INSTITUTO UNIVERSITÁRIO DE LISBOA

Modelling Daily Volatility with External Regressors

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Master in Finance

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Abstract

The main objective of this thesis is to show the importance of including log differences of trading volume and close-to-open negative returns (negative log differences between the closing price of the day before and the opening price) both lagged one time in modelling volatility for the DAX 30, S&P 500 and the Nikkei 225. In order to accomplish this, we use the ARMA (1,1)-EGARCH (1,1), -TGARCH (1,1), -GJR-GARCH (1,1) and -GARCH (1,1), the latter without external regressors. Our models use different error distributions: the student-t, the GED, the skewed student-t and the skew GED distribution. Our sample uses the returns from 02/01/1998 to 29/05/2020 divided into crisis and non-crisis periods. For the out-of-sample analyses we use the last twenty trading days to compare the models estimated with the volatility proxy: the squared returns.

The models that stand out from the others are ARMA (1,1)-EGARCH (1,1) and ARMA (1,1)-TGARCH (1,1) which seem to be the ones that best model and forecast volatility. Despite not reaching a conclusion about the best distribution, we can conclude that the skew version of the distributions performs better in-sample than out-of-sample.

The results show that the log differences of trading volume are an important variable to include in and out-of-sample. Although the close-to-open negative returns are only significant in some periods of analysis and only in ARMA (1,1)-EGARCH (1,1), when they are significant, they yield the best in-sample results.

Keywords: ARMA-GARCH, trading volume, close-to-open negative returns

JEL classification: C32, G17

Resumo

O principal objetivo desta tese é demostrar a importância de incluir as diferenças logarítmicas do volume de trocas e os retornos negativos entre fecho-e-abertura (diferenças logarítmicas entre o preço de fecho do dia anterior e o preço de abertura), ambos com um momento de desfasamento, na modelização da volatilidade para os seguintes índices DAX 30, S&P 500 e Nikkei 225. Para este estudo utilizamos o ARMA(1,1)-EGARCH (1,1), -TGARCH (1,1), -GJR-GARCH (1,1) e -GARCH (1,1) este último sem as variáveis adicionais. Escolhemos ainda as seguintes distribuições: *student-t*, GED, *student-t* assimétrica e GED assimétrica. O período de análise usa os retornos deste 02/01/1998 até 29/05/2020, divididos em tempos de crise e não crise. Para a análise da previsão dos últimos vinte dias comparamos o que o modelo estima com a *proxy* da volatilidade calculada (o quadrado dos retornos).

Os modelos que se destacam são ARMA (1,1)-EGARCH (1,1) e o ARMA(1,1) -TGARCH (1,1) que apresentam os melhores resultados para modelar e estimar a volatilidade. Analisando os resultados, concluímos que a versão assimétrica das distribuições tem um melhor desempenho dentro da amostra.

Os resultados mostram que as diferenças logarítmicas do volume de trocas é uma variável importante a incluir. Os retornos negativos entre fecho-e-abertura, são apenas significativos em alguns períodos de análise e apenas para o modelo ARMA (1,1) -EGARCH (1,1), mas quando são significativos apresentam os melhores resultados dentro da amostra.

Palavras-Chave: ARMA-GARCH, volume de trocas, retornos negativos fecho-abertura

Classificação JEL: C32, G17

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1. Introduction

Volatility is one of the most complex subjects in finance. Changes in prices can be caused by so many factors that although research into finding a way to forecast this began a long time ago, a state-of-the-art model has yet to be arrived at. What is a certainty in one study is not in another.

Volatility in stock markets can be studied for indexes, stocks, commodities, options, exchange rates, etc. However, since there are different characteristics for modelling volatility for each type of financial instrument, a common practice is to study groups of stock markets with the same designation. Our interest here, lies in studying equity indexes, more specifically, we will be modelling the volatility of three main equity indexes: the DAX 30, S&P 500, and the Nikkei 225.

In Engle (1982) we can find the beginning of the ARCH-type models (autoregressive Conditional Heteroskedasticity), with volatility being forecast by past squared errors. However, volatility can also be explained by its past, so Bollerslev (1986) introduced the well known generalized ARCH (GARCH) which takes this further step.

Markets react differently to good news than to bad news, with bad news having higher volatility increases, hence the asymmetric volatility response is also a key behaviour. Nelson (1991), Glosten, Jagannathan and Runkle (1993) and Zakoian (1994) proposed models that include this leverage effect, EGARCH, GJRGARCH and TGARCH respectively. These models are not the only ones to include this effect but they are the ones that are relevant for this dissertation.

Our aim being to model and forecast volatility, we will use the ARMA-GARCH mixture type models. This kind of mixture allows us to take into consideration, the characteristics of price returns and, at the same time, the characteristics of volatility. So, we use four models: ARMA (1,1)-GARCH (1,1), ARMA (1,1)-EGARCH (1,1), ARMA (1,1)-GJR-GARCH (1,1) and finally the ARMA (1,1)-TGARCH (1,1).

The literature is not unanimous with regard to choosing the best distribution for modelling volatility in all moments. Different studies suggest different distributions. For example, Wilhelmsson (2006) considered student-t distribution, while Gao et al. (2012) choose the GED, still more studies can be found with different distributions. So, for computing the volatility for these indexes, we chose to use the student-t, the GED and the skew versions of both.

The main goal of this dissertation is to prove that the inclusion of external regressors in the GARCH equation part (this means that the external variables only impact the conditional volatility and not the ARMA equation part) is significant and produces better results both with in-sample and out-of-sample analyses. For this purpose, the ARMA (1,1)-GARCH (1,1) will not include these external regressors. Since it is a widely used model, with a robust performance like that studied by Wang et al. (2009), it will be a good benchmark.

The external regressors chosen for this study are the log differences of daily trading volume and close-to-open negative returns (only the negative values for the log differences of the closing price of the day before and the opening price) both lagged one time.

For this analysis we consider the period between 01/01/1998 to 29/05/2020. We use all the time periods and different divisions derived from separate All Sample in sub-periods. This historical data was divided into crisis and non-crisis subsamples. The objective here was to see whether the variables have different impacts in different time periods. With regard to the out-of-sample analysis, we will use the last twenty trading days and compare the forecasted volatility given by the models estimated with the volatility proxy, the squared returns. The best models are those that are the least different from reality.

The main contributions of this thesis are diverse. Our dissertation shows that the ARMA (1,1)-EGARCH (1,1) and ARMA (1,1)-TGARCH (1,1) are the best models. While for in-sample, there is a predominance of the ARMA (1,1)-EGARCH (1,1), for out-of-sample, the predominance is for the ARMA (1,1)-TGARCH (1,1) model. As far as error distribution in this thesis is concerned, no conclusion was reached about which distribution is the best. However, by analysing the skewed version of the distributions used against the standard version in the same model (skewed GED versus GED and skewed student-t versus student-t), we conclude that, in general, no version stands out from another. However, dividing this between in and out-of-sample, the results show that the skewed version is better in-sample and the standard one is better out-of-sample, especially for the non-crisis period.

The final contribution of this thesis concerns the external regressors. In modelling daily volatility, the inclusion of the lagged log differences of trading volume is shown to be significant and helpful in explaining volatility. It appears to have a considerable positive impact on the volatility. Hence, when the trading volume increases, the volatility of the equity index increases as well. The lagged close-to-open negative returns are not always statistically significant and the only model where this variable is significant is in ARMA (1,1)-EGARCH (1,1). In general, this has a negative impact (except for one period of analyses), meaning that negative close-to-open returns increase the volatility.

This research is organized as follows. Section 2 reviews the literature about the topic of modelling volatility, such as models, distributions and external regressors. Section 3 gives an explanation of the data used. Section 4 has all the methodology applied for modelling volatility. Section 5 has the results by in and out-of-sample analyses, the presentation of the best models and their respective coefficients for external regressors. Finally, Section 6 is the conclusion of this thesis.

2. Review of literature

As stimulating it is to analyze volatility, there is another great motivation to tackle this subject which is to be able to predict it. The volatility proxy, in order to compare the results, can be calculated in more than one way. We can use implied volatility, which is derived from financial options' prices and is thus a future expectation of volatility, or we can have realized volatility, which is an estimation for daily volatility considering intraday volatility. There are, however, limitations with regard to obtaining this data (Majmudar & Benerjee, 2004). Because of the limitations in this study, we use the squared returns for volatility proxy.

The characteristics of volatility that give it its complex behaviour are precisely why it is such a challenge to model. Large changes tend to be followed by further large changes; the same happens with small changes and this is designated as volatility clustering. Another feature is that periods of high volatility will be followed by a period of normal volatility, and normal volatility will be followed by periods of high volatility (mean reversion). In addition to the two above, there is also the leverage effect, with negative and positive shocks having different impacts on volatility. Lastly, the probability of the returns getting higher values is large. (Engle & Patton, 2001) (Poon & Granger, 2003).

