a CRR policy design and population ageing, the retirement age adjustments are comparatively smaller but still significant. The statutory pension age would have to increase to 68.70 years by 2025 (3.14 years higher than the legislated/predicted 66.75 years and 1.95 years higher than that predicted in a steady state demographic scenario) and to 72.54 years by 2050 (4.12 years higher than the legislated 68.42 years and 3.54 years higher than that predicted in a constant pension scheme's OADR scenario).

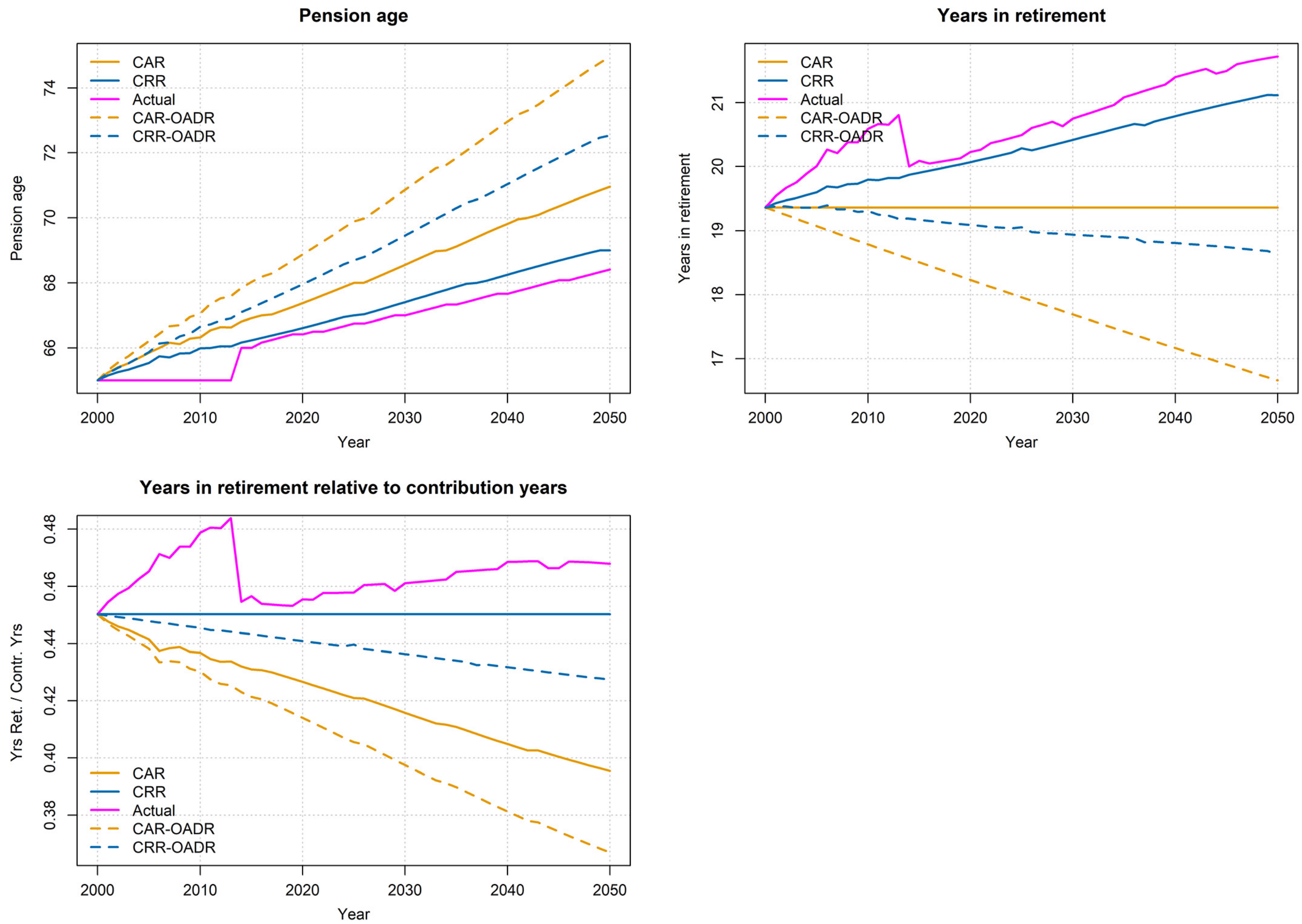

Fig. 7. Portugal - Forecasts of the pension age, years in retirement, and years in retirement relative to contribution years dictated by the CAR and CRR policy designs with and without considering population ageing.

