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# Volatility of Interest Rates After Covid-19: A Multivariate GARCH Analysis for the US Economy

Rodrigo Baggi Prieto Alvarez

Mestrado em Economia Monetária e Financeira

Orientador:
Doutor José Joaquim Dias Curto, Professor Catedrático
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Setembro, 2024

CIÊNCIAS SOCIAIS
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Departamento de Economia Política

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*To the memory of my ancestors, who had the courage to leave Europe and cross the Atlantic in search of a better life in Brazil more than a century ago, in stark contrast to the reverse journey many of us undertake today, I dedicate this humble academic endeavour.*

## Acknowledgments

The development of a dissertation can be an arduous and solitary endeavor, but its challenges can be eased by the support and encouragement of loved ones around us.

To my advisor, Professor José Joaquim Dias Curto, for the trust placed in me and for the expert methodological guidance, since the excellent applied econometrics course at ISCTE. I also extend my gratitude to Professors Sérgio Lagoa, Diptes Bhimjee, and Ricardo Barradas for their advice and feedback throughout the research project and academic work.

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To my wife, for her constant love, the lightness brought to everyday life, the special trips, and for being my person wherever we go, sharing the courage to always create new beginnings and live many lives in one.

**Resumo**

Analisamos a volatilidade da *yield* de 2 anos nos Estados Unidos entre 1994 e 2024, com foco no choque da pandemia de Covid-19, que desencadeou o maior surto inflacionário global em quatro décadas. Isto exigiu do *Federal Reserve* o mais intenso aumento de juros na história, para o maior nível em duas décadas. Ao correlacionar a variância condicional dos juros futuros com indicadores macroeconômicos nos modelos APARCH e DCC-GARCH, constatamos a importância dos choques assimétricos na volatilidade, o *feedback* entre juros futuros, taxa de desemprego e expectativa de inflação, e o efeito *spillover* entre as volatilidades da *Treasury* de 2 anos e das principais variáveis macroeconômicas. Os resultados estão em linha com o duplo mandato do *Fed* e são consistentes com a literatura teórica que relaciona os juros futuros com as expectativas de mercado e com o conteúdo informacional da estrutura a termo das taxas de juros. O estudo oferece insights práticos para a gestão de risco e estratégias de investimento, auxiliando investidores na modelagem de assimetrias na persistência da volatilidade, do risco de taxa de juros frente às condições econômicas e do risco de *downside* na gestão de portfólio.



**Classificação JEL:**

C32; E43; G12.

**Palavras-Chave:**

Volatilidade; Treasuries; APARCH; DCC-GARCH; Choques Assimétricos; Efeito Spillover.

## Abstract

We analyse the volatility of the 2-year U.S. Treasury bond from 1994 to 2024, with particular focus on the shock of the Covid-19 pandemic, which triggered the most significant global inflationary surge in four decades. This led the Federal Reserve to raise interest rates more aggressively than ever before, reaching the highest levels in two decades. By correlating the conditional variance of long-term interest rates with macroeconomic conditions using APARCH and DCC-GARCH models, we were able to assess the significance of asymmetric shocks on volatility, the feedback between the 2-year yield, unemployment rate, and expected inflation, and the spillover effect between Treasury volatility and macroeconomic variables volatility. The findings align with the Fed's dual mandate and are consistent with theoretical literature linking long-term interest rates to market expectations and the information content in the term structure of interest rates. We believe this study provides practical insights for risk management and investment strategies, potentially helping investors more effectively model the asymmetries in volatility persistence, the interest rate risks driven by economic conditions, and downside risks in portfolio management.

**JEL Classification:**

C32; E43; G12.

**Keywords:**

Volatility; US Treasury yields; APARCH; DCC-GARCH; Asymmetric Shocks; Spillover.

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## Introduction

In this study, the purpose is to analyze the long-term interest rates for the United States economy, with particular attention to the period following the Covid-19 pandemic shock. Specifically, we will investigate the volatility of the Treasuries rates between 1994 and 2024. Therefore, the study presents the following general research question:

*What are the determinants of the volatility of long-term interest rates in the US and how important were economic shocks as the Covid-19 according to multivariate GARCH models?*

To our knowledge, there are no studies that analyze the volatility of Treasury rates and relate their evolution to conditions predicted in theoretical macroeconomic models implementing multivariate models from the GARCH family. The research question is important as we focus on the most important asset in the global financial market, particularly during the exogenous shock of the Covid-19 pandemic, which resulted in the major global inflationary outbreak in four decades and required the Federal Reserve to raise interest rates most intensely and rapidly in the history of the US, with rates rising to their highest levels in twenty years.

The theory of informational content (Estrella and Hardouvelis, 1991; Mishkin, 1990; Campbell, 1995) predicts that the yield curve reflects information about future economic conditions, e.g. expectations about growth, inflation, and monetary policy. The expectations theory (Malkiel, 1966; Fama, 1984; Campbell and Shiller, 1991) asserts that the yield curve is based on market forecasts for future short-term interest rates, in a way that long-term interest rates are viewed as an average of current and expected future short-term rates, suggesting that when investors expect future short-term rates to increase, long-term rates will also rise, causing persistence and leading to an upward-sloping yield curve. Both theories support our choice of variables in the specification of the estimated econometric models.

Empirically, we perform multivariate time series analysis to estimate ARIMA-GARCH models, investigating the influence of macroeconomic variables on the interest rate volatility, such as observed and expected inflation rate, economic growth, unemployment rate and public debt as share of GDP. We analyze the importance of structural breaks (e.g. global crises and the lockdown during the pandemic), asymmetric shocks, and the spillover effects in the dynamic conditional correlation (DCC) between those variables. We use data from government bureaus and institutions available by the Federal Reserve Economic Data (FRED) platform, maintained by the research division of Fed St. Louis.

First, the findings confirm a stochastic downward trend for the US Treasuries between 1994 and 2024 and some volatility peaks, especially during the crisis of the Covid-19 pandemic and subsequent inflationary burst (2020-22). The results evidenced GARCH effects, with univariate models confirming persistence for the conditional volatility, in line with the empirical literature (Ribeiro and Curto, 2017; Banerjee, 2021; Ji *et al*., 2022).

Additionally, the GJR-GARCH and APARCH models shows the importance of asymmetry, in which negative shocks (*bad news*) have a greater impact on the interest rates than positive shocks (*good news*). The multivariate analysis trough the Pairwise Granger Causality Test and VAR models for the conditional mean, and by the DCC-GARCH model for the conditional variance corroborate the feedback between 2-year Treasury, unemployment rate and expected inflation, suggesting that the variables are good predictors of the long-term interest rate.

Furthermore, we find a spillover effect between the volatility of long-term interest rates and the volatility of macroeconomic variables, although ruling out asymmetric effects in this case, diverging from previous studies which applied those models to other financial metrics (Ji *et al*., 2022; Curto and Serrasqueiro, 2022). The results seem to be consistent with the dual mandate for the Fed and suggest that, as inflation forecasts or the unemployment rate evolve differently from the Fed's targets (long-term equilibrium), the monetary authority impact the long-term interest rates over the monetary policy usual horizon, affecting the real economy and the future inflation rates. Our findings provide practical insights for risk management and investment strategies, potentially helping investors better model the asymmetries in volatility persistence, interest rate risks driven by economic conditions, and downside risks in portfolio management.

The remainder of this dissertation is organized as follows: Section 2 presents some recent data that contextualize the object of investigation and presents aspects that marked the period of the Covid-19 pandemic and the consequences on the main macroeconomic variables after the end of the lockdown. Section 3 briefly reviews the theoretical and empirical literature, and Section 4 summarizes the econometric methodology developed in the study. Section 5 details the data source and the relevant variables in this investigation, presenting some descriptive statistics and graphically evaluating the trajectory of the indicators from 1994 to 2024. Section 6 presents the tests and results obtained in the estimated econometric models, for which the full detailed statistics are also available in the Appendix. Finally, we conclude with some final considerations on the empirical findings and contributions, stating the limitations of the study and pointing out paths for further research.

## 2. Context and Motivation

### 2.1. Global Relevance of the US Interest Rate Market

The fixed-income public securities market in the US is considered by analysts and economists to be the most important market to monitor daily in finance. The yield curve of future interest rates, composed of the yield rates of Treasuries across various maturities (T-bills, T-notes, and T-bonds), is often referred to as "the mother of all curves" due to its importance in the pricing of almost all global risk assets. Indeed, several factors justify the relevance of this market as an object of study: (i) the current volume of this market is nearly $25 trillion; (ii) the share of the US economy in the world accounts for 25% of the global GDP; (iii) the leadership maintained by the US dollar which comprises 60% of global reserves and 80% of global trade.

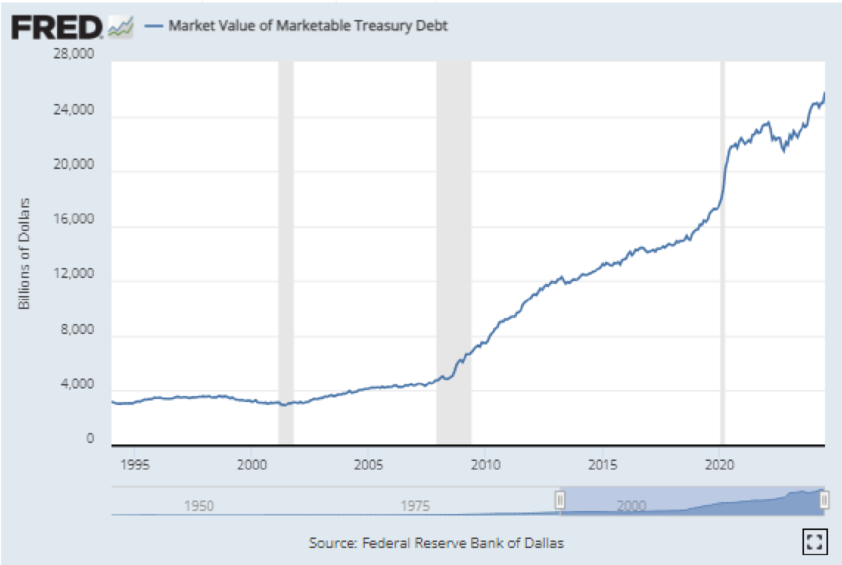


*Figure 1: Total Value of US Treasury Debt (Marketable Treasury Debt) from 1994 to 2024*

Interest rates play a critical role in the financial markets and the overall economy. They influence a wide range of economic activities, e.g. investment, consumption, and government policy. For example, the yield on 2-year US Treasury bonds is closely tied to expectations of near-term economic performance and can be used as indicator of investor sentiment regarding inflation and monetary policy. In turn, the yield on 10-year bonds may reflect investor sentiment about potential economic growth, neutral inflation rate and government debt. It serves as a benchmark for other interest rates, including mortgage rates and corporate bonds. In this study, we focus on key long-term interest rates in the US, which are considered benchmark rates for various financial instruments globally.

## 2.2. The Exogenous Shock of Covid-19

The purpose of this work, beyond simply analysing the evolution of average interest rates, is to evaluate the determinants of the variance in US future interest rates, with a special focus on the recent years marked by the Covid-19 pandemic. This period, particularly in 2020, was characterized initially by a highly stimulative monetary policy, synchronized across major global economies. This involved a rapid and significant reduction of interest rates by central banks, coupled with expansionary fiscal policy and increased government debt.

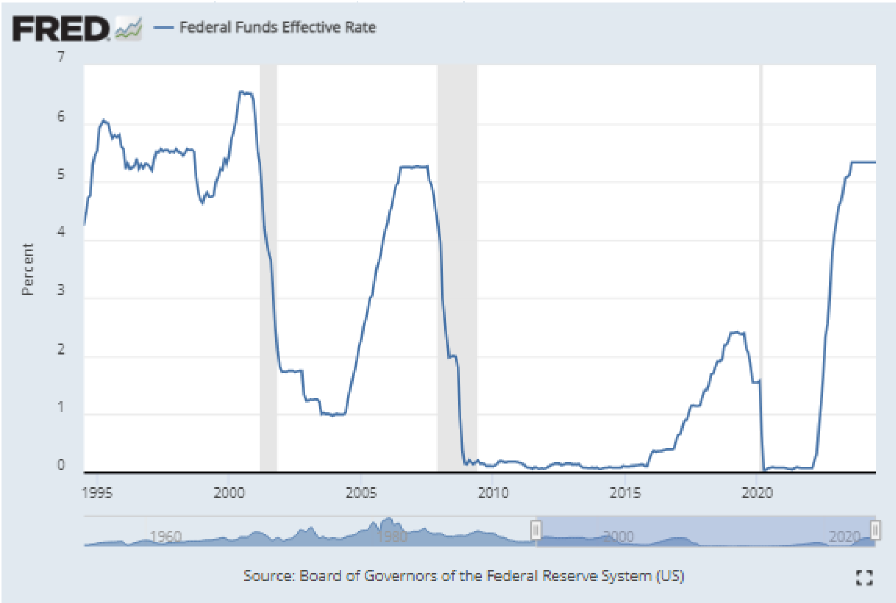


*Figure 2: US Short-Term Basic Interest Rate (Fed Funds Rate) from 1994 to 2024*

In a subsequent phase, from 2021 to 2022, the end of lockdowns, the boost in income through public subsidies, and increased demand for goods, combined with problems in global supply chains and logistical bottlenecks, resulted in the most significant inflationary shock in decades. This scenario required all central banks to completely reverse the previous monetary policy cycle. The Fed implemented an abrupt tightening, raising short-term interest rates from 0.00% to 5.25% p.a. (the highest since the 2000s) at record speed (between March 2022 and July 2023). The speed at which the US central bank raised short-term (spot) interest rates has a significant impact on the level and volatility of yields across other maturities on the yield curve. These drivers, combined with other relevant macroeconomic and financial variables, influence the future interest rate market, which possesses greater liquidity with daily trades essential for determining the cost of capital for companies worldwide—both financial and non-financial.

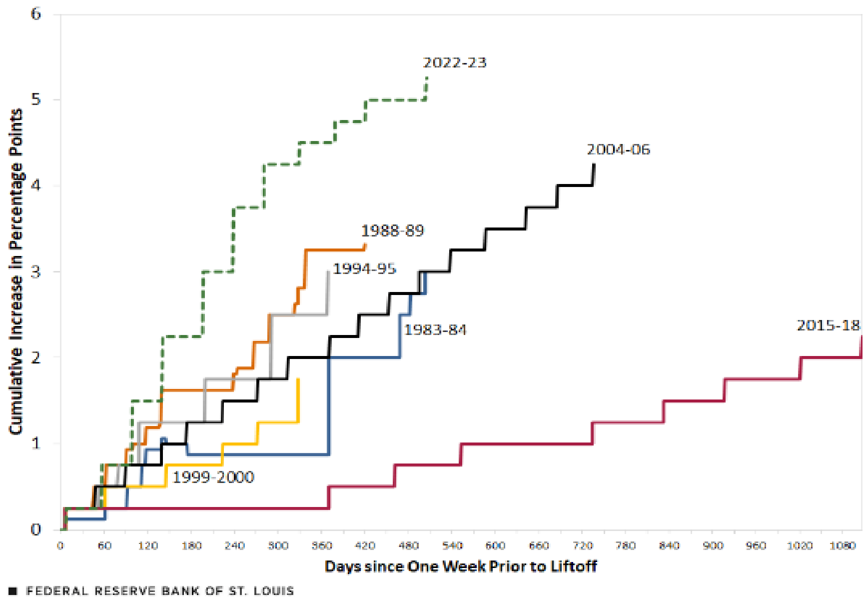


*Figure 3: Magnitude of Nominal Fed Funds Rate Increases in Different Monetary Policy Cycles in the US*

Even before the Fed's monetary tightening, long-term interest rates had already been on an upward trend since 2020. This was possibly a reflection of market expectations regarding rising global inflation, increased government debt, and geopolitical tensions, among other factors. For instance, the yields of 10-year maturity Treasuries were around 2% p.a. before the Covid-19 pandemic, quickly dropping to below 1% p.a. after the lockdown, and then rising sharply in 2022-23 to levels close to 5% p.a., the highest since the pre-global financial crisis of 2007-08.

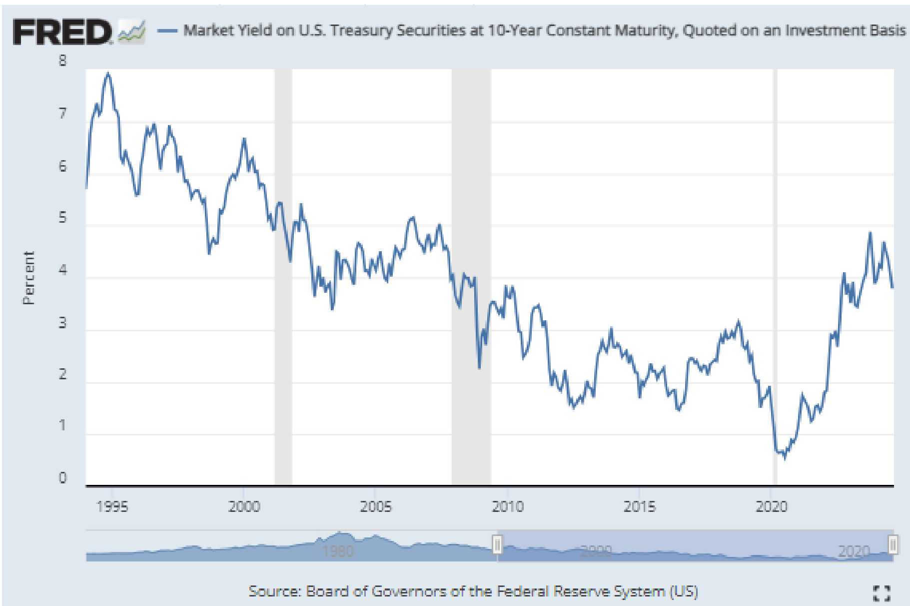


*Figure 4: Yield Rates of 10-Year Maturity Treasuries from 1994 to 2024*

This recent evolution, marked by sharp declines followed by intense increases in 10-year yields, suggests that the realized volatility of future rates also increased during this period. Indeed, according to the Chicago Board Options Exchange (CBOE), the 10-year Treasury Note VIX Index (unfortunately discontinued) also reached the highest levels in its historical series during the pandemic, jumping more than three times. For comparison, this volatility index surpassed the record high during the global financial crisis of 2007-08, as illustrated below.

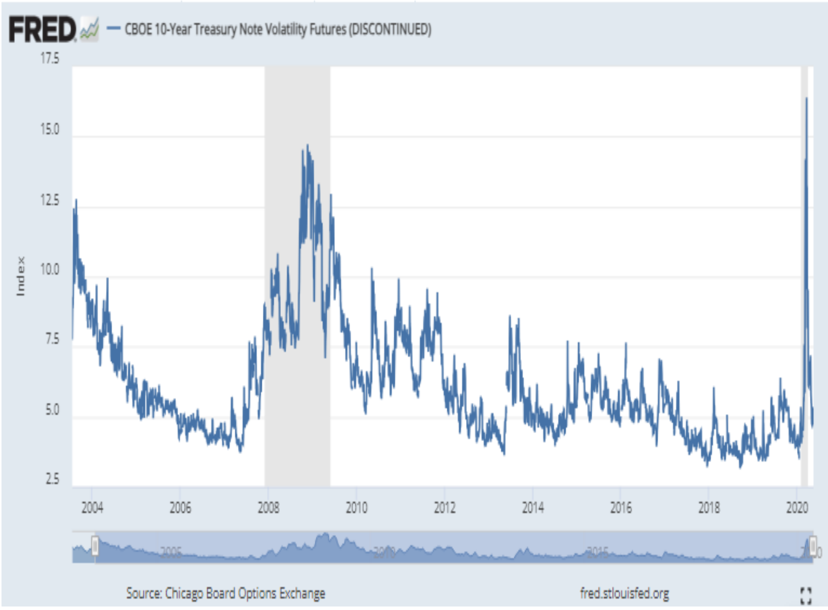


***Figure 5: Volatility Index (VIX) of 10-Year Treasuries from 2004 to 2024 Calculated by the CBOE***

Therefore, this period of financial stress during the Covid-19 pandemic highlights the importance of understanding the volatility in the US future interest rate market. As previously mentioned, given the significance of this market for the pricing of global assets and for the monetary policy decisions of central banks, it is justified the need for improved econometric specifications. These specifications not only aim to model the determinants but also to more accurately predict the volatility of future interest rates, which is the central purpose of this study.

## 3. Literature Review

This section aims to briefly review the theoretical framework and main results found in the empirical literature that are most relevant to understanding the volatility of future interest rates, especially in the face of macroeconomic determinants and structural shocks.

### 3.1. Theoretical Background

The theoretical framework underpinning this study draws on key areas of finance and macroeconomics, particularly the theories related to interest rate drivers and models analysing the information contained in the yield curve.