As previously mentioned, in order to model volatility we have to keep in mind its characteristics. Engle (1982), with the introduction of the ARCH-model (autoregressive Conditional Heteroskedasticity), assumed that conditional variance is a linear function of past squared errors with q number of lags.

The ARCH-model is the simplest model in this thesis. The advantages of this model are that it is easy to estimate and allows the impact of volatility clustering. However, it is precisely because it is a simple model that there are some limitations. One limitation being that in this model, the only thing that affects the current volatility are past error terms, which is probably not true. Another problem is that large negative impacts tend to last different lengths of time from positive shocks and the model presented by Engle (1982) does not consider this impact.

To overcome these limitations of the aforementioned model, Bollerslev (1986) explained the conditional variance by adding the past conditional variances (p) of the series to the ARCH model, producing the Generalised ARCH (GARCH).

By adding the past conditional volatilities to past squared errors, this model is more flexible and can capture both dense tail returns and volatility clustering. The problem with this model, however, is that it is still unable to differentiate between bad news and good news, so negative impacts are not distinct from positive impacts.

Nelson (1991) proposed a model to accommodate this detail in financial time series. The exponential GARCH (EGARCH) model captures both the sign and the size of past residuals. Taking the leverage effect into consideration, the bad news will increase the volatility more than positive shocks will.

The Glosten, Jagannathan and Runkle study (1993) modifies the GARCH (GJR-GARCH) model to include a dummy variable, making it possible to take into account the “leverage” effects in the financial markets. In the case of negative shocks, the dummy is one, and zero otherwise.

Taking a different approach, the TGARCH (threshold GARCH) by Zakoian (1994) uses the conditional standard deviation instead of the conditional variance. The idea comes from a study by Davidian and Carroll (1987) on estimating variance function. Here, they conclude that absolute residuals are more efficient in estimating variance than squared residuals if the distribution of the same is non-normal.

There are other GARCH models, but for a more complete list and details about types of univariate GARCH models read Teräsvirta (2009). A more recent model that is not present in the research is the Flexible Coefficient GARCH (FCGARCH) presented by Medeiros and Veiga (2009). In this last model, it is possible to include more than two limiting regimes besides the nonlinear combinations.

There is a great variety of GARCH-type models but there is no mutual consensus regarding whether one model outperforms the others; some studies conclude it is the simplest GARCH (p,q) that does, while others point to the extensions of GARCH being better.

For the simplest model, and using the data of seven emerging countries, Gokcan (2000) found that the GARCH (1,1) model outperformed the EGARCH (1,1) even when the returns have skewness distributions. The Balaban (2004) study indicates that the GARCH model outperforms other models in forecasting the US dollar – Deutsche mark exchange rate. Hansen and Lunde (2005) compared 330 ARCH-type models and concluded that the standard GARCH (1,1) is the best model for exchange rates, but for IBM stocks this model does not outperform the others. Another conclusion of this study is that the higher order in p and q rarely outperform the lowest 1, 1 combination.

To conclude with regard to the GARCH extensions, Alberg et al. (2008) showed that the best model is the EGARCH, especially with the skewed student-t for Tel Aviv Stock Exchange indexes. Liu and Hung (2010) in their investigation into Standard and Poor’s 100, state that the GJR-GARCH (1,1) obtains the most accurate volatility forecast, and that modelling the asymmetrical component is very important. Lim and Sek (2013) showed that in comparison with EGARCH, the GARCH and TGARCH models perform the best in the pre-crisis period, with the GARCH model working well during the crisis and the TGARCH model working well in the post-crisis period for the stock market in Malaysia.

In addition to all the models presented, a mixture of ARMA (Box & Jenkins, 1976), and GARCH models was adopted to model volatility.

Tang et all. (2003) shows that using the ARMA-GARCH model to predict the daily stock prices of the Cheung Kong Holding and the HSBC Holding produces very good results. Wang et al. (2009) confirmed the explanation power of the in and out-of-sample results of the ARMA-GARCH model for the Dow Jones Industrial Average and S&P 500 indexes. Thorlie et al. (2014) confirmed the performance of the ARMA-GJR-GARCH in predicting the SLL/USD exchange rate.

However, the difficulties involved in modelling and predicting volatility lie not only in finding a model that outperforms the others, but also in choosing a distribution. The use of a normal distribution fails to capture the main stylised characteristics of financial time series such as the presence of excess kurtosis and skewness, but there is no distribution, asymmetric or not, that constantly produces better results than the others. Different studies suggest different distributions.

Verhoeven and McAleer (2004) conclude about the superior results given by the asymmetric distributions in GARCH models. The work of Wilhelmsson (2006) shows that the GARCH model with the student-t distribution is the best for S&P 500 index future returns. Curto et al. (2006), using an AR-GARCH model for the US, German and Portuguese main stock market indexes point to the performance of the stable Pareto distribution and the student-t distribution. Gao et al. (2012) point for GED distribution in a GARCH (1,1) for the Shanghai composite index and the Shenzhen Stock Exchange Component Index. Kosapattarapim et al. (2012), applying a GARCH (p,q) to three emerging South East Asian stock markets, suggest that models with non-normal error distributions tend to provide better out-of-sample results. In a more recent study, Kumar and Basavaraj (2016) demonstrate that the symmetric distribution performs better than asymmetric ones, especially the GED for the S&P 500.

After models and distributions, it is now time to present the topic of external regressors to model volatility. From the models presented, the volatility can be explained by its past volatility, past innovations and leverage effects, but there are other variables than can improve the prediction of volatility. Trading volume and close-to-open negative returns are two of the possible choices to explain volatility.

There is a specific consensus that trading volume is connected with volatility, but agreement on the topic ends when the discussion turns to proving whether or not lagged trading volume affects volatility. There are two distinct theories about this relationship, based on the flow of information coming from the market.

The Mixture of Distribution Hypothesis (MDH) presented in Clark (1973) and Harris (1986), states their perspective that since the news about new prices is received by all investors simultaneously, a new equilibrium is attained immediately and there is no lagged trading volume that helps to predict volatility. On the other hand, Copeland (1976) and Jennings et al. (1981) favour the Sequential Information Arrival Hypothesis (SIAH) where the information reaches traders sequentially and they react in different periods of time, leading to an imbalance. Only when all the traders have reacted to the same information can equilibrium be achieved. According to this theory, therefore, the lagged trading volume can help predict volatility.

In favour of the MDH theory is Brooks (1998), whose results from using GARCH, EGARCH and GJR-GARCH models, suggest the existence of a bidirectional causality between trading volume and volatility. However, they conclude that the inclusion of the lagged trading volume to forecast volatility does not improve the results. Choi et al. (2012) conclude that for the Korean stock market, there is a positive relationship between trading volume and volatility in EGARCH (1,1) and GJR-GARCH (1,1) but the inclusion of the lagged trading volume is not statistically significant enough to explain volatility.

For the SIAH theory, Darrat et al. (2003) use an EGARCH model to analyse all stocks in the Dow Jones Industrial Average (DJIA) and conclude that most of them support the SIAH theory, so lagged trading volume has a causal relationship with volatility. Chiang et al. (2010) reach the same conclusion. Using intraday data from the National Association of Securities Dealers Automated Quotations **(**NASDAQ), they found strong bidirectional nonlinear Granger causality between volatility and lagged trading volume. The results show that the inclusion of lagged values of trading volume improve the prediction of volatility in EGARCH (1,1) and GJR-GARCH (1,1) models. Kambouroudis and McMillan (2016) using six GARCH-type models and data from US, UK and France stock market indexes conclude that the inclusion of the lagged trading volume and VIX (volatility index) contribute towards forecasting volatility despite the low value coefficient.

Not only will trading volume be included but also another variable, namely the close-to-open negative returns, will be added in order to model conditional volatility. Although the literature for this topic is not as complete as the literature of trading volume, we decided to include this variable in our contribution to the literature. To understand the importance of using close-to-open information before talking about their use in models, there are studies like that of Tsiakas (2008) that confirm the substantially predictive power of the inclusion of overnight information, and that separating the negative news from the good improves the performance of stochastic volatility for European and US stock indexes. And Ahoniemi and Lanne (2013) determined that a realised volatility estimator which includes overnight information is more precise in-sample for S&P 500 index, but for the individual stocks the best realized volatility estimator is the one without this additional information.

So, although there are some studies that point to the importance of close-to-open information in modelling volatility, the literature about the application of close-to-open negative returns in GARCH-type models to explain volatility is not extensive. Here, we present some of the studies that include close-to-open returns as external regressors. Gallo and Pacini (1998), using GARCH and EGARCH models, found that the inclusion of close-to-open returns improves the predictability of the conditional volatility for some stock indexes except for the S&P 500. Martens (2002), using a GARCH model, concluded that modelling overnight returns is important in forecasting one-day-ahead volatility, but the effect disappears if the horizon is one week or one month. Chen et.al (2012), focusing on adding more explanatory variables to a GARCH model to increase the predictive power for volatility, concluded that the inclusion of pre-open coefficients is important for in and out-of-sample for the thirty stocks mostly traded on the NASDAQ. Additionally, they state that the inclusion of the overnight squared returns improves the forecast of the conditional volatility.

