GAMES IN CODE FORM VERSUS GAMES IN EXTENSIVE FORM

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1.Introduction

The idea of a new game representation occurred when we analysed a game with 3 players in the normal form. It seemed that a representation that gave us a global picture of the game would be the ideal. As a consequence of this idea, we were tried a game representation that agglutinated the information of extensive form and of normal form. We imagined the codified representation and we verified that this representation has great advantages when the number of players is greater or equal than 3, when the game is an imperfect information one and mainly when it is sequential. We present the code form comparing that representation with the extensive representation.

2.Extensive

The extensive form contains all the information about a game, defining all the moves and when they are performed, what each player knows when he moves, which moves are available to him and where each move leads.

Definition *1* - A game can be described using a tree. This is called the Extensive Form of the game.

The set of players includes the agents taking part in the game. However, in many games there is a place for chance. More concretely, it is necessary to consider the chance that has an uncertainty on some relevant fact. To represent these possibilities we introduce a fictional player: the nature. In terminal nodes do not exist payoff for the nature, and every time a node is allocated to nature a probability distribution over the branches that follow needs to be specified.

Definition *2* - Information set is a collection of nodes $\{m_1,\ldots,m_k\}$, such that: The same player i is to move at each of these nodes; The same moves are available at each of these nodes.

Definition *3* - A strategy of a player i is a list of contingent plans; that is a description of which action he takes at each information set where he is to move.

After some considerations about extensive form we present the aim of this work: a new game representation – the code form. We will introduce this new representation step by step. That is we construct the model on the basis of games with different features and comparing it with the extensive form.

3.Perfect Information

In this section we analyse the dynamic games with perfect information. We have a game of perfect information if when a player has to move he knows everything that happened in the game up to that point: he knows all other players' moves up to that point.

Let there be a game with 3 players with perfect information where there are two strategies for each player: $"D"$ and $"E"$.

The roles of the game are:

- If any player chooses $"E"$ the payoff vector is $(0,0,0)$; If only one player chooses $"D"$, this player obtains a payoff 1 and each of others obtain $-\frac{1}{2}$; If only two players choose $"D"$, then each of them obtain $\frac{1}{2}$ and the other player who chooses $"E"$ obtains -1; If all players choose $"D"$ everyone obtains 0; Order of play is: first player 1, second player 2 and third player 3.

Extensive Form

In figure 1 we model the above situation as an extensive form game.

The labeled nodes, $m_1,\ldots,m_{15}$, represent the various states of the game. In this figure the node labeled 1, m_1, indicate that player 1 makes the first move. The two lines branching from this node indicate that, at this node, player 1 has two choices, $"D"$ and $"E"$. Each of these lines leads to nodes labeled 2, nodes m_2 and m_3, which indicates that after player 1's action it is player 2 turn to play. There are two lines branching from each of the nodes labeled 2. This indicates that in each of the nodes player 2 has two choices, $"D"$ and $"E"$. Each of these actions leads to nodes labeled 3 (nodes m_4, m_5, m_4, and m_7). Hence at these nodes it is player 3 turn to move. The nodes $m_8,\ldots,m_{15}$, are terminal nodes of the game. These nodes represent payoffs of the game.

To find the game equilibrium we use the backward induction method. "The concept of backward induction corresponds to the assumption, that is common knowledge, that each player will act rationally at each node were he moves, even if his rationality would imply that such a node will not be reached".