The results also show that under conditions of population ageing and both a CAR and a CRR policy design, the expected retirement duration in Portugal will have to significantly decline if the pension scheme is to satisfy the requirements of fairness between generations. For instance, the expected retirement duration will have to reduce to 16.66 (18.63) years by 2050 under a CAR (CRR) policy design, versus 20.23 years estimated in 2020. The flat expected retirement duration prescribed by the CAR policy design in a steady state demographic scenario is no longer sufficient to guarantee intergenerational equity in Portugal if the number of contributors per pensioner diminishes. This means that future working generations will have to bear a higher burden of population ageing to achieve long-term affordability and fiscal sustainability.

Similarly, targeting a constant ratio between time spent in work (contributing) and in retirement as prescribed by the CRR policy design in a steady state demographic scenario is not sufficient to fulfil the intergenerational fairness condition under conditions of population ageing. The results show that the expected ratio between time spent in retirement and contributing will have to decline to 37% (43%) years by 2050 under a CAR (CRR) policy design, against the 46% estimated in 2020. Note that the findings obtained for Portugal can be extrapolated to other countries that experience similar demographic and labour market trends.

4. Summary, discussion and conclusions

The goal of indexing a country's normal retirement age and pension benefits in line with the development of life expectancy at pension ages is primarily to mitigate the impact of continuous improvements in longevity on financial sustainability. However, other important objectives should also be considered in assessing the social outcomes and political ramifications of pension reforms. Importantly for this paper, they include targeting intergenerational fairness in universal public pension schemes and instituting demographic, economic, and actuarial rationality for validating reforms. An overriding goal is to reinforce the transparency, credibility, and consistency of pension promises made to younger generations, on which the fulfilment and stability of the intergenerational social contract ultimately reside. Meeting the long-term pension promises – and being expected to do so – is essential to secure social and political trust and the support of the intergenerational contract, particularly under heightened uncertainty.

With these goals in mind, some countries have introduced automatic stabiliser rules to cushion the system from adverse demographic and/or economic events. Unlike discretionary adjustments, which are challenging to approve and carry political risks, especially if they involve major pension entitlement cuts, automatic stabilisers make it clear-cut why changes are needed providing at the same time a transparent and fair mechanism to regulate the size of the adjustment.

To compare countries' policy designs regarding how each country's treatment of life expectancy fulfils the criteria of a good universal pension system, this paper began by giving all countries the same scenario: an earnings-related pension scheme characterised by full proportionality between contributions on earnings benefits paid out. This enabled us to show how key pension parameters (the retirement age, the contribution rate, and the accrual rate) must adapt above all to the changing life expectancy of the pension-age population and population ageing to ensure that the scheme remains actuarially fair and is neutral across generations. Then, considering the normal retirement age as the key policy instrument and automatic stabiliser, we showed how to index the pension age to life expectancy developments and population ageing while respecting the principles of intergenerational actuarial fairness and neutrality among generations. Last, we analysed country outcomes empirically based on their current data and policy design vis-á-vis cohort life expectancy projections incorporating expected mortality and population structure developments.

Our analysis employed two design regimes that encompass essentially all universal public pension schemes. We showed that under stable demographic conditions – a constant pension scheme's old-age dependency ratio – and a CAR policy design, the pension age must be continuously updated to keep the period in retirement constant. This roughly corresponds to the strategy adopted in the Netherlands[16] and Denmark, which is to link the pension age to life expectancy. This said, however, both countries have chosen a period-based longevity measure with the well-known deficiency that basing calculations on period life expectancy systematically underestimates life expectancy when improvement in mortality is occurring continuously, albeit with varied rates of acceleration/deceleration in the rate of increase in mortality over time (e.g., Alho et al., 2013). This is, in fact, a general trend seen in developed economies during recent decades. We note that in both of these countries, the legislated indexation formulas include provisions limiting the increase in the retirement age per period and long indexation lags, generating additional deviation between the actual and the target (intergenerationally fair) number of years spent in retirement.