The use of these external regressors is also important in other models. Here, there is a reference for one study that includes the same external regressors used in this thesis. Wang et al. (2015), using a HAR-RV model (Heterogeneous Autoregressive model of Realized Volatility), noted the importance of negative overnight returns and negative lunch returns in addition to trading volume to predict the volatility of Chinese stock markets, the Shanghai Stock Exchange Composite Index (SHCI) and the Shenzhen Composite Index (SZCI).

3. Data

To analyse the behaviour of volatility in returns, this thesis uses the Close and Open prices adjusted to dividends and the trading volume of the DAX 30, S&P 500 and Nikkei 225. The period of this analysis is between 02/01/1998 and 29/05/2020. All the data in this study were collected from Bloomberg. S&P 500 have 5637, the DAX 30 have 5683 and the Nikkei 225 have 5494 observations.

To calculate the returns in this thesis, we use the Close price of the indexes, the volatility proxy is computed by squaring the returns represented, respectively:

$$r = log\left(\frac{S_t}{S_{t-1}}\right) \tag{1}$$

$$v = r^2 \tag{2}$$

The external regressors used for improving the explanatory power to model and forecast volatility are the close-to-open negative returns (vxreg1) and log differences of trading volume (vxreg2) both lagged one time as represented below:

$$vxreg1 = \min\left(0; log\left(\frac{O_t}{S_{t-1}}\right)\right) \tag{3}$$

$$vxreg2 = log\left(\frac{V_t}{V_{t-1}}\right) \tag{4}$$

With S_t being the Close Price, O_t the Open price and V_t the trading volume.

In this study we divided the whole period into subsamples to group them by crisis and non-crisis periods. The 1st period is between 1998 and 2002 inclusive, in order to catch the dot.com bubble. The 2nd period is from 2003 and 2006 and is a period without a crisis. The 3rd is from 2007 and 2010 to include the financial crisis. The 4th period is from 2011 and 2019 and is a period without a crisis. Finally, the 5th period is for 2020 to include the impact of Covid-19.

Table 3.1 Summary of statistics and normality test for DAX 30 returns

	DAX30					
Period	All sample	1st Period	2nd Period	3rd Period	4th Period	5th Period
Date	1998-2020	1998-2002	2003-2006	2007-2010	2011-2019	2020
Obsrevations	5682	1262	1023	1016	2278	103
Mean	0.000077	-0.000144	0.000358	0.000020	0.000124	-0.000495
Skewness	-0.192630	-0.142548	-0.016519	0.225986	-0.313740	-0.809929
Kurtosis	8.352879	4.540946	6.876736	9.776261	5.762895	8.799227
Jarque-Bera (p-value)	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

Table 3.2 Summary of statistics and normality test for S&P 500 returns

	S&P500					
Period	All sample	1st Period	2nd Period	3rd Period	4th Period	5th Period
Date	1998-2020	1998-2002	2003-2006	2007-2010	2011-2019	2020
Obsrevations	5636	1255	1007	1008	2264	102
Mean	0.000087	-0.000036	0.000206	-0.000052	0.000181	-0.000274
Skewness	-0.363724	0.020120	0.076529	-0.200174	-0.554645	-0.517201
Kurtosis	13.462470	4.645200	4.531975	9.859370	8.062647	6.559177
Jarque-Bera (p-value)	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

Table 3.3 Summary of statistics and normality test for Nikkei 225 returns

	Nikkei225					
Period	All sample	1st Period	2nd Period	3rd Period	4th Period	5th Period
Date	1998-2020	1998-2002	2003-2006	2007-2010	2011-2019	2020
Obsrevations	5493	1231	984	978	2204	97
Mean	0.000030	-0.000196	0.000308	-0.000231	0.000165	-0.000350
Skewness	-0.321858	0.117332	-0.394144	-0.368394	-0.613621	0.299487
Kurtosis	8.884451	4.444408	4.072731	10.074100	9.178797	4.948146
Jarque-Bera (p-value)	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

After analysing the tables, it is clear that the mean of the log returns for the total sample is positive, but the same is not true in different periods. In crisis periods, the mean is negative with the exception of the DAX in the 3rd period, and positive in non-crisis periods.

The skewness is negative in almost all periods of analyses for the different index returns but is positive in the 3rd period for the DAX, in the 1st and 2nd periods for S&P 500 and the 1st and 5th periods for the Nikkei 225. The negative (positive) value means that the left (right) tail of the distribution is longer than the right (left) one, so the distribution is asymmetric. It is important to note that the values of the skewness are greater in absolute value for the negative coefficient than it is for the positive coefficient. It was expected that in crisis periods the skewness would be negative, but that was not the case in the DAX 30 for the 3rd period, for S&P 500 in the 1st period and the 1st and 5th periods of the Nikkei 225. Looking at the Kurtosis, we can confirm the leptokurtic characteristic of the returns, this means that the distribution for all the three indexes have fatter tails and a higher peak around the mean when compared against a normal distribution. The highest value is in the 3rd period, the period of the financial crisis.

The characteristics of the returns in general (asymmetric and leptokurtic) point to the non-normality of the distributions; to test this hypothesis, we use the Jarque and Bera (1987) test:

$$JB = \frac{n}{6}\left(S^2 + \frac{1}{4}(K-3)^2\right) \tag{5}$$

Where n is the number of observations, S is the skewness and K is the kurtosis. The p-value lower than 0.05 indicates strong statistical evidence to reject the null hypothesis and so the conclusion about the returns for all periods and indexes is that they are not normal distributed.

4. Methodology

This chapter is dedicated to explaining all the assumptions and decisions made for modelling the daily volatility of the returns of the three indexes (DAX 30, S&P 500, and Nikkei 225). To do this, we used the R program with the rugarch package.

The models chosen were: ARMA (1,1)-GARCH (1,1), ARMA (1,1)-EGARCH (1,1), ARMA (1,1)-GJRGARCH (1,1), ARMA (1,1)-TGARCH (1,1). The (1,1) combination was chosen because in all these studies, Hansen and Lunde (2005), Wang et al. (2009) and Liu and Hung (2010) are demonstrated the power of explanation and prediction of the lowest combination of p and q.

Firstly, we present the model that gave rise to all the others in the Engle (1982) study. So, let ε_t be the error term that is split into a stochastic part and a time dependent standard deviation.

$$\varepsilon_t = \sigma_t z_t \tag{6}$$

Where z_t is independent and identically distributed (i.i.d.), and the conditional variance given by the ARCH model is:

$$\sigma^2_{t|t-1} = a_0 + \sum_{i=1}^{q} a_i \, \varepsilon^2_{t-i} \tag{7}$$

In this model $a_0 > 0$ and $a_i \geq 0$, $i \in [1, q]$ in order for the conditional mean and variance to be positive. $\sum_{i=1}^{q} a_i < 1$ it is necessary to ensure this process is covariance stationary.

Then Bollerslev (1986) modelled the conditional variance by adding the past conditional variances (p) of the series to the ARCH model producing the Generalised ARCH (GARCH).

$$\sigma^2_{t|t-1} = a_0 + \sum_{i=1}^{q} a_i \, \varepsilon^2_{t-i} + \sum_{i=1}^{p} \beta_i \sigma^2_{t-i} \tag{8}$$

Being $a_0 > 0$; $a_i \geq 0$ with $i \in [1, q]$; $\beta_i > 0$ with $i \in [1, p]$ to guarantee a positive conditional variance and $\sum_{i=1}^{q} a_i + \sum_{i=1}^{p} \beta_i < 1$to ensure a stationary covariance process. By adding the past volatilities, this model is more flexible and captures both dense tail returns and volatility clustering.

Nelson (1991) proposed a model to finally accommodate leverage effects. The exponential GARCH (EGARCH) model captures both the sign and the size of past residuals.

$$\ln\left(\sigma_{t|t-1}^2\right) = a_0 + \sum_{i=1}^{q} a_i \frac{|\varepsilon_{t-i}|}{\sigma_{t-i}} + \sum_{i=1}^{q} a_i \gamma_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}} + \sum_{i=1}^{p} \beta_i \ln\left(\sigma_{t-i}^2\right) \tag{9}$$

With $\gamma_i < 0$ the leverage effect shows that bad news, more than positive shocks, will increase volatility. Since it is a log-conditional variance there is no restriction in the other parameters for a positive conditional variance.