Long-term interest rates include a premium for expected inflation over the bond's maturity, implying that higher inflation expectations lead to higher long-term rates (Fisher, 1930), and models of economic growth predict that the demand for capital during periods of *bull markets* can drive up long-term interest rates (Blanchard and Watson, 1984). Additionally, central bank actions influencing short-term rates can impact long-term rates by altering inflation expectations and risk perceptions (Bernanke and Blinder, 1992), in the same way fiscal policy and shocks on government borrowing can influence long-term rates through increased supply of government bonds (Barro, 1974).

More recent studies (Clarida, Galí, and Gertler, 2000) emphasize the importance of the Fed's policy changes in response to economic shocks, like the fast interest rate adjustments seen during the Covid-19 pandemic. Lastly, international macroeconomic models predict that market risk factors, capital flows and global economic conditions can significantly impact long-term interest rates (Obstfeld and Rogoff, 1996).

In turn, the theory of informational content predicts that the yield curve reflects information about future economic conditions, including expectations about growth, inflation, and monetary policy. A normal upward-sloping curve suggests healthy economic growth, while an inverted curve may indicate a future recession (Estrella and Hardouvelis, 1991; Mishkin, 1990; Campbell, 1995).

On the other hand, the expectations theory asserts that the yield curve is based on market expectations for future short-term interest rates, in a way that long-term interest rates are viewed as an average of current and expected future short-term rates. This theory suggests that if investors expect future short-term rates to increase, long-term rates will also rise, leading to an upward-sloping yield curve (Malkiel, 1966; Fama, 1984; Campbell and Shiller, 1991).

### 3.2. Empirical References

Understanding the volatility of financial market indicators and their transmission effects is essential for investors, policymakers, and academic researchers. Empirically, there is a vast literature in financial econometrics and from different perspectives, e.g., including or not multivariate controls and applying different econometric methodologies. Several studies have investigated the determinants of interest rate volatility using different models. The GARCH model and its multivariate extensions (Engle, 1982; Bollerslev, 1986) are among the most widely used techniques for modelling time-varying volatility, which have since been applied to numerous financial markets, including interest rates.

There are studies from various perspectives, analyzing, for example, the impact of company size on stock price volatility (Antunes, 2021), which suggests that company size plays a role in the transmission of volatility shocks across the market; a study on the role of conditional heteroscedasticity in stock volatility (Curto, 2021), which demonstrates its impact on returns and volatility forecasts, with implications for asset pricing and risk management; and works on multivariate GARCH modeling and spillover effects in the stock market (Marques, 2022). Despite different approaches, the common theme underscores the importance of understanding market volatility, its determinants, and its propagation across assets, using rigorous econometric methodologies that enhance the robustness of the results.

Bauwens *et al*. (2006) provide comprehensive reviews of how these models are used to capture the volatility dynamics of interest rates. Their findings underscore the importance of considering model specification and the inclusion of relevant macroeconomic variables to improve forecasting accuracy. Regarding the inclusion of controls, the study highlights that it has been widely accepted that volatilities move together over time across different assets and markets, and that multivariate modelling leads to better empirical models than univariate models.

In estimating specifically interest rate volatility, some studies use exactly GARCH models, such as Tian and Hamori (2015), Koeda and Kato (2015), and Diaz *et al*. (2011), generally concluding favourably to the method. For example, Tian and Hamori (2015) model short-term interest rate volatility using the realized GARCH. The authors conclude that the RGARCH model is effective for the proposed objective, and the more realized information in the model, the better the fit to the data, volatility forecasts, and accuracy of Value-at-Risk estimates.

Another variant of the model is used in the study by Koeda and Kato (2015), who employ the GARCH-ATSM modeling (Affine Term Structure Model). While they differ from Tian and Hamori (2015) by analyzing long-term interest rates, they confirm the importance of the information set for model accuracy – in this case, the lack of information or the role of uncertainty in the term structure of interest rates and how this can influence market expectations and risk premiums. The study indicates that uncertainty regarding inflation has been the largest contributor to the dynamics of long-term yields since the 1980s, with an important role also reserved for monetary policy.

It is clear, therefore, that models can be applied to both short-term and long-term interest rate volatility. In a more detailed analysis of interest rates, Pérignon and Smith (2007) examine the dynamics of the yield curve in terms of level, slope, and curvature, corroborating that there are significant GARCH components even using the short-term interest rate as a control, as a proxy for unobservable latent volatility.

Nonetheless, as Diaz *et al*. (2011) stress that there can indeed be significant differences in term structure volatility estimates depending on the models used and the assumed structure for error heteroscedasticity conclude that there are significant differences between short-term (< 1 year) and long-term (>10 years) volatility estimates. Markellos and Psychoyios (2018) investigate the volatility of US Treasuries by constructing three implied volatility indices for daily interest rates with maturities of 5 years, 10 years, and 30 years, emphasizing that there is no strong consensus in the empirical literature on how interest rate volatility should be measured and modelled. The authors use indicators based on volatility through the VIX and corroborate the relevance of its modelling as a hedging instrument for interest rate risk for market managers.

In the research on US interest rates, the study by Covarrubias *et al*. (2006) analyses the volatility of 10-year US T-notes using cumulative sums of squares algorithms, contrasting with GARCH models due to endogenously determined regime changes, to better model and predict future interest rate volatility. However, they find that GARCH models, because they are more likely to overestimate volatility (rather than underestimate it), may be more useful to portfolio managers applying Value-at-Risk in risk management, especially under regime changes.

In the use of multivariate GARCH (MGARCH) models, whether from the perspective of stochastic or realized volatility, the authors point out that there are still few studies applying modelling to the relationship between the volatility of financial indicators and real economy variables. There is a dilemma between flexibility and parsimony in this type of modelling.

GARCH models allow conditional variances and covariances to depend on the long-term past but imply a common persistence in all these elements. Dynamic Conditional Correlation (DCC) models, on the other hand, allow different persistence between variances and correlations and make it possible to handle more than just a few series (Bauwens *et al*., 2006).

Precisely to understand how risk spreads, Ribeiro and Curto (2017) confirm the spillover hypothesis by estimating MGARCH models for the interbank funds market between 2006 and 2015, finding a time-varying contagion effect conditional on the intensity of market turbulence, as characterized by the structural shock of Covid-19. This underscores the relevance of variants of the GARCH family of models in this work. For instance, by studying the volatility of technology stocks during the pandemic, Curto and Serrasqueiro (2022) conclude that GJR-GARCH and APARCH models are more accurate as they adapt to rare events and asymmetric impacts of negative shocks on volatility. This reinforces the importance of considering the impacts of asymmetry, spillover, and structural breaks in the proposed analysis of this work.

More closely related to the present research, Ji *et al*. (2022) explore the contagion effects between global financial markets, focusing on the 2008 financial crisis and the Covid-19 pandemic using the DCC-MGARCH model. They find that both crises intensified the correlations between stock markets in China and the United States, highlighting the role of dynamic conditional correlations (DCCs) in measuring such effects. Additionally, they emphasize that contagion effects were more prominent during crises, driven, among other factors, by macroeconomic fundamentals.

Similarly, Banerjee (2021) examines the effects of the pandemic in the futures market using the A-DCC-EGARCH model, finding significant contagion effects during the pandemic from 2015 to 2020, with a particular focus on the shock of Covid-19 in late 2019. The study highlights how futures markets – due to their role in faster price discovery compared to spot markets – serve as an effective path for analysing contagion. On the other hand, Nguyen *et al*. (2022) used similar empirical methodology and verified thar during the global financial crisis, there was significant contagion from the US to both developed and emerging economies, however, the impact was more limited during the Covid-19 crisis.

Although not necessarily regarding the interest rate market, the examples above reinforce the advantages of multivariate GARCH models – in particular the A-DCC-GARCH – for analysing the impacts of asymmetry in volatility and contagion between different explanatory variables in the model, as proposed in this study.

## 4. Methodology

To address the main question of this study, which proposes to investigate the determinants of long-term interest rate volatility in the US, we intend to analyze the future interest rate in terms of its conditional mean and variance. This study will apply multivariate GARCH models, controlling for the effect of macroeconomic explanatory factors and focusing on the impact of the Covid-19– the largest structural break in historical series in finance and economics.

In general, there are three main phenomena to be analyzed in this study regarding the volatility of the financial data: (1) The effect of asymmetry in volatility, which refers to different responses of the variable depending on the positive or negative direction of its past shocks and the shocks of the covariates. For instance, positive (*good news*) and negative (*bad news*) shocks may have different impacts on the future volatility, a phenomenon often observed in financial markets; (2) The existence of spillover effects, which occurs when volatility spreads between different assets and markets. Multivariate models can diagnose and capture the transmission of volatility across various financial variables; and (3) The impact of structural shock on volatility, including here the effect of significant exogenous impacts, such as the Covid-19 pandemic and the consequent global lockdown, which had severe effects on the real economy. Such events can be graphically observed in almost all financial time series, making it essential to control for these breaks in volatility models.

### 4.1. Primary Tests and ARIMA Models

#### 4.1.1. Tests for Stationarity and Normality

To begin the analysis, it is crucial to test the stationarity and normality of the time series data, which ensures the appropriateness of the subsequent models. Stationarity is tested using the Augmented Dickey-Fuller (ADF), the Dickey-Fuller Generalized Least Squares (DF-GLS), the Phillips-Perron (PP) test, and the Zivot-Andrews (ZA) test (Hamilton, 1994). The normality of residuals is assessed using the Jarque-Bera test, which evaluates whether the skewness and kurtosis of the data match a normal distribution.

#### 4.1.2. ARIMA Model Specification

After confirming stationarity, the Autoregressive Integrated Moving Average (ARIMA) models are specified. The model is denoted as ARIMA $(p, d, q)$, where $p$ is the order of the autoregressive process, $d$ is the degree of differencing needed to achieve stationarity, and $q$ is the order of the moving average process (Box, Jenkins, Reinsel, & Ljung, 2015). The model selection criteria include the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), which help to identify the model with the optimal balance between goodness-of-fit and parsimony.

#### 4.1.3. Tests for Residuals Autocorrelation and Heteroscedasticity

Once an ARIMA model is fitted, residual diagnostics are performed to ensure the adequacy of the model. The Box-Ljung test is used to check for autocorrelation in the residuals (Ljung & Box, 1978), testing the null hypothesis that the first $k$ autocorrelations are jointly zero. For testing heteroscedasticity, the ARCH-LM test is employed, regressing the squared residuals on their own lags and evaluating the presence of ARCH effects (Engle, 1982). Finally, the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots are also analyzed to identify any remaining structure in the residuals. A significant pattern in these plots indicates that the model may need further refinement or that a different model specification may be required.

### 4.2. Univariate GARCH Volatility Models

#### 4.2.1 GARCH Specification

The GARCH model is used to model volatility clustering in financial time series data. The standard GARCH (1,1) model specifies the conditional variance $\sigma_t^2$ as a function of past squared residuals and past conditional variances:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{1}$$

where $\alpha_0$ is a constant term, $\alpha_1$ represents the effect of lagged squared residuals (ARCH term), and $\beta_1$ denotes the effect of lagged conditional variances (GARCH term). The parameters $\alpha_1$ and $\beta_1$ capture the short-term persistence in volatility (Bollerslev, 1986).

#### 4.2.2 GJR-GARCH Specification

The Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model extends the standard GARCH model by allowing for asymmetric effects, such as the leverage effect where negative shocks may impact volatility differently than positive shocks (Glosten, Jagannathan, & Runkle, 1993). The model is specified as:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma_{t-1}^2 I_{\{\varepsilon_{t-1}<0\}} + \beta_1 \sigma_{t-1}^2 \tag{2}$$

where $\gamma$ is the parameter capturing the asymmetry, and $I_{\{\varepsilon_{t-1}<0\}}$ is an indicator function that equals 1 if the lagged residual is negative and 0 otherwise, capturing the tendency for volatility to increase more following negative shocks than positive ones of the same magnitude.

#### 4.2.3 APARCH Specification

The Asymmetric Power ARCH (APARCH) model generalizes the GARCH framework by introducing a power parameter $\delta$, allowing for more flexible modeling of volatility dynamics:

$$\sigma_t^{\delta} = \alpha_0 + \alpha_1(|\varepsilon_{t-1}| - \gamma\varepsilon_{t-1})^{\delta} + \beta_1\sigma_{t-1}^{\delta} \tag{3}$$

In this specification, $\gamma$ measures the asymmetry, and $\delta$ determines the power of the innovation term. APARCH provides a versatile approach to capturing various volatility behaviors observed in financial markets (Ding, Granger, & Engle, 1993).

#### 4.2.4 Conditional Variance Analysis

After fitting each GARCH model, the estimated conditional variance is analysed to gain insights into the volatility dynamics of the series, examining the persistence of volatility, the impact of past shocks, and the potential asymmetry in the volatility response. In GARCH models, volatility persistence is measured by the sum of the coefficients of lagged squared residuals ($\alpha_1$) and lagged conditional variances ($\beta_1$). A high sum close to one indicates strong persistence, meaning shocks to volatility have long-lasting effects.

By comparing these parameters across different variants (e.g., GJR-GARCH, APARCH), we can determine the model that best captures the behavior of volatility in the data (Tsay, 2010). Moreover, models like GJR-GARCH, which include asymmetry terms, allow us to observe how negative shocks (*bad news*) impact volatility differently than positive shocks (*good news*). For example, if the asymmetry parameter ($\gamma$) is significantly greater than zero, it indicates a leverage effect, where negative shocks increase volatility more than positive shocks of the same magnitude. Comparing models using information criteria such as the AIC and the BIC, along with likelihood ratio tests, helps identify the best-fitting model.

### 4.3. Multivariate GARCH Volatility Models

#### 4.3.1 Pairwise Granger Causality

Pairwise Granger causality tests are conducted to assess the predictive causality relationships between variables. A time series $X$ Granger-causes another series $Y$ if past values of $X$ contain information that helps predict $Y$ beyond the information contained in past values of $Y$ alone (Granger, 1969). The test is based on the following equations:

$$Y_t = \alpha_0 + \sum_{i=1}^{k} \alpha_i Y_{t-i} + \sum_{j=1}^{k} \beta_j X_{t-j} + \varepsilon_t \tag{4}$$

If the coefficients $\beta_j$ are not jointly significant, we cannot reject the null hypothesis $H_0$, indicating that $X$ does not Granger-causes $Y$.

### 4.3.2 VAR Models

Vector Autoregressive (VAR) models are specified to capture the dynamic relationship between multiple time series. The VAR model is expressed as:

$$Y_t = c + A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots + A_p Y_{t-p} + \varepsilon_t \tag{5}$$

where $Y_t$ is a vector of endogenous variables, $c$ is a vector of constants, $A_i$ are coefficient matrices, and $\varepsilon_t$ is a vector of white noise error terms. VAR models allow for the analysis of the interactions and feedback among the variables over time.

### 4.3.3 Tests for VAR Residuals

The residuals of the VAR models are tested for stability and adequacy using several tests. The CUSUM test checks for structural stability by analyzing the cumulative sum of recursive residuals. If the residuals deviate significantly from zero, it indicates a structural break or instability in the VAR model parameters over time (Enders, 2014). The test is visualized using a plot where the cumulative sum of the residuals is compared against critical bounds. If the plot stays within the bounds, the model is considered stable. The Portmanteau test is used to assess whether the residuals from a VAR model are white noise, meaning they should be uncorrelated over time (Lütkepohl, 2005).

The multivariate ARCH-LM test is used to detect the presence of ARCH effects in the residuals of a VAR model. Identifying such effects is essential because ARCH effects indicate time-varying volatility, which standard VAR models do not capture. If these effects are present, multivariate GARCH models may be more appropriate for modeling the data. The test is based on the idea that if the residuals $\varepsilon_t$ from a VAR model exhibit heteroscedasticity, then the squared residuals should show autocorrelation (Engle & Kroner, 1995). The test involves the following steps:

Estimating the VAR Model: Fit a VAR model to the time series data and obtain the residuals $\varepsilon_t$.

Formulating the Test Regression: Regress the squared residuals $\varepsilon_t \varepsilon_t'$ (the outer product of the residual vector) on their own lags and a constant:

$$\varepsilon_t \varepsilon_t' = \alpha + \sum_{i=1}^{q} \beta_i \, \varepsilon_{t-i} \varepsilon_{t-i}' + u_t \quad (6)$$

where $\alpha$ is a constant matrix, $\beta_i$ are coefficient matrices capturing the lagged effect, and $u_t$ is the error term. The number of lags $q$ is usually chosen based on information criteria or predetermined based on the context.

Computing the Test Statistic: The statistic is based on the $T.R^2$ value from the above regression, where $T$ is the sample size, and $R^2$ is the coefficient of determination from the regression:

$$\text{LM} = T.tr(\widehat{\Omega}^{-1}\hat{S}) \quad (7)$$

In this formula, $\widehat{\Omega}$ represents the estimated variance-covariance matrix of residuals, and $\hat{S}$ is the sum of squared residuals. The test statistic asymptotically follows a chi-square distribution with degrees of freedom equal to the number of restrictions imposed by the lag length $q$ and the number of variables in the VAR model.

Decision Rule: Compare the test statistic to the critical value of the chi-square distribution. If the statistic exceeds the critical value, we reject the null hypothesis of no ARCH effects, indicating the presence of conditional heteroscedasticity in the residuals.

### 4.3.5 DCC-GARCH Specification

The Dynamic Conditional Correlation GARCH (DCC-GARCH) model is used to model time-varying correlations between multiple time series. The DCC-GARCH model combines univariate GARCH models with a dynamic correlation structure (Engle, 2002). The conditional covariance matrix $H_t$ is decomposed as:

$$H_t = D_t R_t D_t \quad (8)$$

where $D_t$ is a diagonal matrix of time-varying standard deviations from univariate GARCH models, and $R_t$ is the dynamic correlation matrix. The DCC-GARCH model provides a flexible framework for capturing changing correlations over time.

### 4.3.6 AG-DCC-GARCH Specification

The Asymmetric Generalized Dynamic Conditional Correlation GARCH (AG-DCC-GARCH) model extends the traditional DCC-GARCH framework by incorporating asymmetries in the correlation dynamics. This model captures how positive and negative shocks affect correlations differently, which is particularly important in financial markets where correlations between assets tend to increase during periods of market stress (Cappiello, Engle, & Sheppard, 2006). The AG-DCC-GARCH model can be expressed as follows:

Univariate GARCH: each asset's volatility is modeled using a univariate GARCH process where $\omega_i$, $\alpha_i$, and $\beta_i$ are parameters for asset $i$, and $\varepsilon_{i,t}$ are the residuals:

$$\sigma_{i,t}^2 = \alpha_0 + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 \quad (9)$$

Dynamic Conditional Correlation Matrix: the conditional correlation matrix $R_t$ is defined as:

$$R_t = (1 - a - b)\bar{R} + a\left(\frac{q_{t-1}}{\sigma_{t-1}\sigma_{t-1}'}\right) + bR_{t-1} + \gamma I_{\{\varepsilon_{t-1}<0\}} q_{t-1} \quad (10)$$

In this equation, $a$ and $b$ are parameters controlling the influence of past shocks and past correlations, $\gamma$ captures the asymmetric response, and $\bar{R}$ is the unconditional correlation matrix. $I_{\{\varepsilon_{t-1}<0\}}$ is an indicator function that equals 1 when $\varepsilon_{t-1}$ is negative, representing periods of market downturns or stress. This model provides a framework to understand how correlation structures evolve over time, especially during crises when the correlations among asset returns tend to increase, often leading to contagion effects.

### 4.3.7 Conditional Time-Varying Correlation Analysis

After fitting the multivariate GARCH models, the estimated conditional correlations are analyzed to understand the dynamic interrelationships between the series. This analysis provides insights into how they evolve over time, especially during periods of market stress (Tsay, 2010). The estimated conditional correlation matrix $R_t$ from the A-DCC-GARCH model is expressed as:

$$R_t = D_t^{-1} H_t D_t^{-1} \quad (11)$$

where $D_t$ is a diagonal matrix of time-varying standard deviations from the univariate GARCH models, and $H_t$ is the conditional covariance matrix. The elements of $R_t$ provide insight into how the correlation between each pair of assets evolves over time. In AG-DCC-GARCH, the conditional correlation matrix additionally captures asymmetric effects, helping to understand how negative economic shocks increase the correlation between asset returns more significantly compared to positive shocks. The time-varying nature of these correlations is analyzed using rolling windows or recursive estimation techniques, allowing for the identification of periods of high financial integration or market segmentation.

## 5. Data and Variables

A favorable aspect of analyzing macroeconomic data and aggregated financial series is the availability of public databases on the websites of central banks and multilateral organizations. The data analyzed here are derived from the Federal Reserve Economic Data (FRED) platform, maintained by the research division of the regional unit of the US central bank in St. Louis (https://fred.stlouisfed.org/), for which we used the R function *getSymbols.FRED* in *RStudio*.

In the initial exploratory research, almost 40 variables were pre-analyzed (Table 1), among potentially candidates for dependent and explanatory according to the empirical literature. Then we reduced to a more restricted group of 19 variables statistically examined before the model specification. In each group of potential variables, we analyzed the descriptive statistics, the correlation matrix and the economic fundamentals to choose each variable for the model to be estimated, in order to avoid possible multicollinearity problems. Finally, we selected 7 specific macroeconomic variables for the econometric estimation, as will be detailed further.