Alternatively, if policymakers wish to pursue a fixed replacement rate (CRR) objective, in which a longer contribution period barely changes pension entitlements, we showed that under stable demographic conditions, the retirement ages must be updated to ensure that the ratio between the number of years spent in work and retirement remains constant over time.

Our empirical scenario estimates for 23 countries have led us to conclude that the pension age increases required to fully accommodate the impact of longevity increases on financial equilibrium and to maintain equity between generations are substantial. And they are well beyond those recently observed and/or legislated. Consequently, the expected duration of retirement (both in absolute terms and relative to the contribution period) is projected to grow in the future. These results have key implications for policymakers since they may trigger a new wave of pension reforms to extend working lives, achieve long-term affordability and fiscal sustainability and restore intergenerational equity. The differences between actual/legislated retirement ages and retirement ages satisfying the requirements of fairness between generations are higher under a CAR policy option than under a CRR design, with gaps steadily increasing and implicit deficits accumulating over time in both cases.

Under conditions of population ageing beyond increasing life expectancy (i.e. insufficient births and net migration), the empirical results show that, for both the CAR and CRR policy designs, the statutory pension age will have to increase at a faster pace to meet the intergenerational equity criteria, transferring a larger fraction of the financial burden of longer lives and an ageing society to working generations.

[16] The Netherlands recently reformed the retirement age indexation formula, limiting the link between retirement age and life expectancy to an 8-month increase rather than a one-year increase per year of life expectancy gains. Consequently, the eligibility age will continue to grow at a slower pace than previously projected.

Indexing the pension age and/or adjusting the length of working lives and consequently career contributory requirements to longevity developments can square pension sustainability and pension adequacy in a scenario with population ageing and later labour market entry ages (adjusting to longer periods of education), rebalancing the number of years spent in work and retirement. What's more, extending working lives to accommodate increasing longevity is preferable to reducing pension levels through so-called sustainability factors or life expectancy coefficients adopted in some countries (e.g., in Finland, Portugal). The latter operate by decreasing the benefit ratio and thus generate increasing old-age (absolute and relative) poverty risks. Moreover, although the sustainability factor design rewards increasingly longer contribution careers, it does not provide for minimum adequacy safeguards. And these are critical for those at the lower end of the income and wealth (i.e., accumulated pension savings) distribution (Bravo and Ayuso, 2021).

Statutory pension ages (and the prevalence of early retirement options) determine the transition into retirement. Despite recent reforms, empirical evidence shows that the average gap between the statutory normal and early retirement ages and the effective retirement age in OECD countries is still significant. And the gap is expected to remain so over the next decades (OECD, 2019a). This, in turn, calls into question to what extent it will be possible for the older working-age population to adjust their behaviour in line with an increasing statutory pension age.

To discuss this, we remind ourselves of some of the key factors affecting labour demand and supply. The propensity of individuals to work up to higher ages depends on many factors: health, the desire for leisure time to pursue other interests, employer policies, trade union policies, care of relatives, and cultural norms. And, generally, as they become older workers will increasingly compare the financial awards of extending their working career with the utility of leisure time.

Lengthy unemployment spells late in the work-life may lead to irreversible and not always intended labour force withdrawal as unemployed elderly typically prefer the certainty of a lower early pension to the uncertainty of an unemployed. Also, it is not uncommon for female spouses to exit the labour force at the same time as their (on average) older male partner. More generally, many opt for early retirement without considering significant pension entitlement losses (Bravo and Herce, 2022). Other factors underlying the decision to retire from the workforce are also significant. These include both explicit and implicit "taxes" on working additional years, as well as other policies that may distort decisions to retire. This is an issue raised by, e.g., Gruber and Wise (1998), Bravo (2016); and Holzmann and Piggott (2018).