Glosten, Jagannathan and Runkle (1993) also modified the GARCH model to include a dummy, thus making it possible to take into account the leverage effect, this being the conditional variance given by:

$$\sigma_{t|t-1}^2 = a_0 + \sum_{i=1}^{q} \left(a_i \varepsilon_{t-i}^2 + \gamma_i \mathrm{I}_{t-i} \varepsilon_{t-i}^2\right) + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \tag{10}$$

The indicator function (I_{t-i}) takes the value of one in the case of $\varepsilon_t < 0$ and 0 otherwise. If the gamma is positive and statistically significant, there is a negative asymmetric volatility response.

The other model that is used in this thesis is the TGARCH (threshold GARCH) by Zakoian (1994). This model is different from the others, since it models the conditional standard deviation instead of the conditional variance:

$$\sigma_{t|t-1} = a_0 + \sum_{i=1}^{q} \left(\alpha_i^+ \varepsilon_{t-i}^+ - \alpha_i^- \varepsilon_{t-i}^-\right) + \sum_{j=1}^{p} \beta_j \sigma_{t-j} \tag{11}$$

Where $\varepsilon_t^+ = \max(\varepsilon_t, 0)$; $\varepsilon_t^- = \min(\varepsilon_t, 0)$. The constraints in the model are $a_0 > 0$; $\alpha_i^+ \geq 0$; $\alpha_i^- \geq 0$; $\beta_j \geq 0$.

For all the models represented above, this thesis uses a combination of the ARMA and GARCH models allowing the capture of more properties of the time series. The ARMA(p`, q`)-GARCH(p,q) representation is:

$$Y_t = \mu + \sum_{i=1}^{p`} \phi_i Y_{t-i} + \sum_{j=1}^{q`} \theta_i \varepsilon_{t-j} + \varepsilon_t \tag{12}$$

$$\sigma_{t|t-1}^2 = a_0 + \sum_{i=1}^{q} a_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \tag{13}$$

Where Y_t is the dependent variable which, in this case, are the index returns of the stock prices. μ is the mean of the time series, ϕ the autoregressive coefficient and θ_i is the moving average coefficient. The conditional volatility (standard deviation in the case of TGARCH) is modelled in accordance with the combination in analysis, this being the 2nd part of the model representation equal to equations (8), (9), (10) and (11) for ARMA-GARCH, ARMA-EGARCH, ARMA-GJR-GARCH and ARMA-TGARCH respectively.

The models used in this thesis have additional regressors in order to pursue the objective of the same, to include external regressors to help predict and model the volatility of some financial assets. The use of this package allows this implementation to be done. The approach simply involves adding $\sum_{j=1}^{m} \xi_j \nu_{jt}$ to the volatility of the models. The ARMA part stays the same, for example in the standard GARCH this will be,

$$\sigma_{t|t-1}^2 = a_0 + \sum_{j=1}^{m} \xi_j \nu_{jt} + \sum_{i=1}^{q} a_i\, \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \tag{14}$$

Where m is the possible number of external regressors ν_j.

It is important to remember that the external regressors used are the logarithmic differences of trading volume lagged one time, and the close-to-open negative returns lagged one time. By using lag versions of the data, we can forecast without needing to depend on data from the day we want to forecast. The choice to use only the negative returns and not the positive ones is because bad news has more impact on volatility than good news does. The variables are in line with Wang et al. (2015), and despite using a different approach, the results for the Chinese stock markets are very interesting.

To compare improvements brought about by the inclusion of these external regressors, the ARMA-GARCH (1,1) does not include any of the two external regressors. In some studies, like that of Wang et al. (2009), this model proves to have good results in-sample and out-of-sample, so it is a good starting point for making comparisons against the other different models with the extra variables.

As the review of the literature suggests, there is no distribution that consistently outperforms the others, so we chose to have four different distributions (student-t, skew student-t, ged, skew ged), described in more detail below. These error distributions of returns can include some of the characteristics present in financial markets. The normal distribution was not used in line with the results explained in the data section.

The density function of a random variable with student-t distribution is:

$$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\beta\nu\pi}\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{(x-\alpha)^2}{\beta\nu}\right)^{-\left(\frac{\nu+1}{2}\right)} \tag{15}$$

Where α is the location, β the scale and ν the shape parameters. Γ is the Gamma function (factorial function extended to complex numbers). Since this distribution is symmetric and unimodal, the location parameter is also the mode, mean and median. This distribution has zero skewness and the excess kurtosis is $\frac{6}{(\nu-4)}$ for $\nu > 4$ and the variance is:

$$Var(x) = \frac{\beta\nu}{(\nu-2)} \tag{16}$$

In Novales and Jorcano (2019), there is the representation of the density functionof a random variable with skew student-t distribution (SSTD):

$$f(x|\xi,\nu) = \frac{2}{\xi+\frac{1}{\xi}}s\left\{g[(sx+m)|\nu]I_{(-\infty,0)}\left(x+\frac{m}{s}\right)+g\left[\frac{(sx+m)}{\xi}|\nu\right]I_{(0,\infty)}(x+\frac{m}{s})\right\} \tag{17}$$

With $g(\cdot\,|\nu)$ being the symmetric student-t density, ξ is the skewness parameter, ν is the degrees of freedom.$I_t = \begin{cases} 1 if x_t \geq -\frac{m}{s} \\ -1 if x_t < -\frac{m}{s} \end{cases}$ and $m and s^2$ are, respectively the mean and the variance of the skewed student-t distribution that compute correspondingly:

$$Mean(x) = M_1(\xi - \xi^{-1}) \tag{18}$$

$$Var(x) = (M_2 - M_1^2)(\xi^2 - \xi^{-2}) + 2M_1^2 - M_2 \tag{19}$$

Where $M_r = 2\int_0^\infty s^r\, g(s)ds$ is the absolute moment generating function.

The density function of a random variable with GED distribution is:

$$f(x) = \frac{\kappa e^{-0.5\left|\frac{x-\alpha}{\beta}\right|^{\kappa}}}{2^{1+\kappa^{-1}}\beta\Gamma(\kappa^{-1})} \tag{20}$$

The α and β represents the same as before, and has the same characteristics as a symmetric and unimodal distribution. κ is now the shape in ged distribution. The variance and kurtosis are:

$$Var(x) = \beta^2 2^{\frac{2}{\kappa}} \frac{\Gamma(3\kappa^{-1})}{\Gamma(\kappa^{-1})} \tag{21}$$

$$Ku(x) = \frac{\Gamma(5\kappa^{-1})\Gamma(\kappa^{-1})}{\Gamma(3\kappa^{-1})\Gamma(3\kappa^{-1})} \tag{22}$$

The density function of a random variable with skewed GED (SGED) distribution is also present in Novales and Jorcano (2019):

$$\begin{aligned} f(x|\xi, \nu) = \frac{2}{\xi + \frac{1}{\xi}} s \left\{ g[\xi(sx + m)|K] I_{(-\infty,0)} \left(x + \frac{m}{s}\right) \right. \\ \left. + g\left[\frac{(sx + m)}{\xi} |K\right] I_{(0,\infty)}(x + \frac{m}{s}) \right\} \end{aligned} \tag{23}$$

With $g(\cdot\,|K)$ is the symmetric GED distribution, ξ is the skewness parameter, K is the shape parameter and I_t follows the same rules as in the skew student-t. the parameter of m and s^2 is also estimated in the same way as above.

For the forecast analysis, a length of twenty trading days for all models is predicted and compared with the volatility proxy (the squared returns). The forecast uses what the model predicts to be the daily volatility based on the information observed in the sample.

The norm in forecasting is to model the behaviour of the returns and then predict it using the observation in All Sample. Here, we take a different approach; we use the behaviour of the last one hundred observations to predict volatility of the next twenty trading days. There are cases where the window size is not big enough and does not converge to fit the data. In this case, we increased the window size by fifty observations up to the limit of three hundred. This limit represents around one year and two months, and serves the purpose of using the most recent past.

The forecast process works from a moving window perspective, where each time we forecast one day the whole process is re-estimated and takes into account the forecasted values to forecast the next one. For example, if we have 1, 2, 3 … 99, 100 values and want to forecast the 101st value, we use all the 1, 2, 3 … 99, 100 for the forecast, and for the 102nd value we use 2, 3, 4 … 100, 101, and so on.

To analyse the results, the evaluation measures chosen are the AIC (Akaike Information Criterion), BIC (Bayesian Information Criterion) for in-sample, and MSE (mean squared error) for in-sample and out-of-sample. This last measure is then multiplied by one million for a better understanding. In all these measures, the lower the value is, the better the model. In the case of the MSE, the closer to reality are the predictions.

$$BIC = \kappa \ln(n) - 2\ln(\hat{L}) \tag{24}$$

$$AIC = 2\kappa - 2\ln(\hat{L}) \tag{25}$$

$$MSE = \left(\frac{1}{n}\sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2\right) * 1000000 \tag{26}$$

Where κ is the number of parameters estimated, $\hat{L}$ represents the maximised value of the likelihood function of the estimated model, n is the sample size and $(Y_i - \hat{Y}_i)$ is the difference between the estimated and the observed.