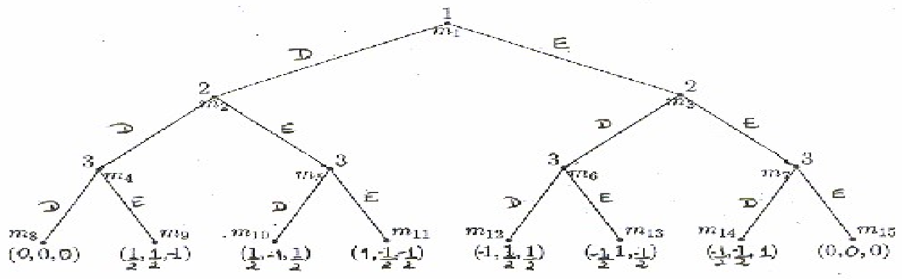

Figure *1* - Perfect information, three players – Extensive Form

In our game we have: in each of the final action nodes of this game player *3* has to take an action. Player *3* chooses an action that maximizes his payoff. Since agent *3* does not have any other node at which he must take an action we conclude that his strategy found by backward induction is $s_3^* = ("D","D","D","D")$. Assigning the payoff vector associated with this move that gives this player the highest payoff to the node at hand, we delete all the moves stamming from this node so that we have a shorter game, where our node is a terminal node. We repeat the process to player *2*. In all the final action nodes of this game agent *2* must choose an action that maximizes his payoff. Hence the strategy of agent *2* obtained through backward induction is $s_2^* = ("D","D")$. The reduced game obtained by replacing the action nodes of agent *2* with a terminal node and associating it with the payoff profile resulting from the optimal action of agent *2* leads us to another reduced game. In this final reduced game we see that agent *1* optimal action is to choose $"D"$. Hence the strategy of agent *1* obtained through backward induction is $s_1^* = ("D")$. As a result we have one strategy profile that is obtained through backward induction: (s_1^*, s_2^*, s_3^*). Hence the play induced by these strategy profiles is: first agent *1* chooses $"D"$ then agent *2* chooses $"D"$ and finally agent *3* chooses $"D"$. As a result the game solution corresponds to the payoffs vector $(0,0,0)$.

Code Form

Figure *2* shows the code form representing the above game.

Code form game idea seats in the game estimated linear reading. We build a table that contains the whole game information. How to organize the information?

Reading from left to the right, the first table column indicates the move number. If the players move simultaneously, this column is filled with the code $"S"$. The following columns mention each one of the players. Last column indicates the payoffs vector in accordance with the strategies chosen by the players. This column identifies the player number and the strategy chosen for it. The order of the players is arbitrary when the moves are simultaneous. If there is a blank field it means that information is the same than in the above field.

1	(1,“D”)			
2		(2,”D”)		
3			(3,”D”)	$(0,0,0)$
			(3,”E”)	$\left(\frac{1}{2},\frac{1}{2},-1\right)$
2		(2,”E”)		
3			(3,”D”)	$\left(\frac{1}{2},-1,\frac{1}{2}\right)$
			(3,”E”)	$\left(1,-\frac{1}{2},-\frac{1}{2}\right)$
1	(1,“E”)			
2		(2,”D”)		
3			(3,”D”)	$\left(-1,\frac{1}{2},\frac{1}{2}\right)$
			(3,”E”)	$\left(-\frac{1}{2},1,-\frac{1}{2}\right)$
2		(2,”E”)		
3			(3,”D”)	$\left(-\frac{1}{2},-\frac{1}{2},1\right)$
			(3,”E”)	$(0,0,0)$

Figure 2 - Perfect information, three players – Code Form

The table is built in accordance with the following algorithm:

Let Γ be a given game with n players. For each $i \in N = \{1,2,...,n\}$, S_i is the set of all strategies that are available to player i.

1. Let j be the starting move of the game
2. Assume player i is the player who has taken an action at j.
3. From S_i select one of the possible actions for player i. Let a_i be this action.
4. If j is a terminal move, select the player number, the action and the payoff vector associated with this move. Change the line and repeat the above steps after step 3. Otherwise, change the line and the column and repeat the above steps, from step 1, with j replaced by j'.
5. If the player who has taken an action at the move j' has no more actions to take select the player who took an action immediately before. Select his column, and repeat the above steps, from step 1, replacing j with the respective move.

6. The process is completed when the player who has taken an action at move j has no more actions to take.

To find the game equilibrium we use the best payoff method. This method has the same philosophy that backward induction method. Mechanically, it is computed as follows:

1. Let i be the player who has taken an action at the terminal move.
2. Consider any move that comes just before terminal move. Select the payoff sets associated with each move.
3. In each of those sets select the best payoff for player i.
4. Delete the column for player i so that we have a shorter game where player i''s move is a terminal move and where player i' is the player who has move just before player i.
5. Repeat the process until the first player is reached.
6. Select the strategies profile induced by the payoff vector obtained.