In the remainder of this discussion, we briefly highlight four key areas that need further elaboration and research in order to make an increase in retirement age a truly relevant and winning proposition to establish financial stability and intergenerational fairness in pension scheme reforms.

Discussion

The role of incentives to work longer Instead of imposing a fixed uniform retirement age for all, in some countries where there is a strict actuarial link between contributions and benefits in the public pension scheme (e.g., Norway's and Sweden's NDC schemes) a minimum pension age is set at the youngest age at which the universal public pension can be claimed – roughly in line with the CAR principle examined in Section 3. From this age, workers are free to draw on a full or partial pension benefit and can continue working full or part-time if they so choose.

In contrast to this, some countries public pension schemes continue to have early retirement provisions that allow individuals to stop working before reaching the statutory retirement age by accepting "penalized" (lower) monthly benefits, sometimes computed in a (non-actuarial) ad hoc way. Incentives that reward additional healthy years of life devoted to continued work reward the individual while at the same time enhancing the country's GDP. Retirement age policies also interact with the consequences of healthy years of life, with other benefit programmes (e.g., unemployment compensation and disability insurance), with work environments and age discrimination and ceilings on legally supported maximum employment ages. Other studies in this area investigate the optimal retirement age and the impact of incentives on individual decisions (Cremer and Pestieau, 2003; Fehr et al., 2003, 2012; Galasso, 2008; Freudenberg et al., 2018; Rabaté, 2019).

Socio-economic heterogeneity in life expectancy How effective and accepted changes to the statutory pension age are depends considerably on the pension system arrangements, social and workplace factors, health, and other reforms (e.g., lifelong learning). Changes may be influenced by heterogeneity among the population. Life expectancy and pension wealth can differ substantially by socio-economic class, income levels, educational attainment, gender, labour market entry age and job, type of work, health condition, and geography (Chetty et al., 2016; OECD, 2017). A large and increasing body of empirical evidence shows that individuals with higher socio-economic status – measured by income, education, or occupation – tend to live longer and enjoy better health than those with lower income, education, and occupational status. And at the other end of the income distribution low life expectancy is strongly correlated with low-income lifestyle risk factors such as smoking, use of narcotics, alcohol abuse, and obesity. This translates into an ex-ante unintended subsidy from lower socio-economic groups to wealthier groups.

In addition to the socioeconomic gradient in longevity, the gender gradient creates an implicit tax/subsidy mechanism redistributing pension wealth from men to women in a retiring birth cohort. Disregarding women's longer average life expectancy when calculating retirement benefits contributes to narrowing the gender pension gap but there are several design features of public pensions and retirement savings plans (e.g., contributions, accrued rights during periods of maternity or parental leave or for the time spent caring for the family, enrolment in pension plans, the accumulation of assets, pay-out phase options, purchasing power mechanisms during retirement, survivor benefits) which are not gender neutral and many tend to disadvantage women. Policy initiatives such as transferring pension entitlements and assets in retirement savings plans between spouses, offering joint life annuities, making old-age survivor pensions the default option for couples, and offering higher child-care credit systems to boost mothers' pension entitlements can contribute to narrowing the gender gap in pensions.

Higher educational attainment delays the labour market entry age but leads to better labour-market outcomes, longer and more stable contribution careers, and is positively correlated with life expectancy (OECD, 2017). Those with lower educational qualifications tend to earn less and are often at greater risk of unemployment. Higher education levels set the ground for improving the socio-economic conditions in which people live and work, facilitate access to better health care, and tend to promote the adoption of healthier lifestyles. This is an important source of heterogeneity to be considered in pension reform.

The general presumption of both economic theory and the rationale behind most countries pension policy is that participants in a pension scheme enter retirement characterized by a random distribution of longevity outcomes among all new retirees – despite abundant evidence that this is not the case. This is clearly an area where additional research is to be welcomed. For example, the results of Chetty et al. (2016) and numerous similar (but smaller) country studies suggest that the remedy for distributional inefficiency is to address the problems that lead to the skewed distribution with targeted social policy (e.g., Palmer and Zhao de Gosson de Varennes, 2020). This is by nature a long-run multi-faceted policy strategy that is outside the domain of the narrow focus of immediate pension policy. Given this restriction, the most viable alternative may be to introduce a tax-transfer structure into the pension pool (see e.g., Holzmann et al., 2020).