In the first group of potentially dependent variables are the US interest rates in the spot market (Fed Funds) and in the futures markets (Treasuries). The second group includes the main candidates for explanatory controls in the multivariate GARCH models. These are the variables that, according to empirical evidence, best explain interest rate volatility (in addition to the lagged interest rate itself).

The variables analyzed comprise the following groups: (i) observed consumer inflation rates; (ii) expected inflation rate; (iii) economic growth; (iv) unemployment rate; (v) fiscal policy metrics, such as public debt and deficit, linked to the country risk and the government's financing costs; and (vi) market risk and uncertainty. It is important to remember that economic activity and inflation rate are both intrinsically linked to the interest rates due to the official mandate of the US central bank. Also, including a proxy of financial market risk in the model help us controlling for unobservable factors, in addition to macroeconomic drivers, which may eventually affect the volatility of the variable of interest.

***Table 1: Potentially Relevant Variables for Estimating GARCH Models***

| Variable | Description and Unit | Source | Frequency | Group |
|---|---|---|---|---|
| *DGS10* | *Market Yield on U.S. Treasury Securities at 10-Year Constant Maturity, Quoted on an Investment Basis, Percent, Daily, Not Seasonally Adjusted* | *Fed* | *Daily* | *Interest Rate* |
| *DGS5* | *Market Yield on U.S. Treasury Securities at 5-Year Constant Maturity, Quoted on an Investment Basis, Percent, Daily, Not Seasonally Adjusted* | *Fed* | *Daily* | |
| *DGS3* | *Market Yield on U.S. Treasury Securities at 3-Year Constant Maturity, Quoted on an Investment Basis, Percent, Daily, Not Seasonally Adjusted* | *Fed* | *Daily* | |
| *DGS2* | *Market Yield on U.S. Treasury Securities at 2-Year Constant Maturity, Quoted on an Investment Basis, Percent, Daily, Not Seasonally Adjusted* | *Fed* | *Daily* | |
| *DFF* | *Federal Funds Effective Rate, Percent, Daily, Not Seasonally Adjusted* | *Fed* | *Daily* | |
| *VIXCLS* | *CBOE Volatility Index: VIX, Index, Daily, Not Seasonally Adjusted* | *CBOE* | *Daily* | *Risk Proxy* |
| *VXTYN* | *CBOE 10-Year Treasury Note Volatility Futures (DISCONTINUED), Index, Daily, Not Seasonally Adjusted* | *CBOE* | *Daily* | |
| *USEPUINDXD* | *Economic Policy Uncertainty Index for United States, Index, Daily, Not Seasonally Adjusted* | *Fed St. Louis* | *Daily* | |
| *CPIAUCSL* | *Consumer Price Index for All Urban Consumers: All Items in U.S. City Average, Index and Percent Change from Year Ago, Monthly, Seasonally Adjusted* | *BLS* | *Monthly* | *Observed Inflation* |
| *CPILFESL* | *Consumer Price Index for All Urban Consumers: All Items Less Food and Energy in U.S. City Average, Index and Percent Change from Year Ago, 1982-1984=100, Monthly, Seasonally Adjusted (This measurement is known as "Core CPI")* | *BLS* | *Monthly* | |
| *PCEPI* | *The Personal Consumption Expenditures Price Index is a measure of the prices that people living in the United States, or those buying on their behalf, pay for goods and services. The change in the PCE price index is known for capturing inflation (or deflation) across a wide range of consumer expenses and reflecting changes in consumer behavior. The PCE Price index is the Federal Reserve's preferred measure of inflation.* | *BEA* | *Monthly* | |
| *PCEPILFE* | *The PCE price index less food excluding food and energy is used primarily for macroeconomic analysis and forecasting future values of the PCE price index.* | *BEA* | *Monthly* | |
| *PCETRIM12M159SFRBDAL* | *Trimmed Mean PCE Inflation Rate, Percent Change from Year Ago, Monthly, Seasonally Adjusted* | *BEA* | *Monthly* | |
| *EXPINF1YR* | *1-Year Expected Inflation, Percent, Monthly, Not Seasonally Adjusted (Their estimates are calculated with a model that uses Treasury yields, inflation data, inflation swaps, and survey-based measures of inflation expectations)* | *Fed Cleveland* | *Monthly* | *Expected Inflation* |
| *EXPINF2YR* | *2-Year Expected Inflation, Percent, Monthly, Not Seasonally Adjusted* | *Fed Cleveland* | *Monthly* | |
| *EXPINF5YR* | *5-Year Expected Inflation, Percent, Monthly, Not Seasonally Adjusted* | *Fed Cleveland* | *Monthly* | |
| *EXPINF10YR* | *10-Year Expected Inflation, Percent, Monthly, Not Seasonally Adjusted* | *Fed Cleveland* | *Monthly* | |
| *MICH* | *University of Michigan: Inflation Expectation, Percent, Monthly, Not Seasonally Adjusted (Median expected price change next 12 months, Surveys of Consumers)* | *Univ. Michigan* | *Monthly* | |
| *USALORSGPNOSTSAM* | *Composite Leading Indicators: Reference Series (GDP) Normalized for United States, Index, Monthly, Seasonally Adjusted* | *OECD* | *Monthly* | *Economic Activity* |
| *BBKMGDP* | *Brave-Butters-Kelley Real Gross Domestic Product, Annualized Percent Change from Preceding Period, Monthly, Seasonally Adjusted* | *Fed Chicago* | *Monthly* | |
| *BBKMCY* | *Brave-Butters-Kelley Cycle Component of GDP, Annualized Percent Change from Preceding Period, Monthly, Seasonally Adjusted* | *Fed Chicago* | *Monthly* | |
| *GDPC1* | *Billions of Chained 2017 Dollars, Quarterly, Seasonally Adjusted Annual Rate* | *BEA* | *Quarterly* | |
| *GDPPOT* | *Billions of Chained 2017 Dollars, Not Seasonally Adjusted* | *BEA* | *Quarterly* | |
| *UNRATE* | *Unemployment Rate, Percent, Monthly, Seasonally Adjusted* | *BLS* | *Monthly* | *Employment* |
| *UEMPMED* | *Median Weeks Unemployed, Weeks, Monthly, Seasonally Adjusted* | *BLS* | *Monthly* | |
| *U6RATE* | *Total Unemployed, Plus All Persons Marginally Attached to the Labor Force, Plus Total Employed Part Time for Economic Reasons, as a Percent of the Civilian Labor Force Plus All Persons Marginally Attached to the Labor Force (U-6), Percent, Monthly, Seasonally Adjusted* | *BLS* | *Monthly* | |
| *UNEMPLOY* | *Unemployment Level, Thousands of Persons, Monthly, Seasonally Adjusted* | *BLS* | *Monthly* | |
| *CE16OV* | *Employment Level, Thousands of Persons, Monthly, Seasonally Adjusted* | *BLS* | *Monthly* | |
| *NROU* | *Noncyclical Rate of Unemployment, Percent, Quarterly, Not Seasonally Adjusted* | *US CBO* | *Quarterly* | |
| *MTSDS133FMS* | *Federal Surplus or Deficit [-], Millions of Dollars, Monthly, Not Seasonally Adjusted* | *US Treasury* | *Monthly* | *Fiscal* |
| *FYFSGDA188S* | *Federal Surplus or Deficit [-] as Percent of Gross Domestic Product, Percent of GDP, Annual, Not Seasonally Adjusted* | *US Budget* | *Monthly* | |
| *GFDEBTN* | *Federal Debt: Total Public Debt, Millions of Dollars, Quarterly, Not Seasonally Adjusted* | *US Treasury* | *Quarterly* | |
| *GFDEGDQ188S* | *Federal Debt: Total Public Debt as Percent of Gross Domestic Product, Percent of GDP, Quarterly, Seasonally Adjusted* | *US Budget* | *Quarterly* | |

### 5.1. Long-term Interest Rates in the US

When analyzing interest rates, the choice of maturity (yield) is not straightforward. As highlighted by Diaz *et al*. (2011), there are differences between the short-term (up to 1-year), medium-term (2-year to 5-year) and long-term (5-year to 10-year yields) volatility estimates, and there may also be differences in results depending on the models used and the assumed structure for error heteroscedasticity. In graph 6 are presented the historical daily interest rates in the US for the 1990s until 2024, an era of significant liberalization in global financial markets, also one of the reasons for the general declining trend in global interest rates.

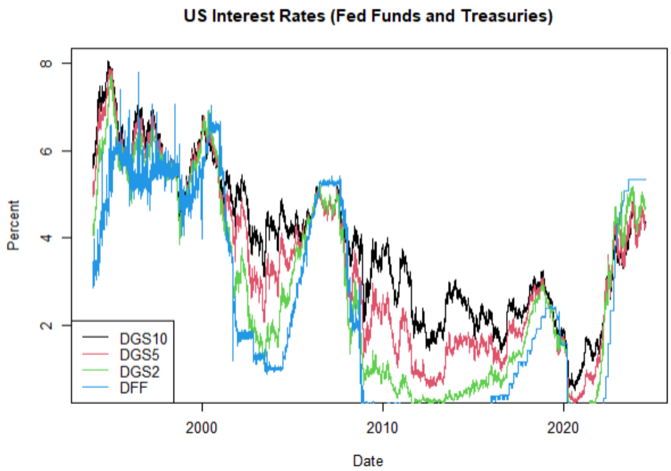


***Figure 6: US Interest Rates: Spot (Fed Funds) and Futures (10, 5, and 2-year Treasuries)***

***Table 2: Descriptive Statistics for the US interest rates***

| *Variable* | *mean* | *sd* | *min* | *max* | *median* | *Q1* | *Q3* | *n* |
|---|---|---|---|---|---|---|---|---|
| *DGS10* | 3.85 | 1.70 | 0.52 | 8.05 | 3.84 | 2.39 | 5.05 | 11137 |
| *DGS5* | 3.36 | 1.92 | 0.19 | 7.9 | 3.1 | 1.66 | 4.79 | 11137 |
| *DGS2* | 2.86 | 2.15 | 0.09 | 7.74 | 2.5 | 0.76 | 4.84 | 11137 |
| *DFF* | 2.51 | 2.24 | 0.04 | 7.8 | 1.81 | 0.17 | 5.08 | 11139 |

It can be observed that, following an oscillation around 6.00% p.a. until 2000, the basic interest rate dropped to levels near 1.50% p.a. after the 2001 recession (the *dot-com* bubble), and rose to about 5.00% p.a. during the economic boom of 2004-2007. Then they fell sharply to levels close to 0% between 2009 and 2017, a period marked by a significant expansion of the Fed's balance sheet (known as Quantitative Easing, or QE). After a brief rise to 2.00% p.a. in 2018-2019, the recessionary shock of the Covid-19 pandemic once again brought spot rates to 0% between 2020-2022. More recently, the global inflationary surge, unprecedented in four decades, compelled the Fed's most fast and intense rate hike ever, reaching the current levels of around 5.50% p.a., the highest rate in 18 years. In parallel to the Fed Funds rates set by the US central bank, futures market rates varied in a similar direction, though with some differences.

The graph 6 shows the Treasuries for 2-year (green), 5-year (red) and 10-year (black). During the recent years, amid 2020-2022, these rates were between 0.25% and 1.00% p.a. shortly after the pandemic, rising rapidly in 2022-2023 reaching the most recent peak in October 2023, the highest in two decades. Two aspects are noteworthy here. First, the phenomenon of the "yield curve inversion", in which 2-year rates became higher than 10-year yields (Graph 7). Historically, rates with longer maturities are, on average, higher, and this inversion has preceded economic recessions, occurring in periods of market stress, particularly when inflation also exceeds the Fed's targets. Also, the historical volatility appears to be inversely related to the yield, being higher for shorter maturities.

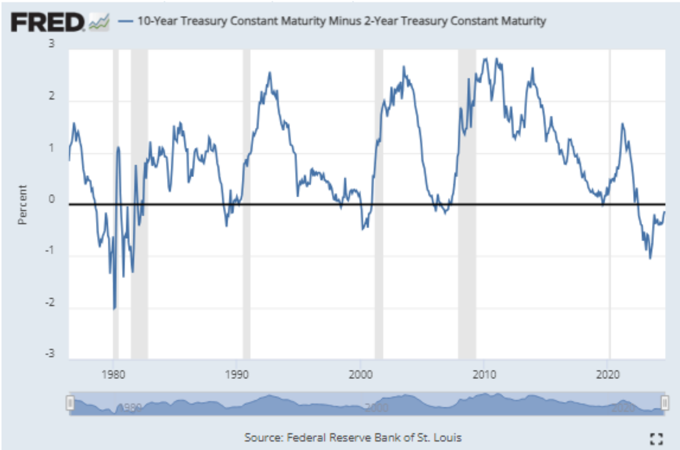


***Figure 7: Historical 10-Year Treasury Constant Maturity Minus 2-Year Treasury Constant Maturity***

Although the different rates have particularities, it is clear the curves have similar trends. In fact, the matrix (Appendix A) shows that 10-year yields have a correlation higher than 0.90 even with 2-year yield, and only slightly lower (0.82) with the Fed Funds spot rate. Considering that the monetary policy action horizon is 18 to 24 months ahead, this analysis led us to decide to choose the 2-year Treasury as the dependent variable for the volatility models. Additional estimates for other yield rates will be presented in the appendix as robustness tests.

### 5.2. Observed Consumer Inflation

The inflationary peak was experienced not only by the US but by all major economies in 2022-2024. The significant rise was due to supply chain disruptions after Covid-19, changes in consumer demand (more goods vs. services during the lockdown), and to the expansive fiscal and monetary policies. The change can be seen in the inflation index trend after 2021.Graph 8 shows different measures of inflation (CPI, Core CPI, PCE and Core PCE), both for the index (in level) and the 12-month rate. In general, the annual rate rose from around 2.00% p.a. to almost 9.00% p.a. for the headline CPI, a rate observed for the US only during the 1970s.

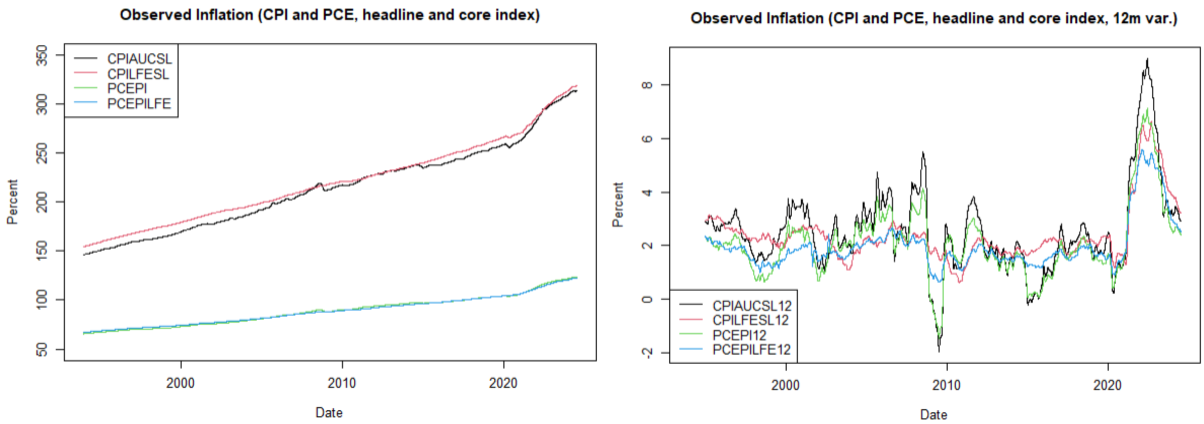


*Figure 8: CPI, Core CPI, PCE and Core PCE inflation index (top) and 12-month rate (bottom) in the US*

The CPI measures the average monthly change in the prices paid by urban consumers for a market basket of consumer goods and services. The Core CPI rate showed a more moderate increase, around 6% p.a. at its peak, because it excludes more volatile prices from the calculations, such as food and energy (which rose above average after the pandemic shock). It provides a clearer view of underlying inflation trends by removing the effects of temporary price shocks. The PCE rate, on the other hand, has historically lower rates than CPI (Table 3), on average, because it uses a methodology that considers the substitution effect of consumers adapting to the price increase in their basket of goods.

*Table 3: Descriptive Statistics for the observed inflation rate*

| *Variable* | *mean* | *sd* | *min* | *max* | *median* | *Q1* | *Q3* | *n* |
|---|---|---|---|---|---|---|---|---|
| *CPIAUCSL12* | 2.54 | 1.63 | -1.96 | 8.99 | 2.30 | 1.65 | 3.17 | 10774 |
| *CPILFESL12* | 2.39 | 1.05 | 0.603 | 6.64 | 2.19 | 1.80 | 2.55 | 10774 |
| *PCEPI12* | 2.09 | 1.34 | -1.47 | 7.12 | 1.97 | 1.38 | 2.55 | 10774 |
| *PCEPILFE12* | 1.98 | 0.93 | 0.626 | 5.57 | 1.76 | 1.48 | 2.10 | 10774 |

Inflation rates are also highly correlated (Appendix A), both for the headline CPI and PCE indexes (0.98) as for the Core CPI and Core PCE (0.92). This suggests a high probability that the conclusions would be similar regardless of which variable is chosen for the models. The PCE is the Fed's preferred indicator for monitoring inflation, and thus we chose it as the explanatory variable representing observed inflation for interest rate volatility models.

### 5.3. Long-term Expected Inflation

The impacts of monetary policy occur are lagged, over a horizon of 18 to 24 months ahead according to empirical evidence. Hence, the Fed's action in a restrictive interest rate cycle, increasing the Fed Funds rate and therefore affecting all Treasuries yields, occurs not only by monitoring current inflation (additionally to economic activity), but mainly by observing inflation *expectations* already priced in by the market in the following quarters.

There are some variables for this purpose. Examples of market-based measures are the TIPS Breakeven Inflation Rate, from the difference between Treasury Inflation-Protected Securities and nominal Treasury bonds, and the Inflation Swaps, derivatives pricing the market willing to pay to receive inflation-linked payments. Other examples are survey-based, as the University of Michigan Inflation Expectations, the consumers' expectations for the short-term (one year) and long-term (five to ten years), and the Survey of Professional Forecasters, led by the Fed Philadelphia, which gathers inflation forecasts from market economists.

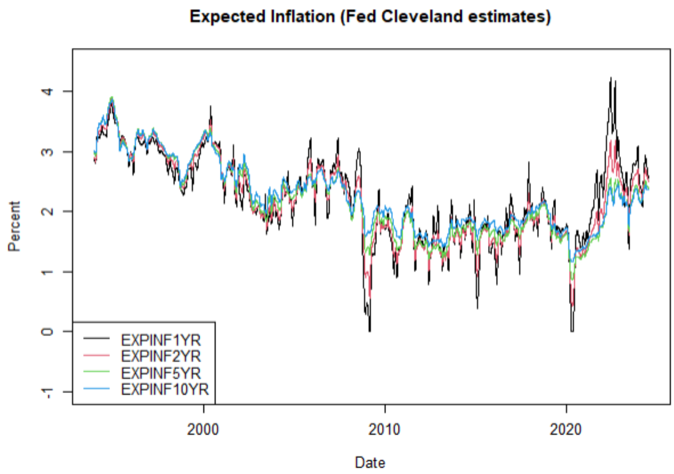


***Figure 9: Inflation Expectation for Different Horizons***

***Table 4: Descriptive Statistics for the expected inflation rate***

| *Variable* | *mean* | *sd* | *min* | *max* | *median* | *Q1* | *Q3* | *n* |
|---|---|---|---|---|---|---|---|---|
| *EXPINF1YR* | 2.23 | 0.724 | 0.000000167 | 4.23 | 2.20 | 1.71 | 2.78 | 11139 |
| *EXPINF2YR* | 2.20 | 0.675 | 0.424 | 3.89 | 2.13 | 1.66 | 2.73 | 11139 |
| *EXPINF5YR* | 2.21 | 0.649 | 0.865 | 3.91 | 2.15 | 1.68 | 2.65 | 11139 |
| *EXPINF10YR* | 2.28 | 0.592 | 1.16 | 3.87 | 2.17 | 1.80 | 2.67 | 11139 |

Here, we consider the variables used by the Fed Cleveland (Graph 9), which estimates a model with Treasury yields, inflation data, inflation swaps, and survey-based measures of inflation expectations. The evolution of expectations is related to observed inflation itself, but there are differences in the magnitude of the ‘overshooting’ of the recent inflationary peak, which partially affects 1-year expectation, but has a much smaller impact on the medium-term and long-term expected inflation.

The correlation matrix (Appendix A) corroborates the analysis and shows that inflation expectations for the different time horizons are also highly correlated, especially between the two metrics for the short term (0.96), and a perfect correlation between the two metrics for the long term. Taking this into account, and the relevant horizon for monetary policy, we decided to use the 2-year expected inflation as the representative variable in the volatility models.