5. Results

5.1 In-Sample

This chapter focuses on presenting the results of the in-sample analysis attained by modelling daily volatility of the returns for the DAX 30, S&P 500, and the Nikkei 225 with the ARMA-GARCH type models. We will present the best models for in-sample analysis by using the three measures explained in the methodology: AIC, BIC and MSE.

Table 5.1.1 Best in-sample models with the respective error distribution

	All sample	1st Period	2nd Period
DAX 30	ARMA(1,1)-TGARCH(1,1)_SSTD	ARMA(1,1)-TGARCH(1,1)_SSTD	ARMA(1,1)-EGARCH(1,1)_SGED
S&P 500	ARMA(1,1)-EGARCH(1,1)_SGED	ARMA(1,1)-EGARCH(1,1)_STD	ARMA(1,1)-EGARCH(1,1)_SGED
Nikkei 225	ARMA(1,1)-EGARCH(1,1)_SSTD	ARMA(1,1)-EGARCH(1,1)_STD	ARMA(1,1)-EGARCH(1,1)_GED

	3rd Period	4th Period	5th Period
DAX 30	ARMA(1,1)-EGARCH(1,1)_STD	ARMA(1,1)-EGARCH(1,1)_SGED	ARMA(1,1)-TGARCH(1,1)_GED
S&P 500	ARMA(1,1)-EGARCH(1,1)_SGED	ARMA(1,1)-EGARCH(1,1)_SGED	ARMA(1,1)-TGARCH(1,1)_SGED
Nikkei 225	ARMA(1,1)-EGARCH(1,1)_SSTD	ARMA(1,1)-EGARCH(1,1)_SSTD	ARMA(1,1)-TGARCH(1,1)_GED

As we can see by looking at table 5.1.1, the best in-sample models are the ARMA (1,1)-EGARCH (1,1) and ARMA (1,1)-TGARCH (1,1) with different error distribution. These models outperform the ARMA (1,1)-GJR-GARCH (1,1) and the ARMA (1,1)-GARCH (1,1) without external regressors, this last being the worst model in almost all time periods and indexes.

However, there is a clear dominance of the ARMA (1,1)-EGARCH (1,1) model, which is similar to the conclusion reached by Alberg et al. (2008). In the 2nd, 3rd and 4th periods, this model is chosen in all indexes, in other words for both non-crisis periods, 2nd and 4th, this model produces better results than the rest. But in the last period the ARMA (1,1)-TGARCH (1,1) is the preferable model.

The table also shows the dominance of the skew error distributions version against the standard one. None outperforms the others but there are more models using the skewed GED than the skewed student-t distribution. It is also important to mention that in crisis periods (1st, 3rd and 4th) there is no distribution that stands out from the others, yet in non-crisis periods (2nd and 4th) there is evidence of the power of the skewed GED error distribution.

It is a fact that in these tables, there is no period where the model and the distribution is the same for all the indexes. All periods, except for All Sample and the 3rd, have two indexes using the same model and distribution, for example in the 1st period the S&P 500 and the Nikkei 225 uses the same ARMA (1,1)-EGARCH (1,1) with the student-t distribution to attain the best results.

To conclude, in the discussion of the models and distributions in-sample, using all the data available in annexes A, B and C, we compared the results attained by the skewed version against the standard distribution and concluded that with in-sample, the skewed distribution for both student-t and GED produce better results around 67% of the time. The approach here was to directly compare student-t and ged against their skewed version and see which one produces the lower value measures (AIC, BIC and MSE) with the same model, just for the in-sample universe. The superior results achieved by the skewed version of the distributions is also demonstrated in Verhoeven and McAleer (2004).

As can be seen, all the models above have external regressors; we will now present the coefficient value for the lagged close-to-open returns with regard to the model chosen above.

Table 5.1.2 Coefficients for lagged close-to-open negative returns of the best models in-sample

	All sample	1st Period	2nd Period	3rd Period	4th Period	5th Period
DAX 30	0.000003	0.000000	22.787662	-19.97323*	-13.41794*	0.000000
S&P 500	-6.164847	0.131647	-99.99773	-74.1382*	8.412821	0.000000
Nikkei 225	0.339163	-9.432042*	14.747098	2.085984	-15.10328*	0.000000

Note: * statistically significant at 5% level

Here is an example to explain the interpretation of this table: the in All Sample for the DAX 30, the coefficient of lagged close-to-open negative returns (vxreg1) in the ARMA (1,1)-TGARCH (1-1) with the skewed student-t is 0.000003, and is not statistically significant at 5% level.

The results, in accordance with the best models, show that in most cases the lagged close-to-open negative returns are not statistically significant in explain the conditional volatility. The table shows that for crisis periods, it is only significant in the 1st period in the Nikkei 225, in the 3rd period it is important for the DAX 30 and S&P 500, and in the last period it is not significant for any of the indexes. In most of the recent non-crisis period (4th), this regressor can help to model volatility for the DAX 30 and the Nikkei 225.

This indicates that in the periods where this variable is statistically significant (for more details see annexes A, B and C) and has a negative coefficient, it produces the best results for the ARMA (1,1)-EGARCH (1,1) compared to the others. It is important to note that this variable is only significant for the ARMA (1,1)-EGARCH (1,1) model.

The negative value of this coefficient means that the lagged close-to-open negative returns increase the conditional volatility of the indexes returns. The positive impact implies that the conditional volatility would be reduced by having close-to-open negative returns. A similar interpretation can be found in Wang et al. (2015), despite the fact that an ARMA-GARCH model was not used, the coefficient for the close-to-open negative returns is also negative.

The positive coefficient it is not in line with the economic theory since bad news produces a higher shock, which means more volatility and not less. But in this table, we can see that the models with the best results never have a positive coefficient for this variable that is statistically significant at a 5% significance level.

After presenting and having discussed the inclusion of the close-to-open negative returns in the model, it is now time to focus our attention on the log differences of the trading volume.

Table 5.1.3 Coefficients for lagged log differences of trading volume of the best models in-sample

	All sample	1st Period	2nd Period	3rd Period	4th Period	5th Period
DAX 30	0.00202*	0.002835*	1.454526*	2.028288*	2.067217*	0.010102*
S&P 500	2.085951*	1.77002*	2.051029*	2.12508*	2.166033*	0.016717*
Nikkei 225	1.968228*	1.335728*	1.114746*	1.70972*	3.041037*	0.009509*

Note: * statistically significant at 5% level

The interpretation of this table is the same as that of the table above. The coefficients represented here are the ones estimated in the best models for each period and index.

For this regressor, the conclusions are different from the external regressor outlined before. For all indexes and periods, the lagged log difference of trading volume is statistically significant at a 5% level. So, the conclusion points in favor of Sequential Information Arrival Hypothesis (SIAH) that lagged differences of trading volume are important to model conditional volatility similar to Kambouroudis and McMillan (2015).

The positive coefficient estimated for this regressor means that the increase in trading volume in one day will increase the conditional volatility for that index the next day, and if the trading volume decreases this will decrease its volatility. With this in mind, supposing that a stock is highly traded and has almost the same trading volume every day, the variation is low between days, so there is no extra value to add to conditional volatility for this stock. But if there is an increase in the trading volume for this stock, this would mean an increase in the conditional volatility of the same.

There is an interesting fact that is not visible here, but that can be seen in more detail in the Annexes (A, B, C). This fact is that the estimation for the impact of this variable in explaining volatility in the ARMA (1,1)-EGARCH (1,1) increases as time passes, making it more important over the years. This can reflect the increase in the high frequency trading (trading that uses algorithms to trade high volumes very quickly).

5.2 Out-of-sample

Now we will present the models that produce the best results for the twenty-day ahead forecast and compare it to the proxy for the volatility chosen, the squared returns. It is also important to remember that this comparison is only based on the MSE value being the lowest value the preferred one.

Table 5.2.1 Best out-of-sample models with the respective error distribution

	All sample	1st Period	2nd Period	3rd Period	4th Period	5th Period
DAX 30	ARMA(1,1)-TGARCH(1,1)_GED	ARMA(1,1)-GARCH(1,1)_SSTD	ARMA(1,1)-EGARCH(1,1)_GED	ARMA(1,1)-TGARCH(1,1)_GED	ARMA(1,1)-TGARCH(1,1)_SGED	ARMA(1,1)-TGARCH(1,1)_GED
S&P 500	ARMA(1,1)-GJRGARCH(1,1)_SSTD[4]	ARMA(1,1)-TGARCH(1,1)_SGED	ARMA(1,1)-EGARCH(1,1)_SSTD[2]	ARMA(1,1)-EGARCH(1,1)_GED	ARMA(1,1)-TGARCH(1,1)_SGED	ARMA(1,1)-TGARCH(1,1)_SSTD
Nikkei 225	ARMA(1,1)-TGARCH(1,1)_SSTD[1]	ARMA(1,1)-TGARCH(1,1)_STD	ARMA(1,1)-EGARCH(1,1)_GED	ARMA(1,1)-GJRGARCH(1,1)_GED	ARMA(1,1)-EGARCH(1,1)_STD	ARMA(1,1)-EGARCH(1,1)_GED

Note: [1, 2, 4] represent 150, 200 and 300 observations used in window size

For the out-of-sample, we do not see just ARMA (1,1)-EGARCH (1,1) and ARMA (1,1)-TGARCH (1,1) in the table. The other two models, (GJR-GARCH and GARCH are present without external regressors). ARMA (1,1)-TGARCH (1,1) is the one that is preferred more often, contrary to the predominance of the ARMA (1,1)-EGARCH (1,1) in-sample. This also indicates that for forecast modelling, the conditional standard deviation produces better results just as in Lim and Sek (2013).