Is there a growing gap between life expectancy and healthy working life expectancy? The minimum pension age should in principle be set at an age that encourages longer working careers in an increasingly more mechanized and worker-friendly working environment. Nevertheless, both poor health and the desire for leisure at increasingly older ages may impede significant extension of working lives, despite increasingly better health.

Interestingly, according to the OECD's Health at a Glance 2019 OECD survey (OECD, 2019b) about 50% of men and women self-report "activity limitation" at age 65. Self-determined activity limitation is, of course, a subjective measure. For example, the life expectancy of French women at age 65 in 2017 was 23.6 years, but only 46% reported "no limitations" in the Healthy Life Year Survey. On the other hand, about 75% of Norwegian and Swedish women – with "only" 21.5 years of life expectancy at age 65 – report "no limitations." This suggests that it may be a subjective as much as objective assessment of healthy life expectancy that drives individual behaviour. This said, social inequalities in health continue to grow (Jivraj et al., 2020). A recent study of English data predicts a widening gap between overall life expectancy and healthy working life expectancy (HWLE), suggesting that working lives are not extending in line with policy goals (Lynch et al., 2022).

Recent demographic evidence in developed countries (e.g., Denmark, Greece, Italy, and Japan) suggests life expectancy improvements may be decelerating (Leon et al., 2019; Raleigh, 2019 Djeundje et al., 2022), followed by a rotation of the age pattern of mortality decline (Li et al., 2013). Djeundje et al. (2022) highlight a notable gender difference, with women experiencing more often lower mortality improvements than projected during the 2011–2017 period. Several hypotheses for the slowdown in mortality improvements have emerged, including worsening trends in diabetes and obesity, socioeconomic inequality in mortality rates, the stabilisation of smoking prevalence rates and cholesterol levels, retrenchment policies following the 2008 economic recession, and public debt problems in some OECD countries, particularly in healthcare and long-term care, or excess winter deaths in some years. The longevity developments in some countries are not, however, consistent with many of these hypotheses suggesting that there may be other factors driving the slowdown. Further research should investigate this topic.

The pandemic has asymmetrically impacted both the health and employment levels of different socioeconomic groups. It is still uncertain the extent to which the COVID-19 pandemic outbreak will permanently affect human longevity and healthy working life expectancy (HWLE) prospects at all ages and hence on future retirement ages. Mortality shocks challenge the reliability of traditional (e.g. Lee-Carter) mortality forecasts for pension schemes as well as life and health insurers, highlighting the importance of departing from a single-model approach toward the use of model combinations to better approximate the actual data generation process and its multiple sources of risk.

Final words on getting life expectancy estimates right for pension policy A weakness in the way the retirement age and pension benefits have been linked to longevity developments is the use of unisex life expectancy measures computed from national statistical office data using period life tables instead of cohort life expectancy measures. This is regrettable since it has been known for some time that period life expectancy lags behind expected longevity improvements – leading to systematic underestimation of remaining life (Alho et al., 2013). Recent empirical studies show that the difference between period and cohort life expectancy measures is sizable, persistent, and still increasing in most countries, translating into an ex-ante unintended financial transfer from future to current generations (Bravo et al., 2021).

As discussed in the text, the present study addresses this issue by applying a stochastic mortality modelling BME approach to project cohort life expectancy. Despite this procedure's higher complexity compared to the traditional single-model approach, the method mitigates the shortcomings of individual learning algorithms, providing a robust statistical framework to produce (and incorporate) plausible future longevity scenarios in policy design.

Further research should investigate the development of the BME approach to mortality forecasting. This includes, for instance, examining alternative methods for constructing the model space, the selection of a specific ensemble learning strategy (e.g., BME, Bagging, Stacking, Boosting, metalearning approaches), the computation of posterior model probabilities (the model weighting scheme), the determination of the training and test sets, the selection of the predictive performance metrics, and the selection of the life table closure method testing alternative approaches such as, for instance, the Kannisto method, the Coale-Kisker method and the Heligman-Pollard Model or Extreme Value Approaches (see, e.g., Huang et al., 2020).