### 5.4. Economic Activity and Output Gap

The Fed's mandate involves balancing the objective of keeping inflation around 2.00% per year while keeping the economy growing close to its natural rate (estimated at around 2.50% by the US government). Clearly, the most appropriate variable to include in the econometric model for this purpose would be GDP itself, as shown previously in Table 1. However, since the variable has a quarterly frequency, we opted for a monthly GDP proxy. The Brave-Butters-Kelley Index (2019) are a byproduct of research originally conducted by the Fed Chicago, maintained by the Indiana University and is constructed from a collapsed dynamic factor analysis of a panel of 490 monthly measures of real economic activity and quarterly real GDP growth.

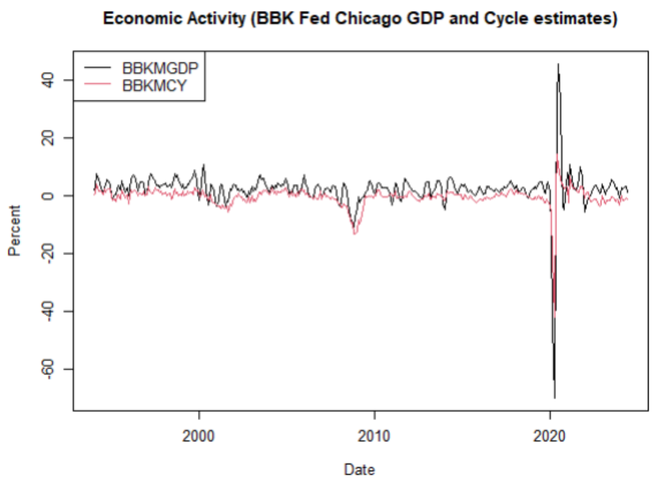


***Figure 10: GDP Growth according to the monthly proxy BBK by the Fed Chicago***

***Table 5: Descriptive Statistics for the economic growth proxy from BBK Index***

| *Variable* | *mean* | *sd* | *min* | *max* | *median* | *Q1* | *Q3* | *n* |
|---|---|---|---|---|---|---|---|---|
| *BBKMGDP* | 3.10 | 3.63 | -0.41 | 4.57 | 2.78 | 1.28 | 4.06 | 11139 |
| *BBKMCY* | 0.19 | 2.63 | -1.55 | 5.22 | 0.23 | 0.61 | 1.33 | 11139 |

The variable consists of the annualized percent change from preceding period and is seasonally adjusted (Graph 10). The BBK monthly GDP growth is indexed to the quarterly estimates of real GDP growth from the US Bureau of Economic Analysis (BEA). In addition to the headline index, it also consists of three sub-indexes: cycle, trend, and irregular components. In practice, assuming the irregular component is zero on average, the cycle component represents the output gap – the growth rate above or below the long-term trend, or the potential economic growth for the US.

Graphically, two points are worth highlighting. First, the unprecedented structural shock that occurred due to the Covid-19 pandemic, which represented a temporary 50% fall in economic activity at the beginning of the lockdown, followed by strong growth in the subsequent months. Second, after 2021, there has been a gradual loss of strength in economic growth, particularly evidenced by negative territory in the economic cycle rate, indicating a negative output gap, expected in a restrictive monetary policy cycle with high interest rates.

The correlation (Appendix A) between the two metrics analyzed is 0.84, indicating that a large part of the variation in the total GDP growth rate corresponds to cyclical variations in addition to the structural growth rate. Nevertheless, from the Fed's perspective, since the most relevant variable for the monetary authority's mandate is *total* economic growth (and not the output gap), we selected the headline GDP proxy as the metric representing economic activity in our group of explanatory variables for the econometric model.

### 5.5. Unemployment Rate and Median Weeks Unemployed

The Phillips Curve, the Okun's Law and the AD-AS macroeconomic model teach us how the inflation rate, the economic growth rate and the unemployment rate are closely linked variables in the framework of analysis by the fiscal and monetary policymaker. In this sense, we include here the labor market indicator as a natural candidate in the group of explanatory variables for the interest rate volatility model. Below (Graph 11), we select two important metrics in the Fed's interest rate decision: the unemployment rate (percentage of the labor force without formal or informal employment) and the average time (in weeks) that a person takes to find a job in the US.

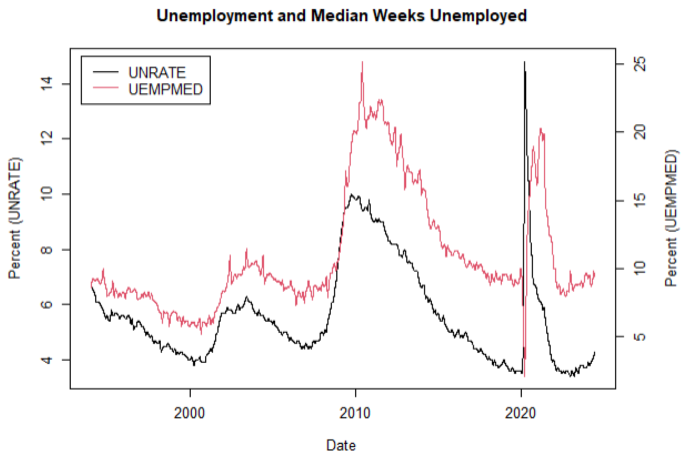


*Figure 11: Unemployed rate as % of workforce (left) and median weeks unemployed (right)*

It should be noted that both variables have evolved very similarly in recent years. The unemployment rate hovered around 4.5% (the neutral rate, or natural equilibrium, estimated by the US government) between 1994 and 2007, rising rapidly to around 10% after the global financial crisis of 2008-09, and then falling back to around 3.5% before the pandemic, a historically very low level that indicated an overheated economy. At that time, the average time for workers to be reemployed was around 10 weeks, also below the average of over 20 weeks observed during the economic crisis.

*Table 6: Descriptive Statistics for the unemployment variables*

| *Variable* | *mean* | *sd* | *min* | *max* | *median* | *Q1* | *Q3* | *n* |
|---|---|---|---|---|---|---|---|---|
| *UNRATE* | 5.60 | 1.79 | 3.4 | 14.8 | 5.18 | 4.34 | 6.1 | 11139 |
| *UEMPMED* | 11.0 | 4.40 | 2.1 | 25.2 | 9.33 | 8.31 | 11.8 | 11139 |

A significant structural shock occurs at the beginning of the lockdown, when the unemployment rate increases almost instantly to 15%. It is worth remembering that, because the labor market in the US is quite liberalized and flexible, the ease of hiring and firing workers makes this variable more volatile than is usually observed in other global economies. For this same reason, the rate falls quickly after the reopening and government stimulus to sustain the economy, returning once again to historic lows of 3.5%. Recently, the rate has already returned to 4.3%, in the context of the economic slowdown and cycle of higher interest rates.

The variables have a high correlation (0.75), as the graphical analysis already suggested (Appendix A). Considering the theoretical and empirical literature, the importance of the unemployment rate in forecasting of inflation and interest rates in the financial market and in the fed Funds rate decision, even detailed in the minutes and statements by the US monetary authority, we chose to maintain this indicator in the group of explanatory variables of the model.

### 5.6. Public Debt and Government Fiscal Deficit

Public debt and government deficit are crucial indicators of a country's fiscal health and economic stability. Fiscal policy has increasingly become a relevant factor in the long-term interest rates. For example, on August 1, 2023, one of the main credit rating agencies (FitchRatings) reduced (slightly) the US long-term rating from AAA to AA+, announcing that:

> *"The rating downgrade of the United States reflects the expected fiscal deterioration over the next three years, a high and growing general government debt burden (...) The repeated debt-limit political standoffs and last-minute resolutions have eroded confidence in fiscal management (...) The debt ratio [of almost 120% of GDP] is over two-and-a-half times higher than the 'AAA' median of 39.3% of GDP and 'AA' median of 44.7% of GDP (...) Over the next decade, higher interest rates and the rising debt stock will increase the interest service burden, while an aging population and rising healthcare costs will raise spending on the elderly absent fiscal policy reforms."*[1]

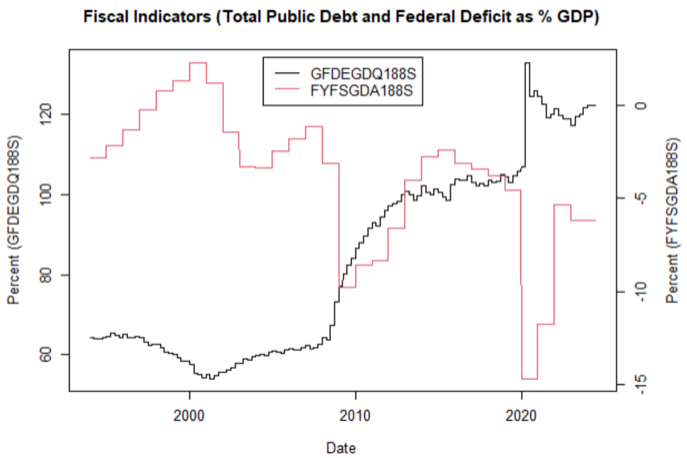


***Figure 12: Public debt as % GDP (left) and federal deficit as % GDP (right)***

In fact, between 1994 and 2024 (Graph 12), the US went from a Debt-to-GDP ratio of around 60% to levels above 100% after the 2008-09 financial crisis – when the government deficit, until then balanced or slightly negative, began to register rates close to 10% of GDP. A new jump occurred after the Covid-19 pandemic, when deficits close to 15% of GDP took the US public debt to around 120% of GDP, precisely the concern stressed by the rating agency.

[1] Fitch Downgrades the United States' Long-Term Ratings to 'AA+' from 'AAA'; Outlook Stable

***Table 7: Descriptive Statistics for the fiscal policy variables***

| *Variable* | *mean* | *sd* | *min* | *max* | *median* | *Q1* | *Q3* | *n* |
|---|---|---|---|---|---|---|---|---|
| *GFDEGDQ188S* | 83.2 | 23.7 | 54.0 | 133 | 80.3 | 61.3 | 103 | 11139 |
| *FYFSGDA188S* | -3.80 | 3.78 | -14.7 | 2.30 | -3.11 | -5.34 | -1.44 | 11139 |

Factors that lead to an increase in interest rates (like inflation or an overheated economy) rise the interest expense on public budget. Also, there is a potential issue of simultaneity, since the high levels of public debt and large fiscal deficits can lead to higher interest rates as investors demand a risk premium for holding government bonds, and this increase the cost of borrowing for the government. This issue will be analyzed through causality tests in the next section. According to FitchRatings, the Interest-to-Revenue ratio in the US is expected to reach 10% by 2025 (compared to 2.8% for the 'AA' median and 1% for the 'AAA' median) due to the higher debt level as well as sustained higher interest rates compared with pre-pandemic levels.

As expected, a negative fiscal result (deficit) leads to higher levels of public debt and, therefore, there is a negative correlation (-0.70) between the two variables (detailed in the Appendix A). Considering the smoother and more persistent behavior of the historical series of Debt-to-GDP ratio, associated with the importance of this variable for risk premium by the rating agencies and the financial market, we chose to include this indicator in the group of explanatory variables of the model.

### 5.7. Proxy of Market Risk

All the macroeconomic explanatory variables chosen will help us test the hypothesis that they are related to future interest rates in the US, and particularly to their recent volatility. On the other hand, volatility *per se* is a measure of risk, but there are also other exogenous risk and uncertainty factors in the markets, not directly related to macroeconomic fundamentals – such as geopolitical events, natural disasters or health epidemics, which is precisely the most recent exogenous shock in the macroeconomic and financial time series.

Therefore, in addition to the selected variables, we included in the group of explanatory variables an indicator that purely reflects the implicit volatility of financial assets, in order to try to control for unobservable factors in the volatility model. This risk proxy may be necessary for understanding the behavior of financial markets, providing insights into investor sentiment, market panics, and potential financial or economic disruptions.

The indicator used here is widely known in the market as the ‘fear index’, provided by the Chicago Board Options Exchange[2] (CBOE). The Volatility Index, or simply VIX (Graph 13) measures market expectation of near-term volatility conveyed by stock index option prices. The indicator's methodology involves calculations that infer the implied value of options, derivatives whose price depends on the probability that the current price of a specific stock will move enough to reach a specific level, called the exercise price or strike price.

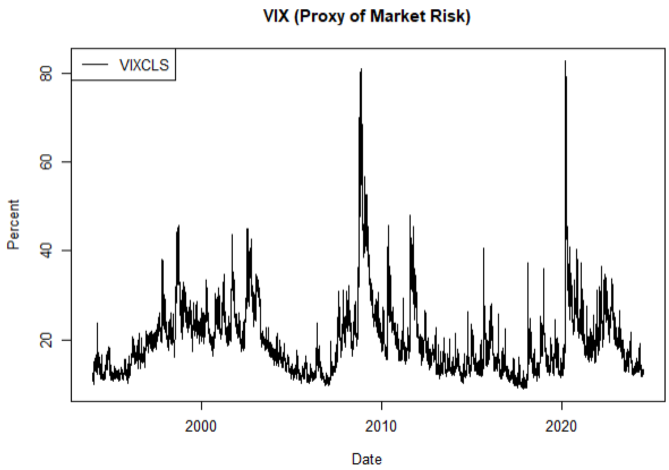


*Figure 13: Risk Indicator by CBOE Volatility Index (VIX)*

The evolution of the VIX shows that there are some frequent but not sharp increases, as in financial crises in the late 1990s, the *dot-com* bubble or the European debt crisis in 2012. On the other hand, two major moments are notable: the peak in the 2008-09 financial crisis and the most recent and historical record high of the index (reaching 82.7, more than four times the historical average), precisely in March 2020, the beginning of the lockdown in the major global economies due to the Covid-19 pandemic, the period of greatest relevance for the present study.

*Table 8: Descriptive Statistics for the Volatility Index (VIX)*

| *Variable* | *mean* | *sd* | *min* | *max* | *median* | *Q1* | *Q3* | *n* |
|---|---|---|---|---|---|---|---|---|
| *VIXCLS* | 19.7 | 8.12 | 9.14 | 82.7 | 17.9 | 13.8 | 23.2 | 11137 |

[2] CBOE Volatility Index Methodology

## 6. Estimation Results

### 6.1. Preliminary Analysis: Stationarity and Normality

In this section, before proceeding with the estimations of the univariate and multivariate volatility models, we present the preliminary tests that allow us to assess the stationarity and normality of the time series, and also the estimation of the models for the conditional mean of the variables selected for the models.

As mentioned in the previous section, in each group of potential variables, prior analysis of descriptive statistics, correlation matrix and economic fundamentals allowed the selection of the 7 variables for the model – since the high correlation between covariates in the same group could imply multicollinearity problems. Even so, we chose to perform unit root tests for all 19 potential variables, as additional evidence of similarity between series belonging to the same macroeconomic group. The Appendix B contains detailed results of the ADF, DF-GLS, PP and ZA tests for the 19 potential variables that were described in detail in the previous section.

In general, in the case of Treasury rates and the Fed Funds rate, the unit root tests did not allow us to reject the null hypothesis, indicating non-stationarity (presence of a stochastic trend), as graphically suggested by the decreasing trend in interest rates since 1994. Similarly, the tests also corroborated the non-stationarity of fiscal policy indicators, confirming the graphical evidence in the case of the government deficit variable (decreasing trend) and public debt (increasing trend since 1994). Thus, subsequent analyses and the estimation of the ARIMA-GARCH family of models for these variables will rely on their log-return transformation (in the first difference, so that we can analyze stationary series).

On the other hand, the unit root tests allowed the rejection of the null hypothesis of non-stationarity (indicating the absence of a stochastic trend) for the variables of observed inflation, expected inflation, monthly proxy for economic growth, unemployment rate and volatility index (proxy for market risk). Therefore, the sequence of analyses in the case of these variables was carried out with the same time series already analyzed in the previous section.

After performing the unit root tests (Appendix B), justifying the choice of variables and the necessary log-return transformations, we consolidate below the 7 macroeconomic variables (Table 9) and their evolution between 1994 and 2024 (Graph 14), data that will be considered for estimating the ARIMA-GARCH models.

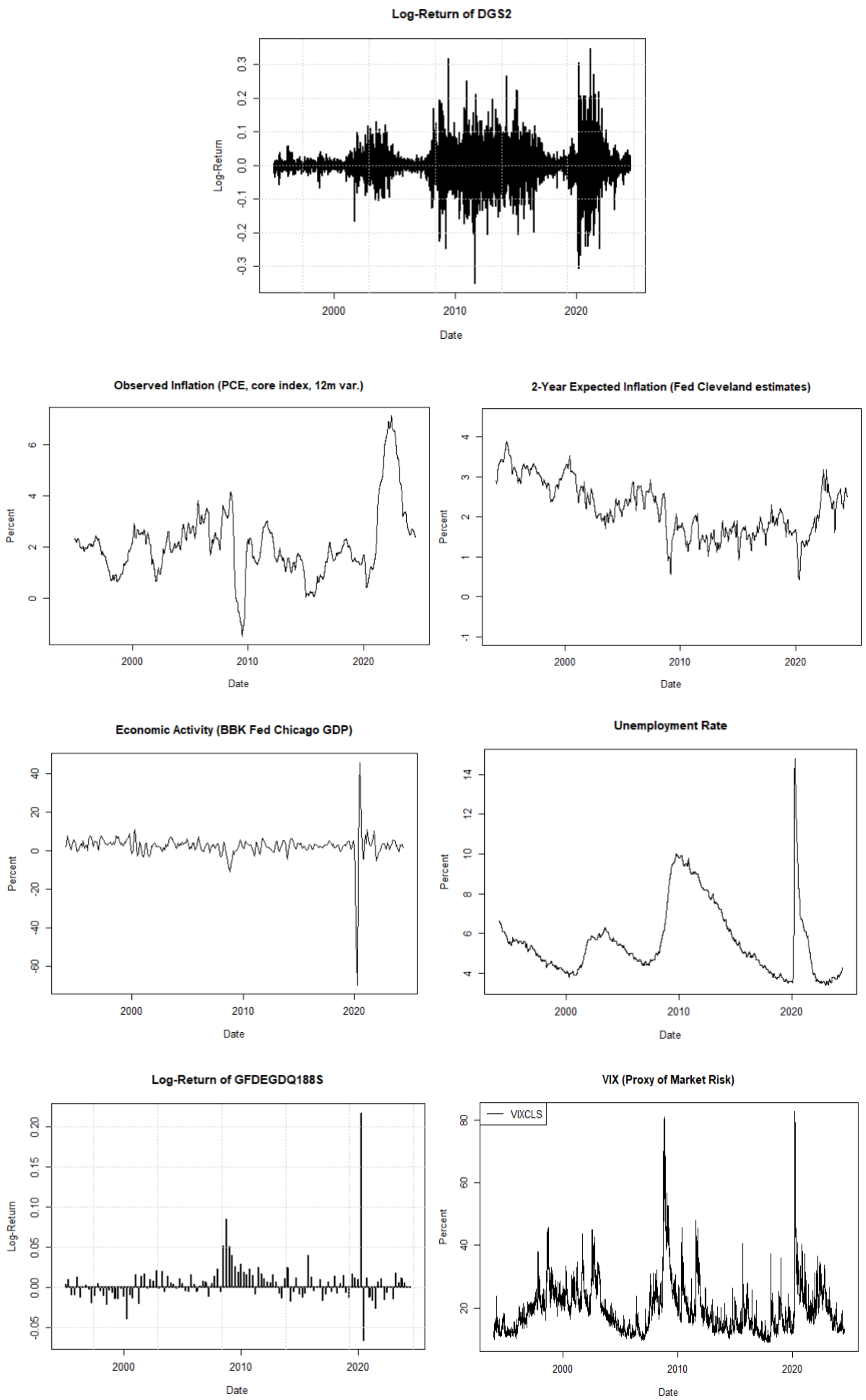


***Figure 14: Stationary historical series of the variables selected for the ARIMA-GARCH models***

***Table 9: Selected Variables for the ARIMA-GARCH Models***

| Variable | Description and Unit | Source | Frequency | Group |
|---|---|---|---|---|
| *LogRet_DGS2* | *Log Return of the Market Yield on U.S. Treasury Securities at 2-Year Constant Maturity, Quoted on an Investment Basis, Percent, Daily, Not Seasonally Adjusted* | *Fed* | *Daily* | *Interest Rate* |
| *PCEPI* | *The Personal Consumption Expenditures Price Index is a measure of the prices that people living in the United States, or those buying on their behalf, pay for goods and services. The change in the PCE price index is known for capturing inflation (or deflation) across a wide range of consumer expenses and reflecting changes in consumer behavior. The PCE Price index is the Federal Reserve's preferred measure of inflation.* | *BEA* | *Monthly* | *Observed Inflation* |
| *EXPINF2YR* | *2-Year Expected Inflation, Percent, Monthly, Not Seasonally Adjusted* | *Fed Cleveland* | *Monthly* | *Expected Inflation* |
| *BBKMGDP* | *Brave-Butters-Kelley Real Gross Domestic Product, Annualized Percent Change from Preceding Period, Monthly, Seasonally Adjusted* | *Fed Chicago* | *Monthly* | *Economic Activity* |
| *UNRATE* | *Unemployment Rate, Percent, Monthly, Seasonally Adjusted* | *BLS* | *Monthly* | *Employment* |
| *LogRet_GFDEGDQ188S* | *Log Return of the Federal Debt: Total Public Debt as Percent of Gross Domestic Product, Percent of GDP, Quarterly, Seasonally Adjusted* | *US Budget* | *Quarterly* | *Fiscal* |
| *VIXCLS* | *CBOE Volatility Index: VIX, Index, Daily, Not Seasonally Adjusted* | *CBOE* | *Daily* | *Risk Proxy* |

It is worth mentioning some points regarding the characteristics of the new transformed log-return series, in the case of 2-year Treasuries and government public debt. For the log-return of interest rates, the initial graphical analysis already suggests that, in addition to volatility not being constant (and therefore capable of being modeled), there are three notable clusters of volatility: in the years following the dot-com bubble (2002-04), between the years of the international financial crisis (2008-09) and the sovereign debt crisis in Europe (2011-14), and again in the recent crisis of the Covid-19 pandemic and subsequent inflationary outbreak (2020-22) – and the latter appears to have been more concentrated in time and intense in volatility.