However, the best model in the 2nd period for all the indexes is still the ARMA (1,1)-EGARCH (1,1). It is also important to note that the ARMA (1,1)-GARCH (1,1) without external regressors does no perform badly in forecast volatility (for details explore Annexes A, B and C).

Looking at the distributions in the table, there is not one that clearly outperforms the others, and we cannot conclude that the skewed version is better than the standard version. For the 3rd period (crisis), the GED distribution is the best for all the three indexes presented in the table yet despite having different models, the distribution is the same.

From a direct comparison of the two tables 5.1.1 and 5.2.1, we can see that there are only two cases where the models with the respective distributions are the same in the 2nd period for the Nikkei 225 using an ARMA (1,1)-EGARCH (1,1) with the GED distribution, and the DAX 30 with the ARMA (1,1)-TGARCH (1,1) also using a GED error distribution.

We can also see that for All Sample, the DAX 30 has the same model in and out-of-sample but different distributions, the same happens in the 2nd period for the DAX 30 and S&P 500, the 3rd period for S&P 500, the 4th period for the Nikkei 225 and the 5th period for S&P 500. This means that the distribution with the best results in-sample does not represent the best results out-of-sample using the same model.

In the out-of-sample analysis, there is another conclusion that indicates the importance of the distribution. For the DAX 30 in the All Sample, the best model for forecasting is the ARMA (1,1)- TGARCH (1,1) with the GED, but the same model using a skewed student-t produces the worst results in this time period analysis. The same happens in the 3rd period for S&P 500, with the ARMA (1,1)-EGARCH (1,1) producing the best results using a GED. However, with the skewed student-t, the forecast is the worst for the period. In the Nikkei, this occurs in the 2nd period where the GED has the best distribution and the skewed version of the GED produces the worst result. In the 4th period, the student-t is the best to use and the worst is the GED, and in the 5th period, the GED outperforms the skewed student-t. All these differences were observed in the ARMA (1,1) -EGARCH (1,1) model.

To analyse whether the skewed version produces better results, we compared the results of the skewed version against the standard one (GED vs skewed GED and student-t against skewed student-t) in the same model. It can be concluded that out-of-sample, the skewed version performs better only 45% of the time. Comparing within the out-of-sample for crisis periods only, the skewed distribution is better 45% of the time and for non-crisis periods, the value drops to 39% (for more detail about results consult the annexes). This shows that the standard version of the GED and student-t are better for modelling the errors of the returns and produce better results in forecasting conditional volatility of the indexes similar to Kumar and Basavaraj (2016).

After analyzing the models and distributions, we now present the coefficients for the external variables. The first one that will be presented is the lagged close-to-open negative returns.

Table 5.2.2 Coefficients for lagged close-to-open negative returns of the best models out-of-sample

	All sample	1st Period	2nd Period	3rd Period	4th Period	5th Period
DAX 30	0.000000	-	22.641238	0.000000	0.000000	0.000000
S&P 500	0.000000	0.000000	-99.668140	-60.04065*	0.000000	0.000000
Nikkei 225	0.000000	0.000000	14.747098	0.000000	-16.15524*	2.209508*

Note: * statistically significant at 5% level

The following is an example to recall the interpretation of the table: in the 3rd period, the model ARMA (1,1)-EGARCH (1,1) with the GED distribution estimated a coefficient for the lagged close-to-open negative returns of approximately -60.04.

The table shows the same conclusion as the in-sample analyses, the lagged close-to-open negative returns are not statistically significant in general. However, in the 3rd period this variable is significant for S&P 500 and in the 4th period for the Nikkei, similar to the in-sample results. It is as yet not observable that when this variable is statistically significant, the model that accounts for this impact produces the best results.

The negative coefficient of this variable means that having close-to-open negative returns would increase the conditional volatility. There is, however, an unexpected behaviour in the 5th period, where the impact is positive, meaning that having a close-to-open negative return on the day before this would lower the conditional volatility for the next day.

This unexpected behaviour only produces the best results in this specific situation (Nikkei 225, 5th Period). In general, when this coefficient is positive, the forecasted values produce the worst results (for detail see annexes A, B and C)

After discussing the results for the lagged close-to-open negative returns, we will present the results for the lagged log differences of trading volume.

Table 5.2.3 Coefficients for lagged log differences of trading volume of the best models out-of-sample

	All sample	1st Period	2nd Period	3rd Period	4th Period	5th Period
DAX 30	0,002067*	-	1,521648*	0,002924*	0,002162*	0,010102*
S&P 500	0,000008*	0,002162*	2,055066*	2,297543*	0,001653*	0,010185*
Nikkei 225	0,003579*	0,003369*	1,114746*	0,000033*	3,059586*	3,262722*

Note: * statistically significant at 5% level

There is not much to be added to this variable. The results are in line with the in-sample analyses. For all the models here, these regressors are statistically significant with the exception of the Dax 30 in the 1st period because the ARMA (1,1)-GARCH (1,1) model does not consider this an external regressor.

The impact is positive and increases over the years, meaning that an increase in the trading volume of the day before will increase the volatility on the next day.

Since the only model that does not account for both external regressors is the ARMA (1,1)-GARCH (1,1) which is the only choice for the 1st period of DAX 30, we can point to the importance of including this regressor in forecasting conditional volatility.

6. Conclusion

Modelling volatility is not a relatively new topic but the importance of constructing a methodology that can track and forecast this behaviour in stock markets is a field that everyone wants to contribute towards. Improving our ability to predict how the markets behave will not only help investors but also help the policymakers maximise the effects of measures taken regarding the economy.

The ARCH model gives rise to a new family of models, with the GARCH now being the most common model used in modelling conditional volatility. The ARMA-GARCH mixture of models is becoming popular; the advantage of this is that it captures both the properties of the returns (ARMA part) and volatility (GARCH part). Here we use this mixture to contribute to the discussion on this topic.

We chose different indexes in order to observe the differences of diverse market locations. Despite the new technology and the literature such as Martens (2001) who concluded that the use of highly frequency data improves the daily forecast, we use the daily returns due to the still limited access. The indexes are the DAX 30, S&P 500 and the Nikkei 225.

The objective was to include external regressors in order to improve the results for model volatility. To achieve this we analysed the coefficients and the significance of each external regressor and also compared the results arrived at through ARMA (1,1)-EGARCH (1,1), -TGARCH (1,1), -GJRGARCH (1,1) with the results of ARMA (1,1)-GARCH (1,1) without external results. Since there was no consensual distribution to use, we estimated volatility using student-t, GED and the skew versions of the same. The reason for including external regressors for modelling volatility was to see whether they are important and how they behave in both crisis and non-crisis periods. In line with the study of Wang et al. (2015) we chose close-to-open negative returns and log differences of trading volume to add to the usual regressors of the GARCH part models.

To evaluate and compare the forecast, the sample was divided into in-sample and out-of-sample. Within the in-sample, the periods are divided into crisis and non-crisis. The out-sample is the last twenty observations of the full sample. To compare this model, the measures chosen are the AIC, BIC and MSE.

Regarding the in-sample analysis, the results for different indexes are almost the same, and reveal that the better results are attained by the ARMA (1,1)-EGARCH (1,1) and by the ARMA (1,1)-TGARCH (1,1) with the the first being selected more often than the last. The ARMA (1,1)-GARCH (1,1) is the model that produces the worst in-sample results.

For the out-of-sample, the conclusion is similar. The ARMA (1,1)-EGARCH (1,1) and ARMA (1,1)-TGARCH (1,1) are the best models, with the ARMA (1,1)-TGARCH (1,1) being the one that produces the best results more often.

Looking at the distributions, there is no clear distribution in this study that outperforms the others. Some patterns can be seen in S&P 500 in-sample with the skew GED distribution, but the same distribution is not always the best for the rest of the indexes.

Comparing the skew version of the distribution against their standard representation (skewed GED vs GED, and the skewed student-t against student-t in the same model) it is possible to conclude that the skew version is better in-sample in both crisis and non-crisis periods. For out-of-sample the same does not happen, the standard version is by far the most preferred being even better in non-crisis periods.