Data availability

Data will be made available on request.

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Appendix A. Stochastic mortality models: technical description

This section draws heavily on Bravo et al. (2021a) and recapitulates the key technical details of the individual stochastic mortality models considered in the Bayesian Model Ensemble approach.

A.1. GAPC stochastic mortality models

Generalised Age-Period-Cohort (GAPC) mortality models are a class of parametric models that link a response variable with a linear or bilinear predictor structure consisting of a series of factors dependent on age of the individual, x; period effects, t; and year of birth (or cohort) effects, $c = t - x$. The structure of GAPC models includes a random component, a systematic component, a (canonical) link function, a set of parameter constraints to ensure identifiability and time series methods for forecasting and simulating the period and cohort indexes (Hunt and Blake, 2021). The random component specifies whether the number of deaths recorded at age x during calendar year t, $D_{x,t}$, follows a Poisson distribution $D_{x,t} \sim \mathcal{P}\left(\mu_{x,t} E^c_{x,t}\right)$, with $\mathbb{E}\left(D_{x,t}/E^c_{x,t}\right) = \mu_{x,t}$, or a Binomial distribution $D_{x,t} \sim \mathcal{B}\left(q_{x,t} E^0_{x,t}\right)$, with $\mathbb{E}\left(D_{x,t}/E^0_{x,t}\right) = q_{x,t}$, where $E^0_{x,t}$ and $E^c_{x,t}$ denote, respectively, the population initially or centrally exposed-to-risk, and $q_{x,t}$ is the one-year death probability for an individual aged x last birthday in year t. The systematic component links a response variable to an appropriate linear predictor $\eta_{x,t}$

$$\eta_{x,t} = \alpha_x + \sum_{i=1}^{N} \beta^{(i)}_x \kappa^{(i)}_t + \beta^{(0)}_x \gamma_{t-x}, \tag{A.1}$$

where $\exp(\alpha_x)$ denotes the general shape of the mortality schedule across age, $\beta^{(i)}_x \kappa^{(i)}_t$ is a set of N age-period terms describing the mortality trends, with each time index $\kappa^{(i)}_t$ contributing in specifying the general mortality trend and $\beta^{(i)}_x$ modulating its effect across ages, and the term $\gamma_{t-x} \equiv \gamma_c$ accounts for the cohort effect c with $\beta^{(0)}_x$ modulating its effect across ages. The age modulating coefficients $\beta^{(i)}_x$ can be preset or nonparametric terms to be estimated. Parameter estimates are obtained using maximum-likelihood methods. The period $\kappa^{(i)}_t$ and the cohort γ_{t-x} indices are treated as stochastic processes and modelled with general univariate ARIMA(p, d, q) methods to generate forecasts of age-specific mortality rates or probabilities. The model specification is complemented with a set of parameter constraints to ensure unique parameter estimates.

A.2. Weighted Hyndman-Ullah method

The Hyndman and Ullah (2007) method combines functional principal component analysis (PCA) with nonparametric penalised regression splines. Assume that the logarithm of the observed mortality rate at age $x \in \left[x_1, x_p\right]$ in year $t \in [t_1, t_n]$, $\log m_{x_i,t} \equiv y_t(x_i)$ is a realization of an underlying continuous and smooth function $f_t(x_i)$ that is observed with error at discrete ages:

$$y_t(x_i) = f_t(x_i) + \sigma_t(x_i)\,\varepsilon_{t,i}, \; i = 1, .., p \;\; t = 1, ..., n, \tag{A.2}$$

where $\sigma_t(x_i)$ allows the amount of noise to vary with x_i in year t, thus rectifying the assumption of homoscedastic error in the LC model, and $\varepsilon_{t,i}$ is an independent and identically distributed standard normal random variable. The log mortality rates are smoothed prior to modelling using penalized regression splines with a partial monotonic constraint. Using functional PCA, the smoothed mortality curves $\mathcal{I} = \{y_1(x), .., y_n(x)\}$ are then decomposed into orthogonal functional principal components and their uncorrelated principal component scores. The original Hyndman-Ullah (HU) method was extended by Shang et al. (2011) using geometrically decaying weights (instead of equal weights) in the estimation of the model parameters. Formally,