As for the log-return of public debt (as a % of GDP), in addition to persistent increases after the recession caused by the international financial crisis, there is also clearly an evident shock due to the increase in government debt to support fiscal stimulus at the beginning of the lockdown caused by the Covid-19 pandemic, as the graphical analysis already suggested from the jumps seen in the original time series.

Finally, the consolidated results of the unit root test (Appendix C) allow us to reject the null hypothesis of non-stationarity for all 7 variables selected for the models. The results of the Jarque-Bera test (Appendix C) allow us to confirm the rejection of the null hypothesis that these time series belong to a normal distribution, including the histogram and the Q-Q plot analysis (Appendix D).

## 6.2. Univariate Analysis: GARCH, GJR-GARCH and APARCH

After preliminary analyses and tests, the objective of this section will be to estimate univariate volatility models. To this end, each of the 7 selected variables will first be modeled in terms of its conditional mean, according to ARIMA modeling. The residuals diagnosis (Appendix E) will allow us to assess the adherence of each time series to the estimated ARIMA model, and then checking the ACF, PACF and squared residuals, together with the Box-Ljung autocorrelation and ARCH-LM heteroscedasticity tests will allow us to proceed with the estimation of the univariate volatility models.

***Table 10: ARIMA models estimated for the selected variables***

| Variable | ar1 | ar2 | ar3 | ar4 | ar5 | ma1 | ma2 | ma3 | ma4 | ma5 | intercept |
|---|---|---|---|---|---|---|---|---|---|---|---|
| LogRet_DGS2 | - | - | - | - | - | -0.130*** | -0.027*** | - | - | - | - |
| VIXCLS | 0.291** | -0.150 | -0.339** | 0.581*** | 0.033** | -0.405*** | 0.151 | 0.322** | -0.682*** | - | - |
| PCEPI12 | 0.980*** | - | - | - | - | 0.027*** | - | - | - | - | - |
| EXPINF2YR | 0.980*** | - | - | - | -0.020** | - | - | - | - | - | - |
| BBKMGDP | -0.596*** | 0.540*** | 0.951*** | - | - | 1.621*** | 1.076*** | 0.067*** | - | - | - |
| UNRATE | 0.963*** | - | - | - | - | 0.019* | 0.018* | 0.018* | 0.017* | 0.016 | - |
| LogRet_GFDEGDQ188S | - | - | - | - | - | - | - | - | - | - | - |

The estimates for the conditional mean (Table 10) were obtained with the *auto.arima* function in the '*forecast*' package of the *RStudio* software. The routine allows obtaining the AR and MA parameters that maximize the likelihood function, being the best model in terms of AIC. The first point to highlight, corroborating the graphical analysis, is the strong persistence that characterizes the observed and expected inflation rate and unemployment rate variables, which are highly inertial, with AR coefficients statistically significant at 1% and close to 1. Likewise, the volatility index (VIX) is also marked by a long-term memory, represented by significant coefficients up to the fifth lag. Finally, it is worth highlighting the importance of the memory of random shocks for the log-return of the 2-year Treasuries, represented in the statistically significant MA parameters.

Next, the results of the Box-Ljung autocorrelation test (Appendix F) indicate that the models were largely successful in eliminating autocorrelation of the residuals – that is, not rejecting the null hypothesis of absence of serial correlation for all variables, with the exception of the economic growth rate proxy and expected inflation, for which even the ARIMA model optimized by the algorithm led to the rejection of the hypothesis that the residuals are not autocorrelated, at least at the 10% significance level.

The analysis of the residuals using the ACF and PACF (presented in the Appendix) indicates that the ARIMA processes were well calibrated to correctly model the conditional variance of the selected variables. In turn, the graphical analysis of the squared residuals correlogram suggests the existence of GARCH effects that can be modeled, especially in the case of the long-term interest rate variables, observed inflation, expected inflation and the volatility index.

Additionally, the ARCH-LM test can help us corroborate the graphical evidence presented in the ACF and PACF. The results (Appendix F) are interesting, although somewhat frustrating, because they allow us to reject the null hypothesis that there is no heteroscedasticity (i.e., confirming that there are GARCH effects to be modeled) only for 2-year Treasuries and the VIX volatility index. However, the results do not seem consistent with the graphical analysis of the squared residuals. In any case, the estimation of the univariate GARCH models can subsequently allow us to draw greater conclusions about the process that characterizes the selected time series.

The first point to be highlighted is that, despite some divergences in the ARCH-LM test and ambiguous signals in the correlogram, in general the results confirmed the existence of ARCH/GARCH effects. The results for the variable of interest *LogRet_DGS2* (Table 21) satisfy the desired properties and suggest high persistence, with future variance being strongly affected by past variance, represented by the coefficients α + β.

***Table 11: Estimation results of the univariate volatility models for the 2-year Treasury***

**Results for variable: LogRet_DGS2**

| Parameter | GARCH | GJR-GARCH | APARCH |
|---|---|---|---|
| μ | 0.000 | 0.000 | 0.000 |
| ω | 0.000 | 0.000 | 0.000 |
| α | 0.043*** | 0.013*** | 0.029*** |
| β | 0.956*** | 0.966*** | 0.966*** |
| γ | | 0.040*** | 0.333*** |
| δ | | | 2.021*** |
| | | | |
| BIC | -4.8454 | -4.8597 | -4.8588 |
| LogLikelihood | 26120.49 | 26202.36 | 26202.35 |

In the GJR-GARCH model, the estimated γ parameter is positive and statistically significant at 1%, indicating the importance of asymmetry in the models. For example, positive shocks impact the volatility of the 2-year Treasury by α. However, negative shocks impact volatility by α + γ, which means that the conditional variance is more sensitive to *bad news*. Finally, in the APARCH model, the positive δ parameter, statistically significant at 1%, and close to 2, suggests that both asymmetry models (GJR-GARCH and APARCH) present equally satisfactory results.

Considering that the LR ratio and the BIC information criterion are very close between the models, and that the parameter that captures asymmetry is also statistically significant in the latter, we chose the APARCH model as the most likely to be used in the data-generating process. The graphical analysis of the conditional variance in the APARCH model (Graph 15) illustrates the results described above, confirming (i) the persistence of volatility for the 2-year Treasury and (ii) the asymmetric peak in volatility in negative shocks (bad news) in times of crisis or stress in the markets, as observed in 2008-09 and more recently in 2020-22.

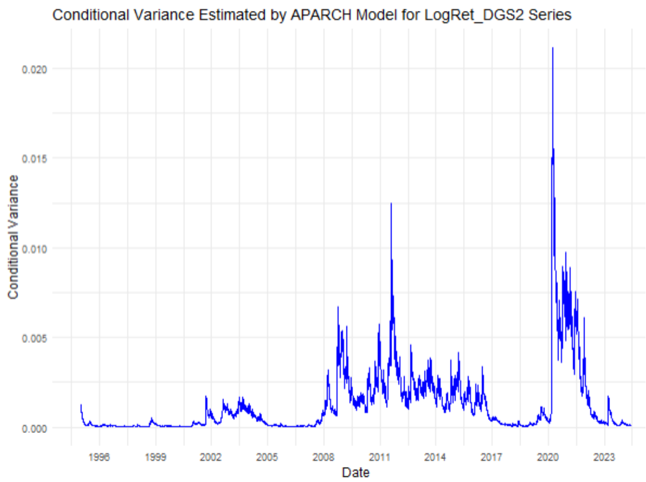


***Figure 15: Conditional variance in the APARCH volatility model for the 2-year Treasury***

We also performed estimations of the GARCH, GJR-GARCH and APARCH models for the explanatory variables, with detailed results in the Appendix G and H. For observed inflation (*PCEPI*) and expected inflation (*EXPINF2YR*), the importance of asymmetry in volatility was not observed, but the ARCH effect was confirmed ($\alpha$ positive and statistically significant).

For economic growth (*BBKMGDP*), the models also confirm persistent conditional variance ($\alpha + \beta$ close to 1 and statistically significant) and no asymmetric effects, while for the unemployment rate (*UNRATE*) the asymmetry coefficient ($\gamma = -0.029$) negative and statistically significant at 1% indicates that its volatility declines with positive shocks (*good news*), which is in line with the predictions of macroeconomic theory.

In turn, for public debt (*LogRet_GFDEGDQ188S*) there is also no relevance of the asymmetric effect, but only the persistence of volatility (positive and statistically significant $\alpha$), while for the volatility index (*VIXCLS*) the APARCH model indicates that all parameters are statistically significant and also an asymmetry coefficient ($\gamma = -0.053$) that is negative and statistically significant at 1%, consistent with the expectation that *good news* tends to reduce the conditional variance of the VIX, in line with empirical evidence.

### 6.3. Multivariate Analysis: DCC-GARCH and AG-DCC-GARCH

Once we have already specified the univariate volatility models for all series, we perform multivariate analyses with the DCC-GARCH and AG-DCC-GARCH models. The objective is to investigate the interrelationship between the volatility of interest rates and the volatility of the explanatory macroeconomic variables, evaluating the *spillover* hypothesis – that is, the time-varying conditional correlation between the variables. To do so, first we perform the Granger causality test, to check the temporal precedence between the variables, and then estimate a VAR model, whose residuals will be the source of the conditional covariance models.

In general, the Pairwise Granger Causality Test (Table 12) confirms a feedback effect between 2-year Treasuries and each of the explanatory variables, with one exception, which indicates that observed (past) inflation (*PCEPI*) does not Granger-cause the interest rate. For the remaining variables, the test rejects the null hypothesis that there is no temporal precedence, suggesting that the chosen indicators are good predictors of the long-term interest rate, in line with the predictions of macroeconomic theory.

***Table 12: Pairwise Granger Causality Test between the 2-year Treasury and each explanatory variable***

| Hypothesis | F_Test | P_value |
|---|---|---|
| H0: LogRet_DGS2 does not Granger-cause PCEPI12 | 10,039 | 0,000 |
| H0: PCEPI12 does not Granger-cause LogRet_DGS2 | 0,412 | 0,745 |
| | | |
| H0: LogRet_DGS2 does not Granger-cause EXPINF2YR | 6,526 | 0,000 |
| H0: EXPINF2YR does not Granger-cause LogRet_DGS2 | 1,928 | 0,044 |
| | | |
| H0: LogRet_DGS2 does not Granger-cause BBKMGDP | 3,864 | 0,002 |
| H0: BBKMGDP does not Granger-cause LogRet_DGS2 | 6,423 | 0,000 |
| | | |
| H0: LogRet_DGS2 does not Granger-cause UNRATE | 15,397 | 0,000 |
| H0: UNRATE does not Granger-cause LogRet_DGS2 | 9,644 | 0,000 |
| | | |
| H0: LogRet_DGS2 does not Granger-cause LogRet_GFDE | 3,170 | 0,001 |
| H0: LogRet_GFDEGDQ188S does not Granger-cause LogF | 14,047 | 0,000 |
| | | |
| H0: LogRet_DGS2 does not Granger-cause VIXCLS | 4,347 | 0,000 |
| H0: VIXCLS does not Granger-cause LogRet_DGS2 | 3,002 | 0,002 |

Next, we estimate a VAR model for *LogRet_DGS2*, using as explanatory variables the set of macroeconomic drivers and the past interest rate itself (all covariates lagged to ensure exogeneity). As an additional analysis, we present in the Appendix I the VAR models estimated for all explanatory variables. The results for the interest rate conditional mean (Table 13) show that, in addition to the autoregressive term, the 2-year Treasuries are explained by the unemployment rate (*UNRATE*) and expected inflation (*EXPINF2YR*).

These estimates not only confirm predictions of monetary policy theory but is also perfectly in line with the mandate established for the Fed, which defines that *"the monetary policy goals of the Federal Reserve are to foster economic conditions that achieve both stable prices and maximum sustainable employment"*.[3] Therefore, the results seem to confirm that, as inflation forecasts or the unemployment rate evolve differently from the Fed's targets (long-term equilibrium), the monetary authority does indeed use the spot rate instrument (Fed Funds), which in turn influences the market pricing of 2-year Treasuries due to the horizon (18 to 24 months) in which monetary policy affects the real economy and can impact the inflation rates.

***Table 13: Results of the VAR model for the 2-year Treasury***

| Variable | Coefficient_Name | Coefficient | Significance |
|---|---|---|---|
| **LogRet_DGS2** | LogRet_DGS2.l1 | -0,140 | *** |
| | VIXCLS.l1 | 0,000 | |
| | PCEPI12.l1 | 0,058 | |
| | EXPINF2YR.l1 | 0,249 | *** |
| | BBKMGDP.l1 | -0,002 | |
| | UNRATE.l1 | -0,112 | *** |
| | LogRet_GFDEGDQ188S.l1 | 0,041 | |
| | LogRet_DGS2.l2 | -0,051 | *** |
| | VIXCLS.l2 | 0,000 | |
| | PCEPI12.l2 | -0,057 | |
| | EXPINF2YR.l2 | -0,250 | *** |
| | BBKMGDP.l2 | 0,002 | |
| | UNRATE.l2 | 0,111 | *** |
| | LogRet_GFDEGDQ188S.l2 | 0,083 | |
| | const | 0,003 | |

From the estimated VAR, we performed the usual residuals diagnostic, which are detailed in the Appendix J. The CUSUM test of model stability, to detect structural changes based on recursive residuals, showed that the critical (lower and upper) lines were not exceeded, therefore the system is stable over time. The Box-Ljung test indicates that the null hypothesis

[3] The Federal Reserve's Dual Mandate (Fed Chicago, 2020)

is not rejected, therefore there is no serial correlation in the residuals. Finally, the multivariate ARCH-LM test on the VAR for *LogRet_DGS2* rejects the null hypothesis, indicating heteroscedasticity in the residuals and, therefore, conditional variance to be modeled.

In this way, we estimate the multivariate conditional volatility models (Table 14), obtained from the functions of the '*rmgarch*' package in *R* (full results are detailed in the Appendix K). The coefficients referring to the time-varying conditional correlation are represented by the parameters $[Joint]dcca1$ and $[Joint]dccb1$. Both represent the temporal dynamics of the correlations between the 7 macroeconomic variables selected for this analysis.

The DCC-GARCH model resulted in both parameters being positive and statistically significant at 1%. The coefficients ($[Joint]dcca1$ = 0.074) and ($[Joint]dccb1$ = 0.897) follow the expected pattern, with the former having a low value and being less than 0.1 and the sum of these parameters being close to or greater than 0.9, a result that confirms that the dynamic correlations are strongly persistent. On the other hand, the AG-DCC-GARCH model, which includes the asymmetric component in the analysis of conditional covariances, did not show statistically significant results, ruling out the hypothesis that the correlations between the series may be influenced by asymmetric shocks. In other words, the temporal pattern of dynamic correlation between the 7 variables does not seem to differentiate *bad news* from *good news*, according to the results obtained. Therefore, the results indicate that the volatilities of the series are persistent, that the correlation dynamics between them are persistent, but the DCC-GARCH model seems to be sufficient to capture these dynamics of conditional correlations.

***Table 14: Estimation results of the complete multivariate volatility model for the 2-year Treasury***

**Results for variable: LogRet_DGS2**

| Parameter | DCC-GARCH (1,1) | AG-DCC-GARCH (1,1) |
|---|---|---|
| [Joint]dcca1 | 0.074*** | 0.164 |
| [Joint]dccb1 | 0.897*** | 0.763*** |
| [Joint]dccg1 | | 0.000 |
| | | |
| BIC | 1792.4 | 1792.4 |
| LogLikelihood | -896.19 | -896.17 |

In addition to estimating the full specifications, with the 7 variables simultaneously, we also estimated the models only with LogRet_DGS2 and each of the explanatory macroeconomic variables (Pairwise estimation). Here, we had three objectives: (i) testing the robustness of the full model to confirm the main conclusions; (ii) attenuating the noise in the covariance matrix; and (iii) reducing the loss of degrees of freedom of the full model, in which we estimated up to 52 parameters, compared to 12 parameters estimated in Pairwise estimation.

The overall results (Table 15) confirm that the temporal dynamics of the correlation are highly persistent also in the models with pairwise variables. First, in all estimations the $[Joint]dccb1$ parameter resulted in estimates greater than 0.9 and always statistically significant at 1%. Furthermore, the results generally corroborate that this persistent dynamic is not influenced by asymmetric shocks, represented by the non-statistical significance (and the low numerical magnitude) of the $[Joint]dccg1$ parameter.

***Table 15: Pairwise DCC-GARCH and A-DCC-GARCH Estimations: LogRet_DGS2 vs. each variable***

| **LogRet_DGS2 and PCEPI12** | | | **LogRet_DGS2 and EXPINF2YR** | | |
|---|---|---|---|---|---|
| Parameter | DCC-GARCH (1,1) | AG-DCC-GARCH (1,1) | Parameter | DCC-GARCH (1,1) | AG-DCC-GARCH (1,1) |
| [Joint]dcca1 | 0.002** | 0.002** | [Joint]dcca1 | 0.002* | 0.002 |
| [Joint]dccb1 | 0.994*** | 0.994*** | [Joint]dccb1 | 0.996*** | 0.996*** |
| [Joint]dccg1 | | 0.000 | [Joint]dccg1 | | 0.000 |
| BIC | -3.3143 | -3.3134 | BIC | -3.9220 | --3.9212 |
| LogLikelihood | 17905.22 | 17905.22 | LogLikelihood | 21178.91 | 21179.07 |

| **LogRet_DGS2 and BBKMGDP** | | | **LogRet_DGS2 and UNRATE** | | |
|---|---|---|---|---|---|
| Parameter | DCC-GARCH (1,1) | AG-DCC-GARCH (1,1) | Parameter | DCC-GARCH (1,1) | AG-DCC-GARCH (1,1) |
| [Joint]dcca1 | 0.003* | 0.002 | [Joint]dcca1 | 0.003*** | 0.003*** |
| [Joint]dccb1 | 0.983*** | 0.983*** | [Joint]dccb1 | 0.992*** | 0.992*** |
| [Joint]dccg1 | | 0.001 | [Joint]dccg1 | | 0.000 |
| BIC | -1.7430 | --1.7422 | BIC | -2.6851 | -2.6843 |
| LogLikelihood | 9440.678 | 9440.961 | LogLikelihood | 14515.78 | 14515.88 |

| **LogRet_DGS2 and LogRet_GFDEGDQ188S** | | | **LogRet_DGS2 and VIXCLS** | | |
|---|---|---|---|---|---|
| Parameter | DCC-GARCH (1,1) | AG-DCC-GARCH (1,1) | Parameter | DCC-GARCH (1,1) | AG-DCC-GARCH (1,1) |
| [Joint]dcca1 | 0.007*** | 0.001 | [Joint]dcca1 | 0.005*** | 0.005*** |
| [Joint]dccb1 | 0.922*** | 0.986*** | [Joint]dccb1 | 0.992*** | 0.992*** |
| [Joint]dccg1 | | 0.001*** | [Joint]dccg1 | | 0.000 |
| BIC | 1783 | 1783 | BIC | 0.87022 | 0.87108 |
| LogLikelihood | -9605113 | -9605110 | LogLikelihood | -4636.792 | -4636.782 |

Finally, the temporal evolution of the dynamic conditional correlation of the pairwise models (Graph 16) also confirms the persistence pattern observed in the full specification, especially with the variables of observed inflation, expected inflation, unemployment rate and the market volatility index. In addition, it confirms the influence of breaks due to market stress.

For example, in the relationship between the volatility of the 2-year Treasury with observed inflation, the slightly positive correlation has its trend changed to close to zero after the global financial crisis in 2008-09, rising sharply in 2019 (when it reaches the maximum in the historical series), followed by a new sharp drop during the Covid-19 pandemic, and more recently returning to values close to zero when the Fed tightened monetary policy. However, for expected inflation, what is observed is an increasing trend, with the volatility between the variables negatively correlated before the financial crisis, subsequently slightly positive and reaching its historical peak in a similar way to the model with observed inflation.