The importance of choosing the correct distribution for modelling volatility is also confirmed in this study where the same models with just a different distribution can produce the worst results especially for out-of-sample analysis.

There are cases where a model with the same distribution performs better in and out-of-sample. This happens in the Nikkei 225 for the 2nd period by ARMA (1,1)-EGARCH (1,1) with the GED distribution, and for ARMA (1,1)-TGARCH (1,1) also with the GED distribution in the 5th period for the DAX 30, but this is not a common result in this study.

Finally, and to conclude with regard to the variables, despite the close-to-open negative returns lagged one time, they are not always statistically significant at a 5% significance level and can only be captured by the ARMA (1,1)-EGARCH (1,1) model. When this regressor is significant, it produces the best results for in-sample analyses, and in some cases for out-of-sample. We can say that it is an important regressor for model volatility in-sample but the impact on out-of-sample is not the best since there are only three cases where the models with this variable outperform the others.

For the log differences of daily trading volume lagged one time, it is significant in all models, in all indexes and time periods. So, the better results attained by the models discussed above can be explained by the inclusion of this variable in comparison with the ARMA (1,1)-GARCH (1,1). The coefficient for this variable shows an increase in the behaviour of the impact to model volatility over the years (in the ARMA (1,1)-EGARCH (1,1) model) that can be explained by the increasing use of high frequency trading in financial markets.

In future research, it would be interesting to see whether the same impacts occur with high frequency data, and whether the use of this type of data improves the forecast. Additionally, it would be interesting to include other variables like the volume of searches in Google and the registration of new investors in platforms like the Robin-hood and eToro since this can increase the number of shares traded.

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Annex A – DAX 30

All Sample

ARMA-GARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000361	0.000260	0.000380	
ar1	0.900035	0.849709	0.880742	
ma1	-0.913498	-0.877965	-0.896684	
omega	0.000000*	0.000000*	0.000000*	
alpha1	0.091626	0.084313	0.093461	
beta1	0.904773	0.910846	0.900157	
gamma1				
vxreg1				
vxreg2				
skew		0.879836		
shape	8.042719	8.723162	1.432218	
AIC	-7.6155	-7.6233	-7.6184	
BIC	-7.6073	-7.6139	-7.6102	
MSE;IN	0.01128162	0.01124960	0.01125708	
MSE;OUT	0.02016395	0.02422567	0.02065045	

ARMA-EGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000282	0.000261	0.000284	0.000262
ar1	0.657575	0.664954	0.653586	0.662823
ma1	-0.697650	-0.710706	-0.693616	-0.708555
omega	-0.122525	-0.126355	-0.121429	-0.125562
alpha1	-0.064010	-0.063961	-0.062053	-0.062264
beta1	0.988840	0.988470	0.988958	0.988564
gamma1	0.112958	0.114149	0.112404	0.114193
vxreg1	-3.840420	-4.051780	-3.774086	-3.989787
vxreg2	1.716359	1.668016	1.697944	1.653559
skew		0.909431		0.909341
shape	36.636754	40.516110	1.847776	1.845374
AIC	-7.7437	-7.7475	-7.7438	-7.7478
BIC	-7.7320	-7.7346	-7.7321	-7.7349
MSE;IN	0.01293185	0.01224502	0.01266199	0.01208286
MSE;OUT	0.05018520	1.34494800	0.04432097	0.03262904

ARMA-GJRGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000249	0.000197	0.000217	0.000140
ar1	-0.089445	-0.126280	-0.174314*	0.617478
ma1	0.077463	0.101404	0.160931*	-0.639335
omega	0.000000*	0.000000*	0.000000	0.000000
alpha1	0.002099	0.016024	0.009766	0.011941
beta1	0.910400	0.908801	0.918460	0.914726
gamma1	0.142160	0.122482	0.104931	0.109220
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.000011	0.000011	0.000009	0.000009
skew		0.886055		0.880597
shape	13.225200	15.195830	1.627365	1.630290
AIC	-7.6840	-7.6904	-7.6774	-7.6863
BIC	-7.6723	-7.6775	-7.6657	-7.6734
MSE;IN	0.01071364	0.01074755	0.01074535	0.01073700
MSE;OUT	0.04556509	0.49165910	0.01738714	0.01776444

ARMA-TGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000240	0.000095	0.000244	0.000172
ar1	0.211565*	0.982907	0.629763	0.675328
ma1	-0.220052*	-0.977152	-0.646819	-0.703714
omega	0.000000	0.000103	0.000118	0.000115
alpha1	0.043369	0.067881	0.071579	0.071416
beta1	0.919583	0.926842	0.919735	0.920901
eta11	0.624079	0.755200	0.588731	0.573642
vxreg1	0.000000*	0.000003*	0.000000*	0.000000*
vxreg2	0.000007	0.002020	0.002067	0.002015
skew		0.896740		0.892234
shape	13.219750	17.201930	1.713233	1.711987
AIC	-7.6685	-7.7130	-7.7075	-7.7139
BIC	-7.6567	-7.7001	-7.6957	-7.7009
MSE;IN	0.01075891	0.01046800	0.01051316	0.01050199
MSE;OUT	0.03049597	20.98657000	0.01707562	0.01840560

1st Period

ARMA-GARCH;(1,1)				
	STD	SSTD	GED	SGED
mu				
ar1	-0.077647*	-0.209786*	-0.086908*	-0.204375*
ma1	0.095985*	0.216424*	0.105015*	0.211231*
omega	0.000001*	0.000001*	0.000001*	0.000001*
alpha1	0.098164	0.097815	0.099772	0.098221
beta1	0.882895	0.884347	0.880787	0.882946
gamma1				
vxreg1				
vxreg2				
skew		0.903490		0.908365
shape	34.39914*	30.36853*	1.922238	1.903906
AIC	-7.0394	-7.0419	-7.0381	-7.0405
BIC	-7.0146	-7.0131	-7.0134	-7.0116
MSE;IN	0.01238976	0.01238481	0.01238962	0.01238352
MSE;OUT	0.02791210	0.02638424	0.02700025	0.02656672

ARMA-EGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	-0.000028*	-0.000023*	-0.000086*	-0.000059*
ar1	0.818199	0.812223	0.826493	0.821908
ma1	-0.836877	-0.835199	-0.846770	-0.845300
omega	-0.273693	-0.268821	-0.296034	-0.294643
alpha1	-0.075234	-0.072765	-0.071662	-0.068992
beta1	0.972972	0.973491	0.970405	0.970625
gamma1	0.175851	0.177732	0.180250	0.181721
vxreg1	-0.317737*	-1.016695*	0.055697*	-0.453193*
vxreg2	1.231656	1.205685	1.255448	1.240894
skew		0.922489		0.949458
shape	99.99088*	59.99996*	2.352414	2.317213
AIC	-7.1095	-7.1092	-7.1156	-7.1149
BIC	-7.0683	-7.0638	-7.0743	-7.0695
MSE;IN	0.01369186	0.01345254	0.01389766	0.01373096
MSE;OUT	0.03576572	8.87429300	0.03357383	0.03750448

ARMA-GJRGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu			-0.000110*	-0.000039*
ar1			0.427351*	0.281300*
ma1			-0.419034*	-0.281738*
omega			0.000001	0.000002
alpha1			0.036900	0.040144
beta1			0.882142	0.874170
gamma1			0.106600	0.107696
vxreg1			0.000000*	0.000000*
vxreg2			0.000020	0.000022
skew				0.929154
shape			2.195081	2.172815
AIC			-7.0946	-7.0961
BIC			-7.0533	-7.0507
MSE;IN			0.01187466	0.01187807
MSE;OUT			0.02863526	0.02818309

ARMA-TGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	-0.000057*	-0.000106*	-0.000102*	-0.000139*
ar1	-0.514808	-0.980991	0.832405	-0.588857
ma1	0.506412	0.972465	-0.845758	0.570279
omega	0.000290	0.000288	0.000307	0.000308
alpha1	0.099431	0.099899	0.102407	0.102982
beta1	0.877668	0.878014	0.872824	0.872294
eta11	0.447913	0.453270	0.406680	0.418209
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.002894	0.002835	0.002955	0.002891
skew		0.916103		0.945618
shape	99.99659*	59.99974*	2.291275	2.268437
AIC	-7.1054	-7.1067	-7.1104	-7.1097
BIC	-7.0642	-7.0614	-7.0691	-7.0643
MSE;IN	0.01166189	0.01164774	0.01168381	0.01170209
MSE;OUT	0.03763669	0.03284988	0.03838424	0.04044340

Note 1: contrary to the text for simplification * means not significant at 5% level
Note 2: [1, 2,3, 4] represent 150, 200, 250 and 300 observations used in window size