$$f_t(x) = \hat{a}^*(x) + \sum_{j=1}^{J} b^*_j(x)\, k_{t,j} + e_t(x), \tag{A.3}$$

where $\hat{a}^*(x)$ is the weighted functional mean age function estimated by:

$$\hat{a}^*(x) = \frac{1}{n} \sum_{j=1}^{J} w_t f_t(x), \quad \sum_{j=1}^{J} w_t = 1, \tag{A.4}$$

where $\{w_t = \pi(1-\pi)^{n-t},\ t=1,...,n\}$ denotes a set of weights, and $\pi \in (0,1)$ refers to the geometrically decaying weight parameter, with the optimal value chosen so as to minimise an overall forecast error measure within the validation data; $\mathcal{B}^* = \left\{b_j^*(x)\right\}\ j=1,...,J$ is a set of weighted first J functional principal components with uncorrelated principal component scores $\{k_{t,j}\}$ derived by functional PCA from the set of weighted curves $\{w_t[f_t(x) - \hat{a}^*(x)];\ t=1,...,n\}$; $e_t(x)$ is the residual function with mean zero and variance $\upsilon(x)$ estimated by averaging $\{e_1^2(x),...,e_n^2(x)\}$, $e_t(x) \sim \mathcal{N}(0,\upsilon(x))$; and $J<n$ is the number of principal components used.

A.3. CP-splines model

Camarda's (2019) CP-spline model extends the two-dimensional P-splines model by incorporating demographic constraints to ensure that future mortality over the whole age range follows a plausible and well-behaved demographic profile when estimated from past data. Consider a mortality dataset comprising deaths and exposure-to-risk arranged in two $m \times n$ matrices, $\boldsymbol{Y} = (d_{ij})$ and $\boldsymbol{E} = (E_{ij})$, respectively, with rows and columns classified by single age at death (x, $m \times 1$) and single year of death (t, $n \times 1$), respectively. The approach assumes that the number of deaths d_{ij} at age i in year j is Poisson-distributed with mean $\mu_{ij}E_{ij}$, i.e., $d_{ij} \sim \mathcal{P}(\mu_{ij}E_{ij})$. The goal is to model and forecast mortality over both age and time combining (fixed knot) B-splines with a roughness penalty to achieve a compromise between fitting accuracy and smoothness. Let $\boldsymbol{B}_x$, $m \times k_x$ and $\boldsymbol{B}_t$, $n \times k_t$ be the B-splines over ages and years, respectively. The log mortality is described as a linear combination of B-splines and associated coefficients ($\boldsymbol{\alpha}$):

$$\ln[\mathbb{E}(\boldsymbol{Y})] = \ln(\boldsymbol{E}) + \boldsymbol{B\alpha} \tag{A.5}$$

where $\ln(\boldsymbol{E})$ is the offset and $\boldsymbol{\eta}=\boldsymbol{B\alpha}$ is the linear predictor. The regression matrix for the two-dimensional model is given by the Kronecker product of the k equally spaced B-splines bases for age x and year t, $\boldsymbol{B} = \boldsymbol{B}_t \otimes \boldsymbol{B}_x$, where $\otimes$ denotes the Kronecker product of two matrices. The two-dimensional penalty is given by

$$\boldsymbol{P} = \lambda_x\left(\boldsymbol{I}_{k_t} \otimes \boldsymbol{D}_x'\boldsymbol{D}_x\right) + \lambda_t\left(\boldsymbol{D}_t'\boldsymbol{D}_t \otimes \boldsymbol{I}_{k_x}\right), \tag{A.6}$$

where λ_x and λ_t are the smoothing parameters used for age and year, respectively; $\boldsymbol{I}_{k_x}$ and $\boldsymbol{I}_{k_t}$ are identity matrices of dimension k_x and k_t, respectively; and $\boldsymbol{D}_x$ and $\boldsymbol{D}_t$ are difference matrices over the rows (ages) and columns (years) of the coefficient matrix. The model includes shape constraints and asymmetric penalties on the rate of aging (relative derivatives of the age mortality profile), $\boldsymbol{D}_x^t$, and on the rate of change of mortality rates over time, $\boldsymbol{D}_t^t$, to enforce mortality patterns over age and time.