In the pairwise model between the 2-year Treasury and the unemployment rate, there is a relatively cyclical pattern before the 2008-09 crisis that is followed by a declining trend, which reached historical lows with the negative correlation after the Covid-19 pandemic. On the other hand, in the case of VIX, the positive correlation with the volatility of future interest rates during the pandemic period reaches the highest values in more than 2 decades.

For the pairwise models with the economic growth rate proxy and % public debt/GDP, there is no such clear pattern, even though the parameters estimated in DCC-GARCH confirmed the persistence of the dynamic conditional correlation. For the latter, it was the only case in which AG-DCC-GARCH suggested the relevance of the asymmetric component in the dynamic correlation of volatility, even though graphically historical correlation very close to zero.

In general, it can be said that the models that capture the spillover effect between the volatility of future interest rates and macroeconomic drivers confirm the hypothesis of persistence in the relationship between the conditional variance of the most relevant variables in the monetary policy decision by the Fed in the US.

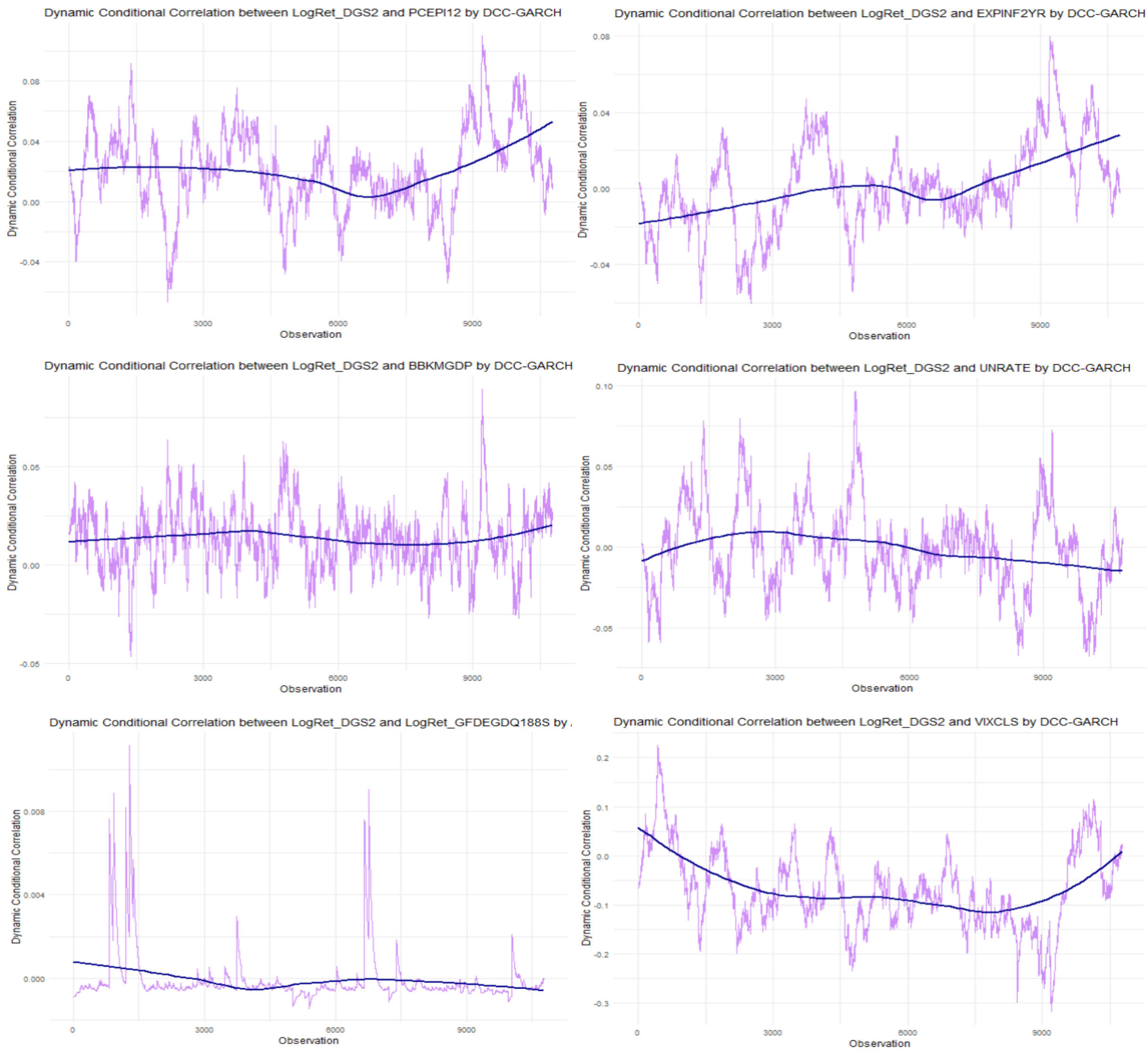


***Figure 16: Dynamic Correlations in the Pairwise DCC-GARCH Estimations (LogRet_DGS2 vs each variable)***

## Final Considerations and Further Research

This study has provided an analysis of the volatility and correlation dynamics of U.S. Treasury 2-year yield and macroeconomic determinants using univariate and multivariate GARCH models. While the findings offer significant insights, they also highlight topics for further analysis. This section summarizes the key takeaways, acknowledges the limitations, and suggests directions for future research.

First, we observed the stochastic trend for interest rates and fiscal variables in the US between 1994 and 2024. While the former shows a decreasing trend, public debt as % of GDP shows an increasing trend in recent decades. On the other hand, we verified stationarity for observed and expected inflation, economic growth rate, unemployment rate and market risk index. Graphically, we evidenced volatility peaks in the years following the *dot-com bubble* (2002-04), between the years of the international financial crisis (2008-09) and the sovereign debt crisis in Europe (2011-14), and again in the recent crisis of the Covid-19 pandemic and subsequent inflationary outbreak (2020-22) and the statistical tests confirmed ARCH/GARCH effects, for which the univariate models corroborated high persistence, with future variance being strongly affected by past variance. This evidence is in line with the empirical literature that shows the relevance of shocks, such as Covid-19, for the persistence of volatility among financial variables (Ribeiro and Curto, 2017; Banerjee, 2021; Ji et al., 2022).

The multivariate GJR-GARCH and APARCH models allow to conclude on the importance of asymmetry – that is, negative shocks (*bad news*) have a greater impact on the conditional volatility of future interest rates than positive shocks (*good news*). Furthermore, the DCC-GARCH model identified the spillover effect between the volatility of interest rates and the volatility of the explanatory macroeconomic variables – i.e. the time-varying conditional correlation between the variables. On the other hand, the AG-DCC-GARCH model ruled out the hypothesis that the dynamic correlations between the variables' variances may be influenced by asymmetric shocks. The results for volatility spillover, although confirming the persistence of conditional variance in multivariate models, diverge from empirical studies (Ji *et al.*, 2022; Curto and Serrasqueiro, 2022) that found a statistically significant impact of asymmetric shocks on the dynamic conditional correlation between variables.

In addition to estimating the full specifications for DCC-GARCH with the 7 variables simultaneously, we also estimated the models only for 2-year Treasuries and each of the explanatory macroeconomic variables, and the results confirmed that the temporal dynamics between variables´ volatilities are highly persistent also in this pairwise estimation. The results of the multivariate conditional volatility models corroborate the evidence from the Pairwise Granger Causality Test and the VAR model for the conditional mean, confirming the feedback effect between 2-year Treasuries and the unemployment rate and expected inflation and suggesting that the variables are good predictors of the long-term interest rate.

Therefore, the present study confirms predictions of monetary policy theory in line with the Fed's mandate, suggesting that, as inflation forecasts or the unemployment rate evolves differently from the Fed's targets (long-term equilibrium), the monetary authority does indeed use the spot rate instrument (Fed Funds), which in turn influences the market pricing of 2-year Treasuries due to the horizon (18 to 24 months) in which monetary policy affects the real economy and can impact the inflation rates. In general, it can be said that the models that capture the spillover effect between the volatility of future interest rates and macroeconomic drivers confirm the hypothesis of persistence in the relationship between the conditional variance of the most relevant variables for the monetary policy decision by the Fed in the US.

The empirical results in this analysis are consistent with theoretical models that relate long-term interest rates to market expectations for inflation and the labor market (economic activity). For example, the expectations hypothesis in the information theory of the term structure of interest rates (Estrella and Hardouvelis, 1991; Mishkin, 1990; Campbell, 1995) predicts that the yield curve reflects information about future economic conditions, including expectations about growth, inflation, and monetary policy. In addition, the temporal persistence observed in the conditional mean and variance for the long-term interest rate is supported by theoretical models that predict that investors anticipate future increases in the short-term interest rate in the spot market for long-term interest rates, implying a positively sloped yield curve (Fama, 1984; Campbell and Shiller, 1991).

In this sense, this empirical study brings practical insights for risk management and investment strategies: (i) the results of the APARCH model help investors in modeling asymmetry and the importance of accounting for downside risk in portfolio management, especially in contracts whose underlying assets are interest rates and their volatility; (ii) investors can use insights from the DCC-GARCH models to better manage interest rate risk by understanding how different maturities respond to economic conditions.

Naturally, there are some limitations to this study that indicate paths for future research and improvements in the empirical analysis. For example, methodologically, we can test alternatives to the DCC-GARCH model that allow capturing nonlinear dynamics (e.g. ANST-GARCH, ARCH-NNH), or even modern models based on machine learning algorithms (e.g. NN, LSTM, Ensemble Methods), which would allow a comparative extension of the results. In addition, it is possible to test whether daily proxies in high-frequency data (more granular than the traditional monthly periodicity of macroeconomic variables) could increase the accuracy of long-term interest rate volatility models.

Regarding the choice of variables and sample interval, additional tests can be performed, such as estimations in sub-samples or rolling windows, in order to analyze the robustness of the model parameters – and it could eventually be useful to assess the impact of the Covid-19 shock on the stability of the estimated coefficients. Another potential advance could be to expand the sample beyond the US, establishing a comparative panel of different countries and central banks (e.g. EU, UK, Japan), which would provide insights into the global financial interconnection and the transmission of shocks and volatility not only between different macroeconomic variables, but also across different markets. Such results could improve public policies that promote financial stability and effective coordination in the joint action of central banks.

Lastly, specifications for different maturities of the yield curve could also be tested, even though the choice of 2-year Treasuries was based on graphical analysis, on the correlation matrix with other maturities and on the macroeconomic fundamentals regarding the relevant horizon for monetary policy. Nevertheless, the present study applies a robust and referenced econometric methodology in the literature for modeling the dynamic conditional volatility of long-term interest rates and brings some empirical contributions to the real investment decisions and to the validation of theoretical predictions of the monetary policy and interest rate models.

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## Appendix

## A. Correlation matrix for the variables in each macroeconomic group

*Table 16: Intra-group correlation matrix for the selected variables*

| | DGS10 | DGS5 | DGS2 | DFF |
|---|---|---|---|---|
| DGS10 | 1.00 | 0.98 | 0.91 | 0.82 |
| DGS5 | 0.98 | 1.00 | 0.97 | 0.90 |
| DGS2 | 0.91 | 0.97 | 1.00 | 0.96 |
| DFF | 0.82 | 0.90 | 0.96 | 1.00 |

| | CPIAUCSL12 | CPILFESL12 | PCEPI12 | PCEPILFE12 |
|---|---|---|---|---|
| CPIAUCSL12 | 1.00 | 0.77 | 0.98 | 0.86 |
| CPILFESL12 | 0.77 | 1.00 | 0.75 | 0.92 |
| PCEPI12 | 0.98 | 0.75 | 1.00 | 0.90 |
| PCEPILFE12 | 0.86 | 0.92 | 0.90 | 1.00 |

| | EXPINF1YR | EXPINF2YR | EXPINF5YR | EXPINF10YR |
|---|---|---|---|---|
| EXPINF1YR | 1.00 | 0.96 | 0.88 | 0.84 |
| EXPINF2YR | 0.96 | 1.00 | 0.98 | 0.96 |
| EXPINF5YR | 0.88 | 0.98 | 1.00 | 1.00 |
| EXPINF10YR | 0.84 | 0.96 | 1.00 | 1.00 |

| | BBKMGDP | BBKMCY |
|---|---|---|
| BBKMGDP | 1.00 | 0.84 |
| BBKMCY | 0.84 | 1.00 |

| | UNRATE | UEMPMED |
|---|---|---|
| UNRATE | 1.00 | 0.75 |
| UEMPMED | 0.75 | 1.00 |

| | GFDEGDQ188S | FYFSGDA188S |
|---|---|---|
| GFDEGDQ188S | 1.00 | -0.70 |
| FYFSGDA188S | -0.70 | 1.00 |

## B. Unit root tests for all potential variables for the econometric models

***Table 17: Unit root tests for the group of interest rate variables***

| *Variable* | *Test* | *Statistic* | *Significance* | *Conclusion* |
|---|---|---|---|---|
| *DGS10* | ADF with drift and trend | -1.8637788 | | Do not reject H0 |
| *DGS10* | DF-GLS with drift and trend | -1.9347603 | | Do not reject H0 |
| *DGS10* | PP with intercept and trend | -1.6788435 | | Do not reject H0 |
| *DGS10* | Zivot-Andrews | -4.6791477 | | Do not reject H0 |
| *DGS5* | ADF with drift and trend | -1.2708334 | | Do not reject H0 |
| *DGS5* | DF-GLS with drift and trend | -1.4346468 | | Do not reject H0 |
| *DGS5* | PP with intercept and trend | -1.1332908 | | Do not reject H0 |
| *DGS5* | Zivot-Andrews | -3.4558368 | | Do not reject H0 |
| *DGS2* | ADF with drift and trend | -0.3925527 | | Do not reject H0 |
| *DGS2* | DF-GLS with drift and trend | -0.7764834 | | Do not reject H0 |
| *DGS2* | PP with intercept and trend | -0.8188756 | | Do not reject H0 |
| *DGS2* | Zivot-Andrews | -3.1111215 | | Do not reject H0 |
| *DFF* | ADF with drift and trend | -2.0876695 | | Do not reject H0 |
| *DFF* | DF-GLS with drift and trend | -1.2877419 | | Do not reject H0 |
| *DFF* | PP with intercept and trend | -0.6689535 | | Do not reject H0 |
| *DFF* | Zivot-Andrews | -4.9639289 | * | Do not reject H0 |

***Table 18: Unit root tests for the group of observed inflation rate variables***

| *Variable* | *Test* | *Statistic* | *Significance* | *Conclusion* |
|---|---|---|---|---|
| *CPIAUCSL12* | ADF with drift and trend | -4.711312 | *** | Reject H0 |
| *CPIAUCSL12* | DF-GLS with drift and trend | -4.343843 | *** | Reject H0 |
| *CPIAUCSL12* | PP with intercept and trend | -1.590310 | | Do not reject H0 |
| *CPIAUCSL12* | Zivot-Andrews | -9.146127 | *** | Reject H0 |
| *CPILFESL12* | ADF with drift and trend | -3.114828 | | Do not reject H0 |
| *CPILFESL12* | DF-GLS with drift and trend | -2.561367 | | Do not reject H0 |
| *CPILFESL12* | PP with intercept and trend | -1.242943 | | Do not reject H0 |
| *CPILFESL12* | Zivot-Andrews | -22.624434 | *** | Reject H0 |
| *PCEPI12* | ADF with drift and trend | -4.071908 | *** | Reject H0 |
| *PCEPI12* | DF-GLS with drift and trend | -3.721875 | *** | Reject H0 |
| *PCEPI12* | PP with intercept and trend | -1.416869 | | Do not reject H0 |
| *PCEPI12* | Zivot-Andrews | -9.224379 | *** | Reject H0 |
| *PCEPILFE12* | ADF with drift and trend | -2.838150 | | Do not reject H0 |
| *PCEPILFE12* | DF-GLS with drift and trend | -2.280301 | | Do not reject H0 |
| *PCEPILFE12* | PP with intercept and trend | -1.379178 | | Do not reject H0 |
| *PCEPILFE12* | Zivot-Andrews | -18.653946 | *** | Reject H0 |

***Table 19: Unit root tests for the group of expected inflation rate variables***

| *Variable* | *Test* | *Statistic* | *Significance* | *Conclusion* |
|---|---|---|---|---|
| *EXPINF1YR* | ADF with drift and trend | -7.1405239 | *** | Reject H0 |
| *EXPINF1YR* | DF-GLS with drift and trend | -6.6089069 | *** | Reject H0 |
| *EXPINF1YR* | PP with intercept and trend | -3.1840814 | * | Do not reject H0 |
| *EXPINF1YR* | Zivot-Andrews | -3.5895577 | | Do not reject H0 |
| *EXPINF2YR* | ADF with drift and trend | -5.1224862 | *** | Reject H0 |
| *EXPINF2YR* | DF-GLS with drift and trend | -4.7732404 | *** | Reject H0 |
| *EXPINF2YR* | PP with intercept and trend | -1.9584064 | | Do not reject H0 |
| *EXPINF2YR* | Zivot-Andrews | -3.6168713 | | Do not reject H0 |
| *EXPINF5YR* | ADF with drift and trend | -4.0435916 | *** | Reject H0 |
| *EXPINF5YR* | DF-GLS with drift and trend | -3.8032601 | *** | Reject H0 |
| *EXPINF5YR* | PP with intercept and trend | -1.0601381 | | Do not reject H0 |
| *EXPINF5YR* | Zivot-Andrews | -4.8429066 | ** | Reject H0 |
| *EXPINF10YR* | ADF with drift and trend | -4.0209751 | *** | Reject H0 |
| *EXPINF10YR* | DF-GLS with drift and trend | -3.7966206 | *** | Reject H0 |
| *EXPINF10YR* | PP with intercept and trend | -0.9839935 | | Do not reject H0 |
| *EXPINF10YR* | Zivot-Andrews | -2.4289818 | | Do not reject H0 |

***Table 20: Unit root tests for the group of economic activity variables***

| *Variable* | *Test* | *Statistic* | *Significance* | *Conclusion* |
|---|---|---|---|---|
| *BBKMGDP* | ADF with drift and trend | -13.007247 | *** | Reject H0 |
| *BBKMGDP* | DF-GLS with drift and trend | -12.012966 | *** | Reject H0 |
| *BBKMGDP* | PP with intercept and trend | -5.057529 | *** | Reject H0 |
| *BBKMGDP* | Zivot-Andrews | -3.462997 | | Do not reject H0 |
| *BBKMCY* | ADF with drift and trend | -75.508324 | *** | Reject H0 |
| *BBKMCY* | DF-GLS with drift and trend | -2.371308 | | Do not reject H0 |
| *BBKMCY* | PP with intercept and trend | -8.156835 | *** | Reject H0 |
| *BBKMCY* | Zivot-Andrews | -8.383846 | *** | Reject H0 |

***Table 21: Unit root tests for the group of unemployment rate variables***

| *Variable* | *Test* | *Statistic* | *Significance* | *Conclusion* |
|---|---|---|---|---|
| *UNRATE* | ADF with drift and trend | -4.536259 | *** | Reject H0 |
| *UNRATE* | DF-GLS with drift and trend | -4.187106 | *** | Reject H0 |
| *UNRATE* | PP with intercept and trend | -1.899741 | | Do not reject H0 |
| *UNRATE* | Zivot-Andrews | -3.690309 | | Do not reject H0 |
| *UEMPMED* | ADF with drift and trend | -2.902779 | | Do not reject H0 |
| *UEMPMED* | DF-GLS with drift and trend | -2.780433 | * | Do not reject H0 |
| *UEMPMED* | PP with intercept and trend | -1.053633 | | Do not reject H0 |
| *UEMPMED* | Zivot-Andrews | -6.578283 | *** | Reject H0 |

**Table 22: Unit root tests for the group of fiscal variables**

| *Variable* | *Test* | *Statistic* | *Significance* | *Conclusion* |
|---|---|---|---|---|
| *GFDEGDQ188S* | ADF with drift and trend | -2.266073 | | Do not reject H0 |
| *GFDEGDQ188S* | DF-GLS with drift and trend | -1.078789 | | Do not reject H0 |
| *GFDEGDQ188S* | PP with intercept and trend | -2.264851 | | Do not reject H0 |
| *GFDEGDQ188S* | Zivot-Andrews | -4.607713 | | Do not reject H0 |
| *FYFSGDA188S* | ADF with drift and trend | -2.691248 | | Do not reject H0 |
| *FYFSGDA188S* | DF-GLS with drift and trend | -2.306084 | | Do not reject H0 |
| *FYFSGDA188S* | PP with intercept and trend | -2.699744 | | Do not reject H0 |
| *FYFSGDA188S* | Zivot-Andrews | -4.583887 | | Do not reject H0 |

**Table 23: Unit root tests for the market risk variable**

| *Variable* | *Test* | *Statistic* | *Significance* | *Conclusion* |
|---|---|---|---|---|
| *VIXCLS* | ADF with drift and trend | -8.257436 | *** | Reject H0 |
| *VIXCLS* | DF-GLS with drift and trend | -6.646767 | *** | Reject H0 |
| *VIXCLS* | PP with intercept and trend | -7.599831 | *** | Reject H0 |
| *VIXCLS* | Zivot-Andrews | -9.813662 | *** | Reject H0 |