In our game we have:

Player 3 is the player who takes an action in the last move. He has 4 payoffs sets:

$$\left\{(0,0,0),\left(\frac{1}{2},\frac{1}{2},-1\right)\right\},\ \left\{\left(\frac{1}{2},-1,\frac{1}{2}\right),\left(1,-\frac{1}{2},-\frac{1}{2}\right)\right\},\ \left\{\left(-1,\frac{1}{2},\frac{1}{2}\right),\left(-\frac{1}{2},1,-\frac{1}{2}\right)\right\} \text{ and } \left\{\left(-\frac{1}{2},-\frac{1}{2},1\right),(0,0,0)\right\}.$$

In each of those sets select the best payoff for player 3. They are:

$$(0,0,0),\ \left(\frac{1}{2},-1,\frac{1}{2}\right),\ \left(-1,\frac{1}{2},\frac{1}{2}\right) \text{ and } \left(-\frac{1}{2},-\frac{1}{2},1\right).$$

Now consider the shorter games and repeat the process for player 2. Player 2 is the player who takes an action in the last move for this shorter game. He has 2 payoffs sets:

$$\left\{(0,0,0),\left(\frac{1}{2},-1,\frac{1}{2}\right)\right\} \text{ e } \left\{\left(-1,\frac{1}{2},\frac{1}{2}\right),\left(-\frac{1}{2},-\frac{1}{2},1\right)\right\}.$$

In each of those sets select the best payoff for player 2. They are: $(0,0,0)$ and $\left(-1,\frac{1}{2},\frac{1}{2}\right)$.

Now consider the shortest games and repeat the process for player 1. Player 1 is the player who takes an action in the last move for this shortest game. He has one set of payoffs:

$$\left\{(0,0,0),\left(-1,\frac{1}{2},\frac{1}{2}\right)\right\}.$$

In this set we select the best payoff for player 1. This payoff is $(0,0,0)$. The solution for this game results from all players choosing strategy $"D"$.

4. Imperfect Information

We have a game of imperfect information if when a player has to move he does not know all the choices that other players have made.

Let there be a game with 3 players with imperfect information where there are two strategies for each player: $"a"$ e $"b"$.

Suppose: Player 1 prefers $"a"$ to $"b"$; Player 2 prefers $"b"$ to $"a"$; Player 3 prefers $"a"$ to $"b"$. The roles of the game are:

- Player 1 chooses and communicates to the other players his decision. Then the other players choose simultaneously. The option with the highest number of votes is chosen.

Extensive Form

Figure 3 gives the above described situation extensive form representation.

The dashed line connecting nodes m_4 and m_5 indicates that player 3 does not know whether the game is at a state represented with m_4 or m_5. That is, player 3 does not know if player 2 chooses $"a"$ or $"b"$. The nodes m_4 and m_5 are not connected to any of the nodes m_4 and m_7 (at which player 3 most vote), because player 3 knows the choice of player 1 when it is his turn to take an action. Similarly, the dashed line connecting the nodes m_4 and m_7 indicates that player 3 does not know whether the game is at a state represented with node m_4 or m_7. We will say that the nodes connected with dashed lines belong to the same information set of a player. When it is a player's turn to play, the player knows the information set that contains the node representing the state of the game, but does not know which of the nodes in the information set it is (unless there is only one node in the set). Player 1 has only one information set: $\{m_1\}$. Player 2 has two information sets: $\{m_2\}$ and $\{m_3\}$. Player 3 has two information sets: $\{m_4, m_5\}$ and $\{m_6, m_7\}$. Thus when it is player 2 turn to move he knows if the game is at a state represented by m_2 or at a state represented by m_3. When it is player 3 turn to move player 3 knows if the game is at a state represent by a node in the set $\{m_4, m_5\}$ or $\{m_6, m_7\}$, but he does not know whether the state is m_4, m_5, m_4, or m_7.