A.4. Regularized SVD model

Huang et al. (2009) and Zhang et al. (2013) extend one-way functional PCA to two-way functional data by introducing regularisation of both left and right singular vectors in the singular value decomposition (SVD) of the data matrix. The authors assume the regularized SVD (RSVD) fits the following model for explaining the mortality rate in terms of period t and age x

$$m(x,t) = \sum_{j=1}^{q} d_j U_j(t) V_j(x) + \varepsilon(x,t), \tag{A.7}$$

where d_q is the singular value, $U_i(\cdot)$ and $V_j(\cdot)$ are smooth functions of period and age, respectively, and $\varepsilon(x,t)$ is a mean zero random noise. The model is fitted iteratively. The first pair of singular vectors of a data matrix $\boldsymbol{X} = (m_{x,t})_{n\times p}$, $U_1(t)$ and $V_1(x)$, whose discretized realisations are, respectively, denoted as $\boldsymbol{u}_1 \equiv (U_1(t_1),...,U_1(t_n))^T$ and $\boldsymbol{v}_1 \equiv (V_1(x_1),...,V_1(x_p))^T$, is obtained by solving a least squares problem as

$$(\hat{\boldsymbol{u}},\hat{\boldsymbol{v}}) = \underset{(\boldsymbol{u},\boldsymbol{v})}{\arg\min}\left\|\boldsymbol{X} - \boldsymbol{u}\boldsymbol{v}^T\right\|_F^2, \tag{A.8}$$

where $\|\cdot\|_F$ is the Frobenius norm (sometimes called the Euclidean norm) of a matrix. Subsequent pairs are extracted sequentially by removing the effect of preceding pairs. For two-way functional data, the RSVD of Huang et al. (2009) defines the regularised singular vectors as

$$(\hat{\boldsymbol{u}},\hat{\boldsymbol{v}}) = \underset{(\boldsymbol{u},\boldsymbol{v})}{\arg\min}\left\{\left\|\boldsymbol{X} - \boldsymbol{u}\boldsymbol{v}^T\right\|_F^2 + \mathcal{P}_\lambda(\boldsymbol{u},\boldsymbol{v})\right\}, \tag{A.9}$$

where $\mathcal{P}_\lambda(\boldsymbol{u},\boldsymbol{v})$ is a regularisation penalty

$$\begin{aligned}\mathcal{P}_\lambda(\boldsymbol{u},\boldsymbol{v}) &= \lambda_{\boldsymbol{u}}\boldsymbol{u}^T\boldsymbol{\Omega}_u\boldsymbol{u}\cdot\|\boldsymbol{v}\|^2 + \lambda_{\boldsymbol{v}}\boldsymbol{v}^T\boldsymbol{\Omega}_v v\cdot\|\boldsymbol{u}\|^2 \\ &\quad + \lambda_{\boldsymbol{u}}\boldsymbol{u}^T\boldsymbol{\Omega}_u\boldsymbol{u}\cdot\lambda_{\boldsymbol{v}}\boldsymbol{v}^T\boldsymbol{\Omega}_v v,\end{aligned} \tag{A.10}$$

whereby $\boldsymbol{\Omega}_u$ $(n\times n)$ and $\boldsymbol{\Omega}_v$ $(p\times p)$ are symmetric and nonnegative definite domain-specific penalty matrices, whose purpose is to balance goodness-of-fit against smoothness; λ is a vector of regularization parameters optimally estimated based on generalized cross-validation (GCV) criterion. To forecast mortality rates and derive confidence intervals, the time functions $U_i(t)$ are treated as time series and modelled using general univariate ARIMA processes, rescaling the pairs in (A.7) by the ratio d_i/d_1, $i=2,...,q$.

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