# C. Consolidated unit root and normality tests after log-return transformation

**Table 24: Unit root tests for the selected variables after log-return transformation**

| *Variable* | *Test* | *Statistic* | *Significance* | *Conclusion* |
|---|---|---|---|---|
| *LogRet_DGS2* | ADF with drift and trend | -81.131332 | *** | Reject H0 |
| *LogRet_DGS2* | DF-GLS with drift and trend | -49.464680 | *** | Reject H0 |
| *LogRet_DGS2* | PP with intercept and trend | -118.827360 | *** | Reject H0 |
| *LogRet_DGS2* | Zivot-Andrews | -117.852856 | *** | Reject H0 |
| *VIXCLS* | ADF with drift and trend | -8.190756 | *** | Reject H0 |
| *VIXCLS* | DF-GLS with drift and trend | -6.559699 | *** | Reject H0 |
| *VIXCLS* | PP with intercept and trend | -7.551413 | *** | Reject H0 |
| *VIXCLS* | Zivot-Andrews | -9.680056 | *** | Reject H0 |
| *PCEPI12* | ADF with drift and trend | -4.071908 | *** | Reject H0 |
| *PCEPI12* | DF-GLS with drift and trend | -3.721875 | *** | Reject H0 |
| *PCEPI12* | PP with intercept and trend | -1.416869 | | Do not reject H0 |
| *PCEPI12* | Zivot-Andrews | -9.224379 | *** | Reject H0 |
| *EXPINF2YR* | ADF with drift and trend | -5.122486 | *** | Reject H0 |
| *EXPINF2YR* | DF-GLS with drift and trend | -3.340406 | ** | Reject H0 |
| *EXPINF2YR* | PP with intercept and trend | -2.336328 | | Do not reject H0 |
| *EXPINF2YR* | Zivot-Andrews | -4.061507 | ** | Reject H0 |
| *BBKMGDP* | ADF with drift and trend | -12.769622 | *** | Reject H0 |
| *BBKMGDP* | DF-GLS with drift and trend | -11.115194 | *** | Reject H0 |
| *BBKMGDP* | PP with intercept and trend | -4.976103 | *** | Reject H0 |
| *BBKMGDP* | Zivot-Andrews | -3.537394 | | Do not reject H0 |
| *UNRATE* | ADF with drift and trend | -4.448177 | *** | Reject H0 |
| *UNRATE* | DF-GLS with drift and trend | -4.237738 | *** | Reject H0 |
| *UNRATE* | PP with intercept and trend | -1.832627 | | Do not reject H0 |
| *UNRATE* | Zivot-Andrews | -3.690309 | | Do not reject H0 |
| *LogRet_GFDEGDQ188S* | ADF with drift and trend | -73.442215 | *** | Reject H0 |
| *LogRet_GFDEGDQ188S* | DF-GLS with drift and trend | -22.014810 | *** | Reject H0 |
| *LogRet_GFDEGDQ188S* | PP with intercept and trend | -103.849255 | *** | Reject H0 |
| *LogRet_GFDEGDQ188S* | Zivot-Andrews | -103.945105 | *** | Reject H0 |

**Table 25: Normality test for the group of selected variables after log-return transformation**

| *Variable* | *Test Statistic* | *Significance* | *Conclusion* |
|---|---|---|---|
| *LogRet_DGS2* | 8.617609e+04 | *** | Reject H0 - The data is not normally distributed |
| *VIXCLS* | 3.511356e+04 | *** | Reject H0 - The data is not normally distributed |
| *PCEPI12* | 6.538183e+03 | *** | Reject H0 - The data is not normally distributed |
| *EXPINF2YR* | 3.050661e+02 | *** | Reject H0 - The data is not normally distributed |
| *BBKMGDP* | 1.632220e+06 | *** | Reject H0 - The data is not normally distributed |
| *UNRATE* | 4.651129e+03 | *** | Reject H0 - The data is not normally distributed |
| *LogRet_GFDEGDQ188S* | 5.142752e+09 | *** | Reject H0 - The data is not normally distributed |

## D. Histograms and Q-Q plots for the selected variables for the econometric models

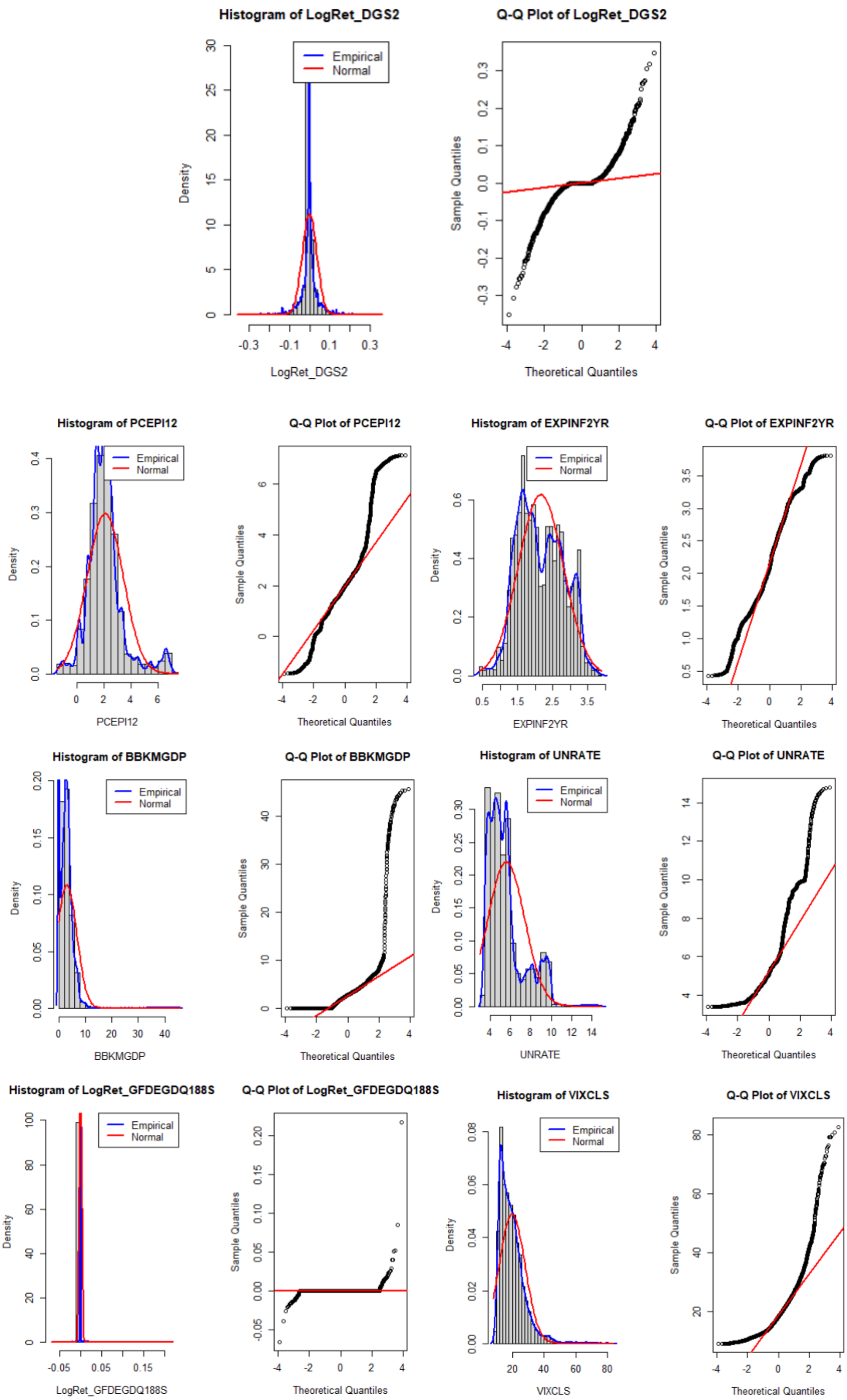


*Figure 17: Analysis of the normality of the variables selected for the volatility models*

## E. ACF, PACF and squared residuals from the ARIMA models for the variables

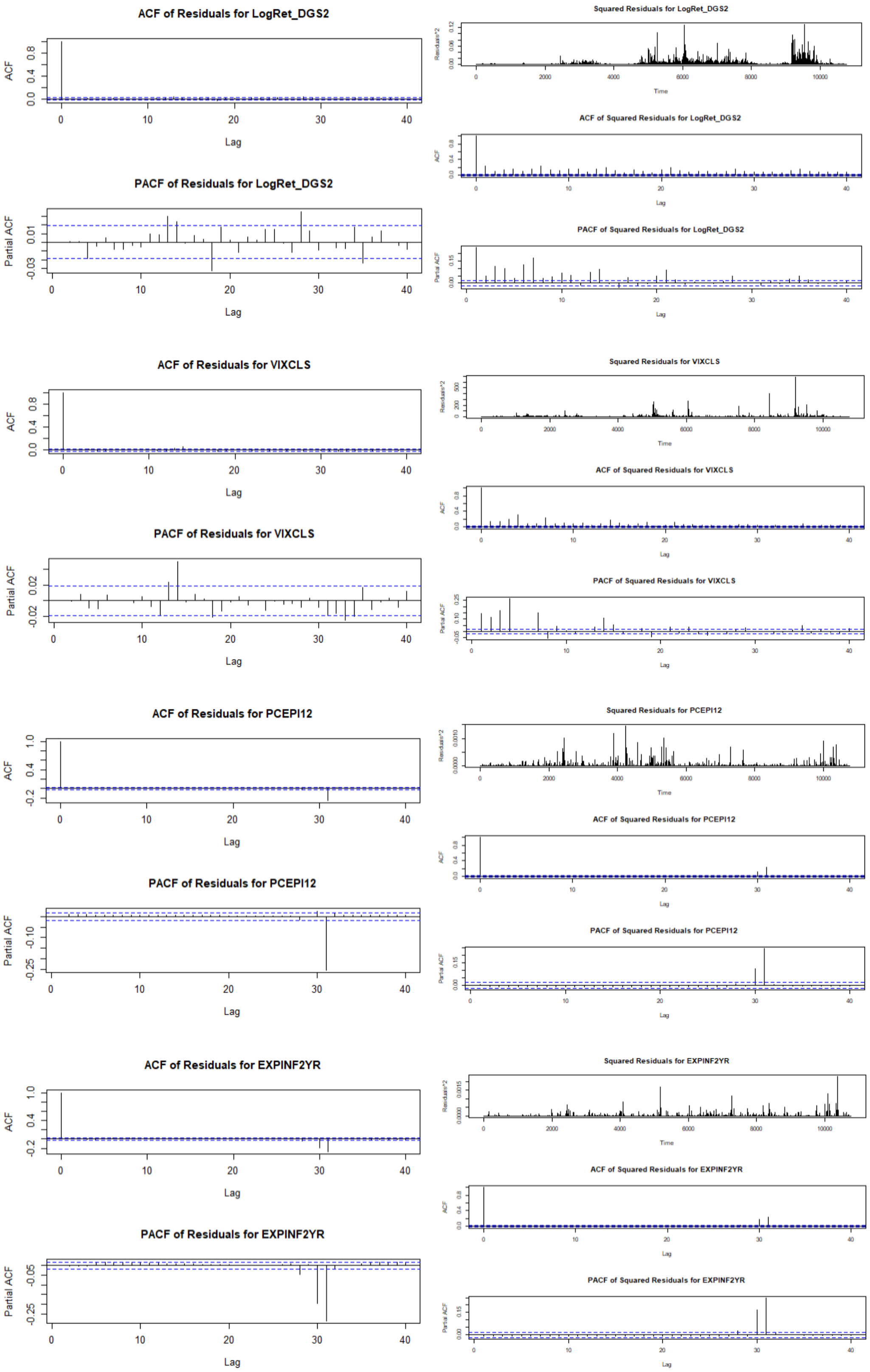


*Figure 18: ACF, PACF and squared residuals of the ARIMA models for the variables*

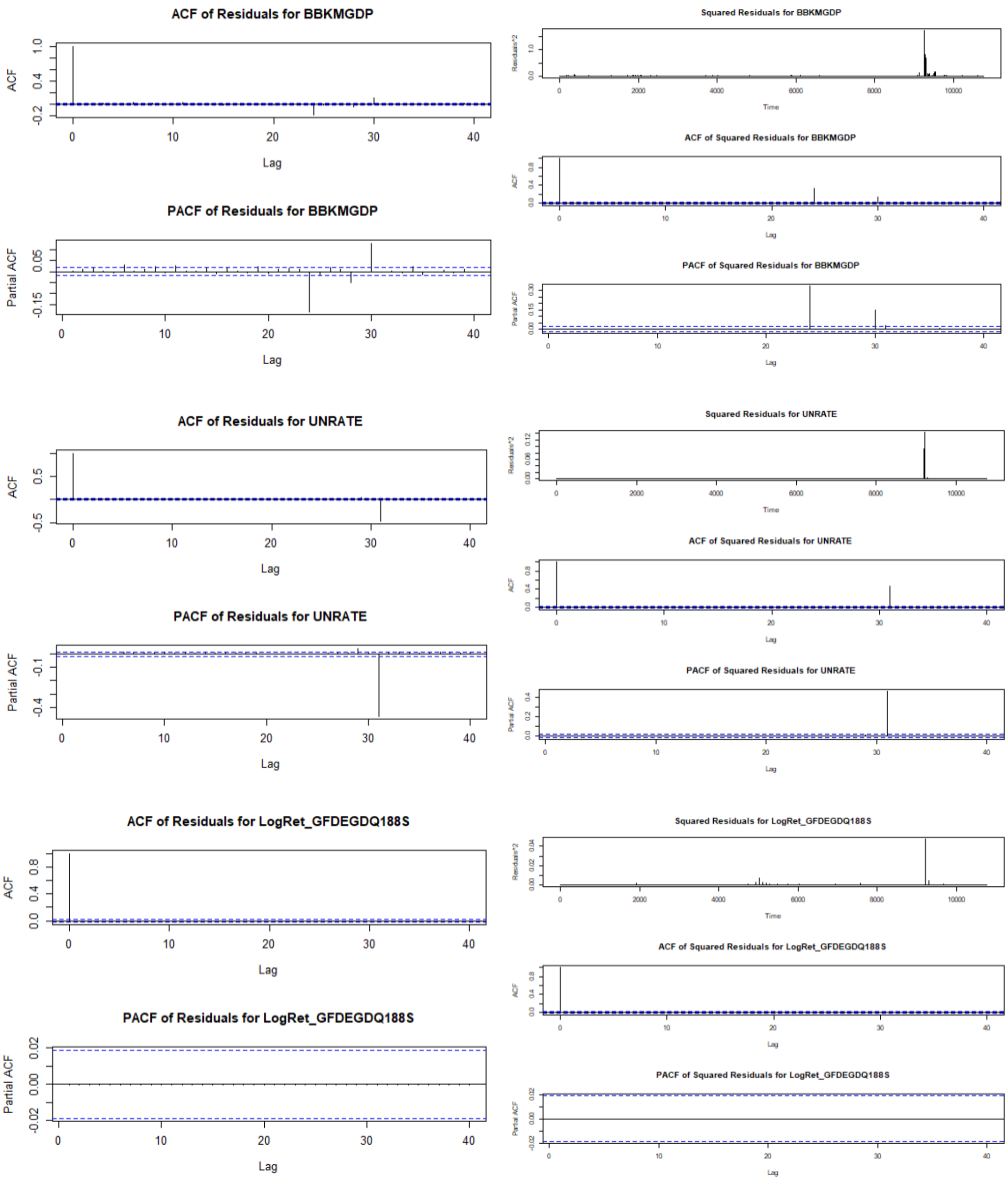


***Figure 18: ACF, PACF and squared residuals of the ARIMA models for the variables***

## F. Residuals diagnostics after ARIMA: tests for autocorrelation and heteroskedasticity

*Table 26: Box-Ljung Test: autocorrelation in residuals from ARIMA models for the selected variables*

| Variable | Statistic | Significance | Conclusion |
|---|---|---|---|
| **LogRet_DGS2** | 6,033 | | Do not reject H0 |
| **PCEPI12** | 7,544 | | Do not reject H0 |
| **EXPINF2YR** | 16,793 | * | Reject H0 |
| **BBKMGDP** | 22,117 | ** | Reject H0 |
| **UNRATE** | 11,619 | | Do not reject H0 |
| **LogRet_GFDEGDQ188S** | 0,021 | | Do not reject H0 |
| **VIXCLS** | 3,797 | | Do not reject H0 |

*Table 27: ARCH-LM Test: heteroscedasticity in residuals from ARIMA models for the selected variables*

| Variable | Statistic | Significance | Conclusion |
|---|---|---|---|
| **LogRet_DGS2** | 1386,689 | *** | Reject H0 |
| **PCEPI12** | 8,181 | | Do not reject H0 |
| **EXPINF2YR** | 5,625 | | Do not reject H0 |
| **BBKMGDP** | 0,300 | | Do not reject H0 |
| **UNRATE** | 0,082 | | Do not reject H0 |
| **LogRet_GFDEGDQ188S** | 0,011 | | Do not reject H0 |
| **VIXCLS** | 1589,143 | *** | Reject H0 |

## G. Estimation of GARCH, GJR-GARCH and APARCH for the explanatory variables

***Table 28: Results for PCEPI, EXPINF2YR, BBKMGDP, UNRATE LogRet_GFDEGDQ188S and VIXCLS***

**Results for variable: PCEPI12**

| Parameter | GARCH | GJR-GARCH | APARCH |
|---|---|---|---|
| μ | 1.972*** | 1.972*** | 1.972*** |
| ω | 0.000 | 0.000 | 0.000 |
| α | 0.862*** | 0.863*** | 0.865 |
| β | 0.137 | 0.137 | 0.142 |
| γ | | -0.002 | 0.000 |
| δ | | | 1.976 |
| BIC | 1.5299 | 1.5308 | 1.5316 |
| LogLikelihood | -8223.061 | -8223.059 | -8222.85 |

**Results for variable: EXPINF2YR**

| Parameter | GARCH | GJR-GARCH | APARCH |
|---|---|---|---|
| μ | 1.907*** | 1.907*** | 1.907*** |
| ω | 0.000*** | 0.000*** | 0.000 |
| α | 0.999*** | 0.999*** | 0.999*** |
| β | 0.000 | 0.000 | 0.000 |
| γ | | 0.001 | 0.000 |
| δ | | | 1.695*** |
| BIC | 0.92136 | 0.92223 | 0.92302 |
| LogLikelihood | -4944.819 | -4944.819 | -4944.471 |

**Results for variable: BBKMGDP**

| Parameter | GARCH | GJR-GARCH | APARCH |
|---|---|---|---|
| μ | 2.846*** | 2.846*** | 2.846*** |
| ω | 0.002*** | 0.002*** | 0.002 |
| α | 0.918*** | 0.916*** | 0.924*** |
| β | 0.081*** | 0.081*** | 0.083*** |
| γ | | 0.004 | 0.001 |
| δ | | | 1.974*** |
| BIC | 3.0861 | 3.0870 | 3.0878 |
| LogLikelihood | -16606.24 | -16606.23 | -16605.96 |

**Results for variable: UNRATE**

| Parameter | GARCH | GJR-GARCH | APARCH |
|---|---|---|---|
| μ | 5.000*** | 5.000*** | 5.000*** |
| ω | 0.000 | 0.000 | 0.002* |
| α | 0.740*** | 0.736*** | 0.104*** |
| β | 0.259 | 0.260 | 0.892*** |
| γ | | 0.006 | -0.029*** |
| δ | | | 0.744*** |
| BIC | 2.1584 | 2.1593 | 2.2842 |
| LogLikelihood | -11608.91 | -11608.86 | -12277.07 |

**Results for variable: LogRet_GFDEGDQ188S**

| Parameter | GARCH | GJR-GARCH | APARCH |
|---|---|---|---|
| μ | 0.000 | 0.000** | 0.000 |
| ω | 0.000 | 0.000 | 0.000 |
| α | 0.047 | 0.049 | 0.050 |
| β | 0.904** | 0.906*** | 0.900 |
| γ | | 0.039 | 0.052 |
| δ | | | 2.001 |
| BIC | -12.808 | -10.046 | -10.533 |
| LogLikelihood | 69013.26 | 54138.65 | 56771.28 |

**Results for variable: VIXCLS**

| Parameter | GARCH | GJR-GARCH | APARCH |
|---|---|---|---|
| μ | 15.273*** | 15.286*** | 15.295*** |
| ω | 0.574*** | 0.591*** | 0.527*** |
| α | 0.902*** | 0.959*** | 0.822*** |
| β | 0.097* | 0.090* | 0.112** |
| γ | | -0.124*** | -0.053*** |
| δ | | | 1.391*** |
| BIC | 5.7256 | 5.7248 | 5.724 |
| LogLikelihood | -30825.05 | -30816.49 | -30807.09 |