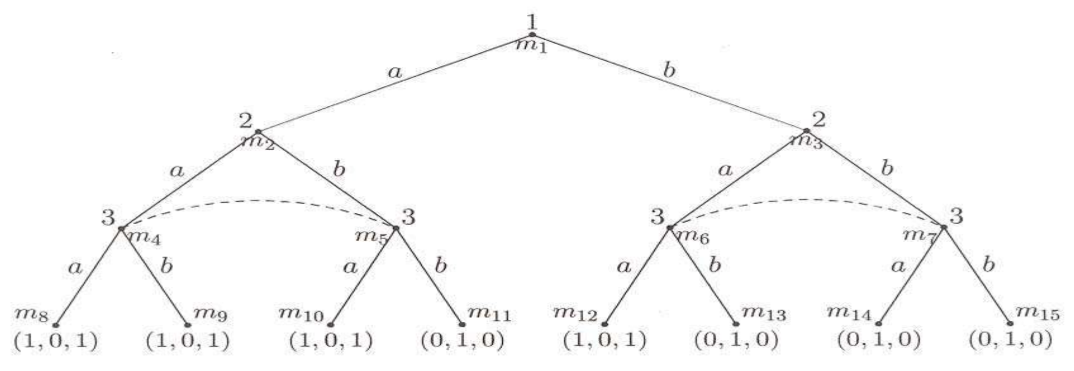

Figure 3 - Imperfect information, three players – Extensive Form

To find game equilibrium we cannot apply the backward induction method because we cannot apply it beyond perfect information games with a finite horizon. Subgame perfect equilibrium is a generalization of the backward induction equilibrium to extensive form games with imperfect information. Our game has three subgames, namely Γ_{m1}, Γ_{m2} e Γ_{m3}. Γ_{m1} is the game Γ and is called the trivial subgame of Γ. The other two subgames Γ_{m2} and Γ_{m3} (shown in Figure 4) are called nontrivial subgames.

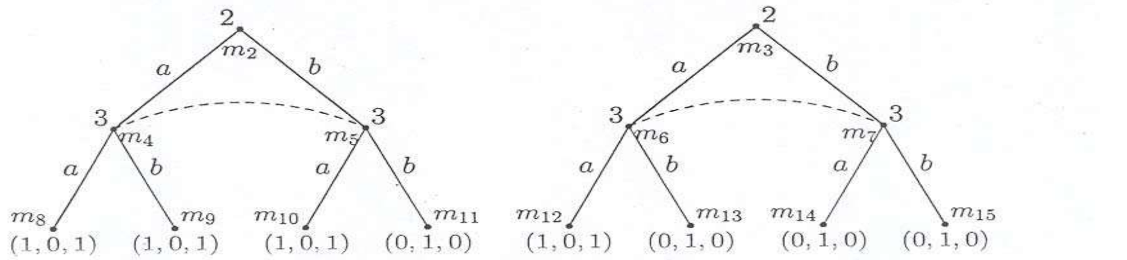

Figure 4 - Subgames – Extensive Form

Definition 4 - A strategic profile s^* in an extensive form game Γ is a subgame perfect equilibrium (SPE) if the restriction of s^* to any subgame of Γ is a subgame equilibrium.

To find the subgame perfect solutions of a game Γ we start with a subgame of Γ which does not contain any other subgames. We find the solutions of this subgame. Then we truncate the original game Γ to a game Γ' by replacing the subgame with a terminal node and associating with it the payoff profile resulting from the play of one of the solutions of the subgame. Then we repeat the same steps with Γ replaced by Γ' until all the subgames are eliminated. Analysing subgames J_{m2} e J_{m3} we verify that $(0,1,0)$ corresponds to J_{m3} subgame perfect equilibrium and $(1,0,1)$ corresponds to J_{m2} subgame perfect equilibrium. Thus the perfect equilibrium in reduced subgame corresponds to payoff vector $(1,0,1)$.

Code Form

The figure 5 represents the game in code form.

1	(1,"a")			
2 - S		(2,"a",1,3)	(3,"a",1,2)	(1,0,1)
			(3,"b",1,2)	(1,0,1)
		(2,"b",1,3)	(3,"a",1,2)	(1,0,1)
			(3,"b",1,2)	(0,1,0)
1	(1,"b")			
2 - S		(2,"a",1,3)	(3,"a",1,2)	(1,0,1)
			(3,"b",1,2)	(0,1,0)
		(2,"b",1,3)	(3,"a",1,2)	(0,1,0)
			(3,"b",1,2)	(0,1,0)

Figure 5 - Imperfect information, three players – Code Form

When we construct this representation we verify that one more code is necessary to indicate that there is imperfect information. In the column of the player who has imperfect information we add code $"I"$ and the player number to whom the imperfect information refers. This code only exists for moves with imperfect information.

The best payoff algorithm used to find the solution, referred above, most likely works for games of this type. Nevertheless, as it has not been completely tested, the SPE was used to find the payoff - $(1,0,1)$.

5. Conclusion

- Although both representations provide a global view of the gameCode form does this more clearly
- Furthermore, code form provides more information about the game than extensive form
- For example, using code form simultaneous moves can be readily observed
- The solution in code form can be computed more linearly
- Because code form solution has not been thoroughly tested, the SPE can be used to find the actual solution
- The solution of a game in the codified form is perfect in subgames
- Considering that the importance of a representation rests on its ease of interpretation, ease of solving a game and truthful information about the game, code form representation is the only one, which fulfils all these requirements.

6. Summary

The aim of this paper is to present and discuss a new representation form games with more than 2 players. This formalization centres the consequences of different strategies without suppressing the meticulousness of the extensive form.

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