## H. Conditional variance in the APARCH volatility model for the explanatory variables

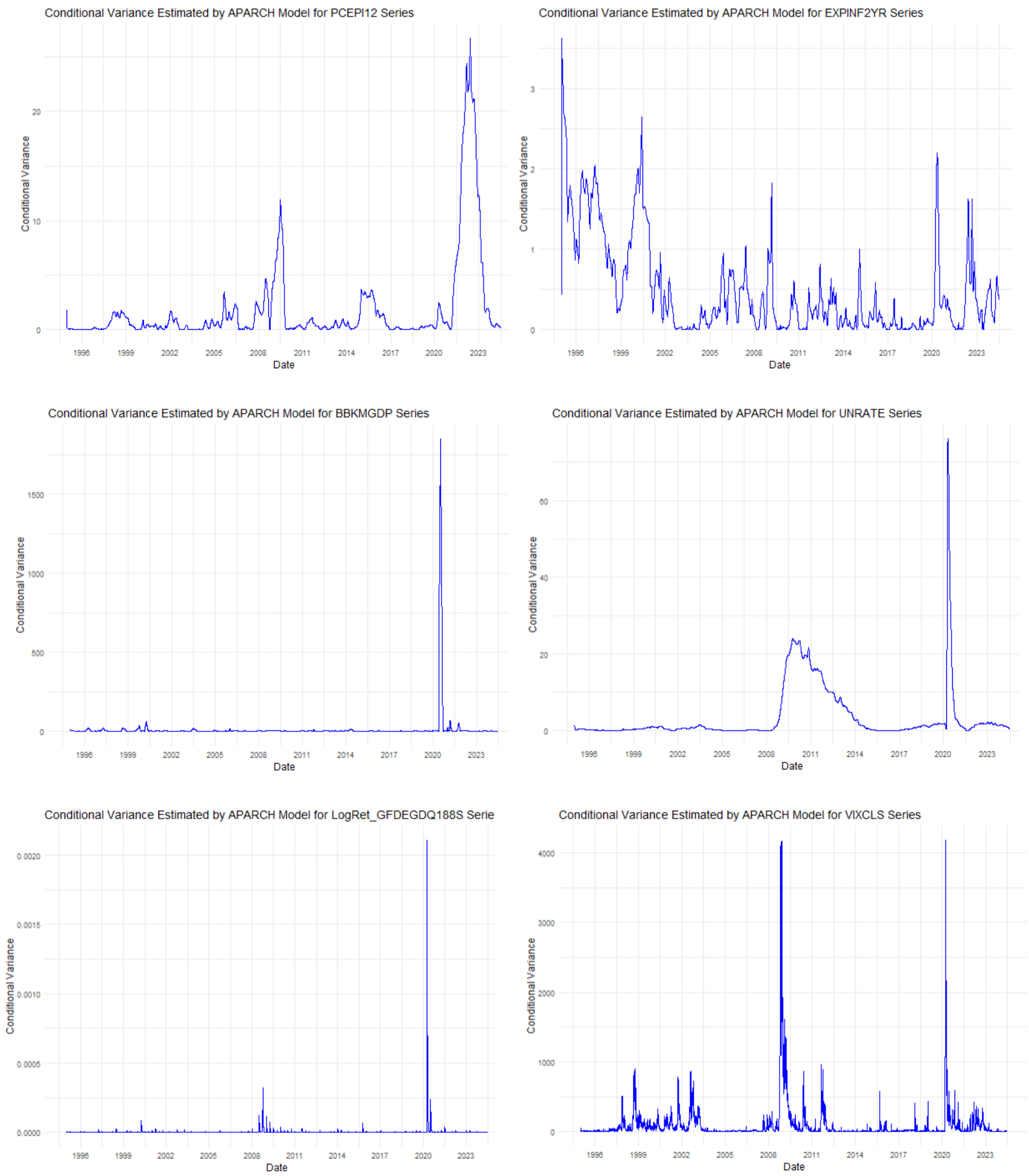


*Figure 19: Conditional variance in the APARCH volatility model for the selected variables*

# I. Estimation results in the VAR models for the explanatory variables

*Table 29: Results of the VAR model for the explanatory variables*

| Variable | Coefficient_Name | Coefficient | Significance | Variable | Coefficient_Name | Coefficient | Significance |
|---|---|---|---|---|---|---|---|
| **PCEPI12** | LogRet_DGS2.l1 | 0,000 | | **EXPINF2YR** | LogRet_DGS2.l1 | -0,001 | * |
| | VIXCLS.l1 | 0,000 | | | VIXCLS.l1 | 0,000 | |
| | PCEPI12.l1 | 1,981 | *** | | PCEPI12.l1 | 0,006 | *** |
| | EXPINF2YR.l1 | -0,001 | | | EXPINF2YR.l1 | 1,960 | *** |
| | BBKMGDP.l1 | 0,000 | | | BBKMGDP.l1 | 0,000 | |
| | UNRATE.l1 | 0,000 | | | UNRATE.l1 | -0,002 | * |
| | LogRet_GFDEGDQ188S.l1 | 0,014 | ** | | LogRet_GFDEGDQ188S.l1 | 0,052 | *** |
| | LogRet_DGS2.l2 | 0,000 | | | LogRet_DGS2.l2 | 0,000 | |
| | VIXCLS.l2 | 0,000 | | | VIXCLS.l2 | 0,000 | |
| | PCEPI12.l2 | -0,981 | *** | | PCEPI12.l2 | -0,006 | *** |
| | EXPINF2YR.l2 | 0,001 | | | EXPINF2YR.l2 | -0,960 | *** |
| | BBKMGDP.l2 | 0,000 | | | BBKMGDP.l2 | 0,000 | |
| | UNRATE.l2 | 0,000 | | | UNRATE.l2 | 0,002 | * |
| | LogRet_GFDEGDQ188S.l2 | -0,001 | | | LogRet_GFDEGDQ188S.l2 | -0,001 | |
| | const | 0,000 | | | const | 0,000 | *** |
| **BBKMGDP** | LogRet_DGS2.l1 | 0,0215 | *** | **UNRATE** | LogRet_DGS2.l1 | -0,0019 | |
| | VIXCLS.l1 | -0,0001 | | | VIXCLS.l1 | 0 | |
| | PCEPI12.l1 | 0,0047 | | | PCEPI12.l1 | -0,0066 | |
| | EXPINF2YR.l1 | 0,0359 | | | EXPINF2YR.l1 | -0,008 | |
| | BBKMGDP.l1 | 1,972 | *** | | BBKMGDP.l1 | -0,0001 | |
| | UNRATE.l1 | -0,0608 | *** | | UNRATE.l1 | 1,9804 | *** |
| | LogRet_GFDEGDQ188S.l1 | 0,7609 | *** | | LogRet_GFDEGDQ188S.l1 | -0,9456 | *** |
| | LogRet_DGS2.l2 | -0,0233 | *** | | LogRet_DGS2.l2 | 0 | |
| | VIXCLS.l2 | 0,0001 | | | VIXCLS.l2 | 0 | |
| | PCEPI12.l2 | -0,0049 | | | PCEPI12.l2 | 0,0065 | |
| | EXPINF2YR.l2 | -0,0356 | | | EXPINF2YR.l2 | 0,0078 | |
| | BBKMGDP.l2 | -0,9729 | *** | | BBKMGDP.l2 | 0 | |
| | UNRATE.l2 | 0,0611 | *** | | UNRATE.l2 | -0,9805 | *** |
| | LogRet_GFDEGDQ188S.l2 | -0,0623 | | | LogRet_GFDEGDQ188S.l2 | -0,0064 | |
| | const | 0,0002 | | | const | 0,0009 | *** |
| **LogRet_GFDEGDQ188S** | LogRet_DGS2.l1 | 0,0007 | | **VIXCLS** | LogRet_DGS2.l1 | 0,0406 | |
| **LogRet_GFDEGDQ188S** | VIXCLS.l1 | -0,0001 | *** | **VIXCLS** | VIXCLS.l1 | 0,8847 | *** |
| **LogRet_GFDEGDQ188S** | PCEPI12.l1 | -0,001 | | **VIXCLS** | PCEPI12.l1 | -4,3358 | *** |
| **LogRet_GFDEGDQ188S** | EXPINF2YR.l1 | -0,0035 | | **VIXCLS** | EXPINF2YR.l1 | -6,7021 | *** |
| **LogRet_GFDEGDQ188S** | BBKMGDP.l1 | -0,0002 | | **VIXCLS** | BBKMGDP.l1 | 0,0804 | |
| **LogRet_GFDEGDQ188S** | UNRATE.l1 | 0,0177 | *** | **VIXCLS** | UNRATE.l1 | 3,3276 | *** |
| **LogRet_GFDEGDQ188S** | LogRet_GFDEGDQ188S.l1 | -0,0203 | ** | **VIXCLS** | LogRet_GFDEGDQ188S.l1 | -8,7812 | * |
| **LogRet_GFDEGDQ188S** | LogRet_DGS2.l2 | -0,0021 | *** | **VIXCLS** | LogRet_DGS2.l2 | -0,288 | |
| **LogRet_GFDEGDQ188S** | VIXCLS.l2 | 0,0001 | *** | **VIXCLS** | VIXCLS.l2 | 0,0974 | *** |
| **LogRet_GFDEGDQ188S** | PCEPI12.l2 | 0,001 | | **VIXCLS** | PCEPI12.l2 | 4,3444 | *** |
| **LogRet_GFDEGDQ188S** | EXPINF2YR.l2 | 0,0034 | | **VIXCLS** | EXPINF2YR.l2 | 6,7052 | *** |
| **LogRet_GFDEGDQ188S** | BBKMGDP.l2 | 0,0002 | | **VIXCLS** | BBKMGDP.l2 | -0,0767 | |
| **LogRet_GFDEGDQ188S** | UNRATE.l2 | -0,0176 | *** | **VIXCLS** | UNRATE.l2 | -3,3121 | *** |
| **LogRet_GFDEGDQ188S** | LogRet_GFDEGDQ188S.l2 | -0,0046 | | **VIXCLS** | LogRet_GFDEGDQ188S.l2 | -12,2562 | ** |
| **LogRet_GFDEGDQ188S** | const | -0,0003 | * | **VIXCLS** | const | 0,2341 | ** |

## J. VAR Residuals Diagnostics

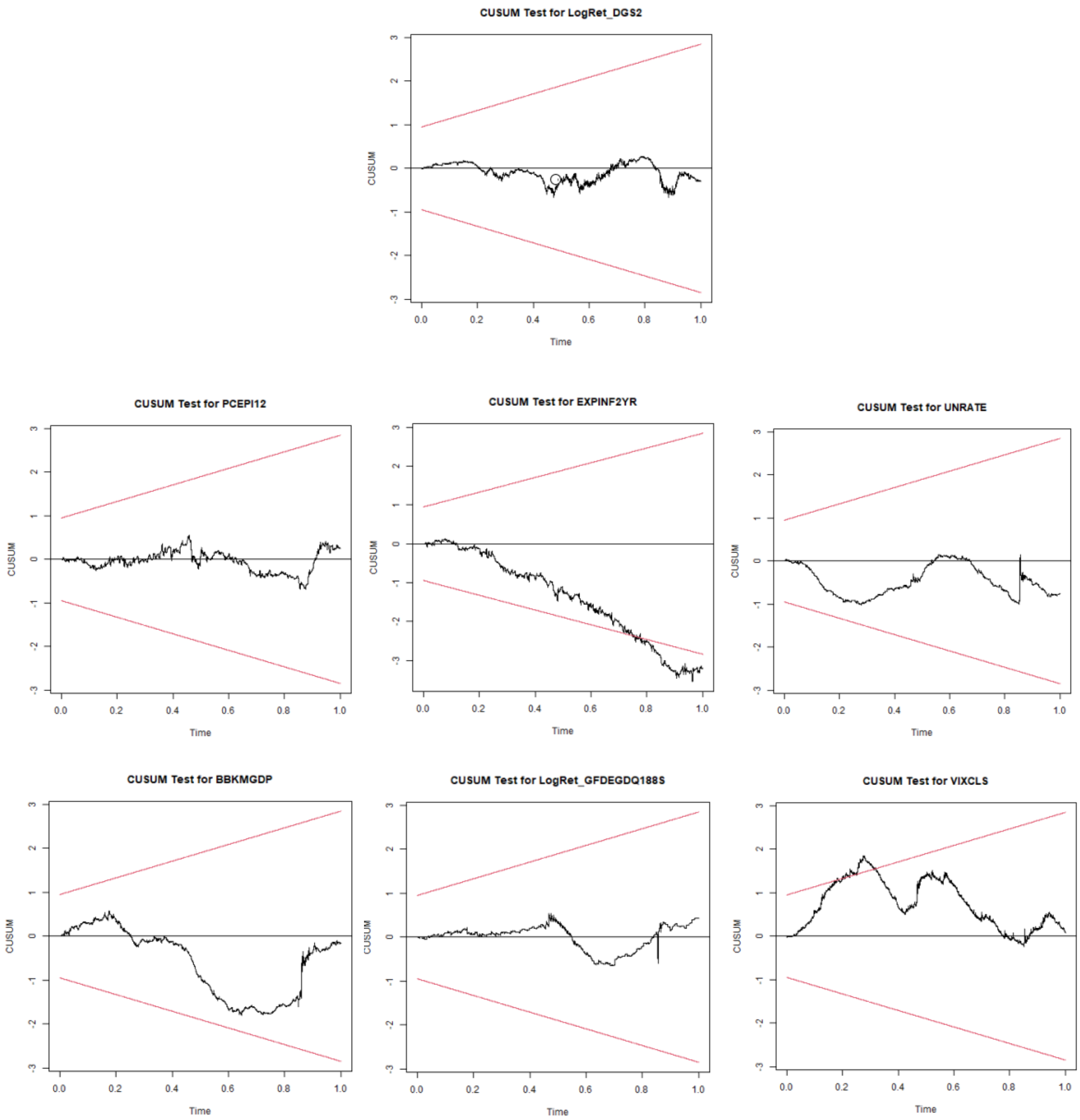


*Figure 20: CUSUM test for stability in the VAR models for the selected variables*

*Table 30: Box-Ljung (autocorrelation) and ARCH-LM (heteroscedasticity) - VAR model for LogRet_DGS2*

```
Ljung-Box Statistics:
        m       Q(m)     df    p-value
[1,]   1.0      52.7    49.0     0.33
[2,]   2.0     102.1    98.0     0.37
```

```
ARCH-LM Test Results for LogRet_DGS2

        ARCH LM-test; Null hypothesis: no ARCH effects

data:  residuals
Chi-squared = 635.68, df = 2, p-value < 2.2e-16
```

## K. Complete results of the multivariate volatility model for the 2-year Treasuries

*Table 31: Estimation results of the DCC-GARCH model for LogRet_DGS2*

```
DCC-GARCH Model Results

*---------------------------------*
*          DCC GARCH Fit          *
*---------------------------------*

Distribution         :  mvnorm
Model                :  DCC(1,1)
No. Parameters       :  51
[VAR GARCH DCC UncQ] : [0+28+2+21]
No. Series           :  7
No. Obs.             :  10774
Log-Likelihood       :  -9655529
Av.Log-Likelihood    :  -896.19

Optimal Parameters
-----------------------------------
                              Estimate  Std. Error   t value Pr(>|t|)
[LogRet_DGS2].mu              0.000082    0.000129 6.3782e-01 0.523592
[LogRet_DGS2].omega           0.000001    0.000000 1.9359e+00 0.052881
[LogRet_DGS2].alpha1          0.042574    0.001427 2.9832e+01 0.000000
[LogRet_DGS2].beta1           0.956426    0.000748 1.2790e+03 0.000000
[VIXCLS].mu                  15.273232    0.275873 5.5363e+01 0.000000
[VIXCLS].omega                0.573706    0.105485 5.4387e+00 0.000000
[VIXCLS].alpha1               0.901666    0.029251 3.0826e+01 0.000000
[VIXCLS].beta1                0.097334    0.033077 2.9426e+00 0.003254
[PCEPI12].mu                  1.971775    0.004742 4.1578e+02 0.000000
[PCEPI12].omega               0.000008    0.000019 4.3317e-01 0.664888
[PCEPI12].alpha1              0.862226    0.211102 4.0844e+00 0.000044
[PCEPI12].beta1               0.136774    0.225281 6.0713e-01 0.543768
[EXPINF2YR].mu                1.907324    0.011340 1.6819e+02 0.000000
[EXPINF2YR].omega             0.000044    0.000007 6.2241e+00 0.000000
[EXPINF2YR].alpha1            0.999000    0.040686 2.4554e+01 0.000000
[EXPINF2YR].beta1             0.000000    0.041087 1.0000e-06 0.999999
[BBKMGDP].mu                  2.845990    0.017280 1.6470e+02 0.000000
[BBKMGDP].omega               0.001514    0.000270 5.6148e+00 0.000000
[BBKMGDP].alpha1              0.917733    0.018456 4.9724e+01 0.000000
[BBKMGDP].beta1               0.081267    0.020384 3.9869e+00 0.000067
[UNRATE].mu                   4.999947    0.063778 7.8396e+01 0.000000
[UNRATE].omega                0.000000    0.000005 4.3755e-02 0.965100
[UNRATE].alpha1               0.739578    0.048708 1.5184e+01 0.000000
[UNRATE].beta1                0.259422    0.050153 5.1726e+00 0.000000
[LogRet_GFDEGDQ188S].mu       0.000069    0.000003 1.9679e+01 0.000000
[LogRet_GFDEGDQ188S].omega    0.000000    0.000000 3.1600e-04 0.999748
[LogRet_GFDEGDQ188S].alpha1   0.047279    0.005885 8.0342e+00 0.000000
[LogRet_GFDEGDQ188S].beta1    0.903535    0.003142 2.8756e+02 0.000000
[Joint]dcca1                  0.073963    0.017000 4.3507e+00 0.000014
[Joint]dccb1                  0.896573    0.043408 2.0655e+01 0.000000
```

*Table 32: Estimation results of the AG-DCC-GARCH model for LogRet_DGS2*

```
A-DCC-GARCH Model Results

*---------------------------------*
*          DCC GARCH Fit          *
*---------------------------------*

Distribution          :  mvnorm
Model                 :  aDCC(1,1)
No. Parameters        :  52
[VAR GARCH DCC UncQ] : [0+28+3+21]
No. Series            :  7
No. Obs.              :  10774
Log-Likelihood        :  -9655383
Av.Log-Likelihood     :  -896.17

Optimal Parameters
-----------------------------------
                             Estimate  Std. Error   t value Pr(>|t|)
[LogRet_DGS2].mu             0.000082    0.000121 6.7970e-01 0.496698
[LogRet_DGS2].omega          0.000001    0.000000 1.8052e+00 0.071037
[LogRet_DGS2].alpha1         0.042574    0.001684 2.5282e+01 0.000000
[LogRet_DGS2].beta1          0.956426    0.000817 1.1702e+03 0.000000
[VIXCLS].mu                 15.273232    0.282796 5.4008e+01 0.000000
[VIXCLS].omega               0.573706    0.106435 5.3902e+00 0.000000
[VIXCLS].alpha1              0.901666    0.029192 3.0888e+01 0.000000
[VIXCLS].beta1               0.097334    0.033328 2.9205e+00 0.003495
[PCEPI12].mu                 1.971775    0.004765 4.1377e+02 0.000000
[PCEPI12].omega              0.000008    0.000019 4.2634e-01 0.669861
[PCEPI12].alpha1             0.862226    0.216363 3.9851e+00 0.000067
[PCEPI12].beta1              0.136774    0.230872 5.9242e-01 0.553568
[EXPINF2YR].mu               1.907324    0.011345 1.6812e+02 0.000000
[EXPINF2YR].omega            0.000044    0.000007 6.0514e+00 0.000000
[EXPINF2YR].alpha1           0.999000    0.040090 2.4919e+01 0.000000
[EXPINF2YR].beta1            0.000000    0.040532 1.0000e-06 0.999999
[BBKMGDP].mu                 2.845990    0.017349 1.6405e+02 0.000000
[BBKMGDP].omega              0.001514    0.000294 5.1476e+00 0.000000
[BBKMGDP].alpha1             0.917733    0.017269 5.3145e+01 0.000000
[BBKMGDP].beta1              0.081267    0.019479 4.1719e+00 0.000030
[UNRATE].mu                  4.999947    0.063375 7.8894e+01 0.000000
[UNRATE].omega               0.000000    0.000005 4.3518e-02 0.965288
[UNRATE].alpha1              0.739578    0.049371 1.4980e+01 0.000000
[UNRATE].beta1               0.259422    0.051269 5.0600e+00 0.000000
[LogRet_GFDEGDQ188S].mu      0.000069    0.000003 1.9732e+01 0.000000
[LogRet_GFDEGDQ188S].omega   0.000000    0.000000 2.7200e-04 0.999783
[LogRet_GFDEGDQ188S].alpha1  0.047279    0.005889 8.0278e+00 0.000000
[LogRet_GFDEGDQ188S].beta1   0.903535    0.003140 2.8776e+02 0.000000
[Joint]dcca1                 0.163482    0.123073 1.3283e+00 0.184068
[Joint]dccb1                 0.763414    0.241553 3.1604e+00 0.001575
[Joint]dccg1                 0.000000    0.036432 7.0000e-06 0.999994
```