2nd Period

ARMA-GARCH;(1,1)				
	STD[3]	SSTD[2]	GED	SGED
mu				
ar1	-0.938366	0.746100	-0.560207	0.638398
ma1	0.922201	-0.785731	0.521880	-0.677705
omega	0.000000*	0.000000*	0.000000*	0.000000*
alpha1	0.064225	0.064772	0.065705	0.065233
beta1	0.926126	0.928556	0.922956	0.926664
gamma1				
vxreg1				
vxreg2				
skew		0.828827		0.848580
shape	10.472220	8.814331	1.481938	1.432632
AIC	-7.9092	-7.9237	-7.9131	-7.9269
BIC	-7.8798	-7.8894	-7.8837	-7.8927
MSE;IN	0.00442186	0.00440647	0.00441672	0.00440605
MSE;OUT	0.00017159	0.00017234	0.00014627	0.00020003

ARMA-EGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000467	0.000456	0.000488	0.000447
ar1	0.780709	0.749284	0.777281	0.755297
ma1	-0.828046	-0.804527	-0.825623	-0.811472
omega	-0.113085	-0.114679	-0.112133	-0.113968
alpha1	-0.068521	-0.074611	-0.070019	-0.073258
beta1	0.989252	0.988987	0.989340	0.989089
gamma1	0.085439	0.089374	0.086189	0.089184
vxreg1	19.82103*	23.277841*	22.641238*	22.787662*
vxreg2	1.520747	1.458792	1.521648	1.454526
skew		0.868903		0.871941
shape	64.261583*	59.998427*	1.768639	1.825533
AIC	-8.0251	-8.0326	-8.0270	-8.0339
BIC	-7.9761	-7.9788	-7.9780	-7.9801
MSE;IN	0.00408302	0.00406074	0.00408520	0.00405915
MSE;OUT	0.00011623	0.00037741	0.00008911	0.00026214

ARMA-GJRGARCH;(1,1)				
	STD[4]	SSTD	GED	SGED
mu	0.000442	0.000362	0.000494	0.000388
ar1	0.844345	0.803042	0.829606	0.838220
ma1	-0.879953	-0.854935	-0.865700	-0.882816
omega	0.000000	0.000000*	0.000000*	0.000000*
alpha1	0.026761	0.019329	0.022430	0.024228
beta1	0.935925	0.943152	0.938870	0.941510
gamma1	0.043196	0.048463	0.042576	0.040483
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.000006	0.000006	0.000006	0.000006
skew		0.854790		0.870520
shape	11.962960	12.781250	1.533277	1.565630
AIC	-7.9626	-7.9730	-7.9662	-7.9744
BIC	-7.9136	-7.9192	-7.9172	-7.9205
MSE;IN	0.00429522	0.00428321	0.00431571	0.00429519
MSE;OUT	0.00015041	0.00013770	0.00013194	0.00013673

ARMA-TGARCH;(1,1)				
	STD	SSTD[1]	GED	SGED
mu	0.000491	0.000434	0.000497	0.000435
ar1	0.825429	0.808826	0.833051	0.824445
ma1	-0.862242	-0.851281	-0.870226	-0.865485
omega	0.000106	0.000098	0.000076	0.000068
alpha1	0.058242	0.056431	0.058954	0.056042
beta1	0.935108	0.938078	0.934914	0.939260
eta11	0.555603	0.576569	0.398067	0.426834
vxreg1	0.071797*	0.065520*	0.000000*	0.000000*
vxreg2	0.001732	0.001641	0.001804	0.001727
skew		0.877551		0.898389
shape	12.489370	13.225180	1.562481	1.580744
AIC	-7.9827	-7.9892	-7.9814	-7.9858
BIC	-7.9338	-7.9353	-7.9324	-7.9320
MSE;IN	0.00425424	0.00424948	0.00427242	0.00426672
MSE;OUT	0.00035732	0.00017938	0.00044595	0.00031783

3rd Period

ARMA-GARCH;(1,1)				
	STD	SSTD	GED	SGED
mu				
ar1	0.420471*	0.599049*	-0.987296	0.586228
ma1	-0.431407*	-0.633451	0.976332	-0.611574
omega	0.000001*	0.000001*	0.000001*	0.000001*
alpha1	0.093187	0.092102	0.096401*	0.094470
beta1	0.895013	0.896825	0.890416	0.893220
gamma1				
vxreg1				
vxreg2				
skew		0.895783		0.899081
shape	7.246646	7.522055	1.361495	1.365561
AIC	-7.3685	-7.3731	-7.3744	-7.3780
BIC	-7.3389	-7.3386	-7.3449	-7.3435
MSE;IN	0.02131321	0.02120938	0.02137562	0.02124474
MSE;OUT	0.00019291	0.00023078	0.00018333	0.00019250

ARMA-EGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000385	0.000354	0.000383	0.000353
ar1	0.486670	0.444719	0.502142	0.467954
ma1	-0.547146	-0.514086	-0.556684	-0.530919
omega	-0.248180	-0.242763	-0.233906	-0.235130
alpha1	-0.034913*	-0.040195*	-0.033869*	-0.03995*
beta1	0.977963	0.978418	0.979295	0.979144
gamma1	0.146571	0.145327	0.144647	0.144230
vxreg1	-19.97323	-19.023980	-19.165630	-18.597550
vxreg2	2.028288	1.960483	2.014267	1.949299
skew		0.922957		0.924917
shape	37.57810*	49.61084*	1.841705	1.864136
AIC	-7.6182	-7.5120	-7.5117	-7.5123
BIC	-7.5686	-7.4578	-7.4624	-7.4582
MSE;IN	0.01633426	0.01617741	0.01629056	0.01629056
MSE;OUT	0.00020767	0.00020178	0.00023499	0.00022318

ARMA-GJRGARCH;(1,1)				
	STD	SSTD	GED[1]	SGED[4]
mu	0.000282*	0.000260	0.000444	0.000222*
ar1	0.366125*	0.327231	-0.166661*	0.304863
ma1	-0.384235*	-0.359460	0.160909*	-0.337239
omega	0.000001*	0.000001*	0.000001	0.000001
alpha1	0.000079*	0.000048*	0.000091*	0.000064*
beta1	0.898007	0.901089	0.887278	0.897669
gamma1	0.156804	0.133346	0.136885	0.156824
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.000016	0.000030	0.000036	0.000016
skew		0.896526		0.890846
shape	12.882140	18.98442*	1.766946	1.652443
AIC	-7.4470	-7.4847	-7.4896	-7.4545
BIC	-7.3978	-7.4306	-7.4404	-7.4003
MSE;IN	0.01929080	0.01932700	0.01942946	0.01928142
MSE;OUT	NA	NA	0.00018598	0.00016255

ARMA-TGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000304	0.000268*	0.000258	0.000231
ar1	0.980389	0.338228	0.969285	-0.982053
ma1	-0.979578	-0.379612	-0.961790	0.978036
omega	0.000089	0.000095	0.000081	0.000098
alpha1	0.067813	0.069475	0.054307	0.069093
beta1	0.929791	0.927230	0.942033	0.927487
eta11	0.635973	0.623216	0.999994	0.685853
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.003023	0.002927	0.002924	0.002881
skew		0.902783		0.920877
shape	19.33754*	25.8994*	1.681816	1.704518
AIC	-7.4832	-7.4872	-7.5821	-7.5955
BIC	-7.4340	-7.4330	-7.5325	-7.5410
MSE;IN	0.01913120	0.01905849	0.01897492	0.01906662
MSE;OUT	0.00016704	NA	0.00014706	0.00016065

Note 1: contrary to the text for simplification * means not significant at 5% level
Note 2: [1, 2,3, 4] represent 150, 200, 250 and 300 observations used in window size

4th Period

ARMA-GARCH;(1,1)				
	STD	SSTD	GED	SGED
mu				
ar1	-0.777550*	-0.822693*	-0.850542	0.828983
ma1	0.79024*	0.832633*	0.857114	-0.853875
omega	0.000000*	0.000000*	0.000000*	0.000000*
alpha1	0.091063	0.089173	0.086687	0.077417
beta1	0.901369	0.904439	0.900222	0.911730
gamma1				
vxreg1				
vxreg2				
skew		0.892853		0.874362
shape	6.186960	6.001363	1.337173	1.292283
AIC	-7.9219	-7.9301	-7.9284	-7.9472
BIC	-7.9067	-7.9123	-7.9132	-7.9242
MSE;IN	0.00297249	0.00297330	0.00296282	0.00295130
MSE;OUT	0.00035982	0.00038254	0.00033131	0.00036158

ARMA-EGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000273	0.000263	0.000286	0.000269
ar1	0.757066	0.748665	0.740779	0.739180
ma1	-0.791964	-0.785688	-0.777383	-0.777307
omega	-0.372714	-0.363026	-0.375119	-0.364053
alpha1	-0.087574	-0.088817	-0.088171	-0.089573
beta1	0.967091	0.967927	0.966996	0.967938
gamma1	0.153598	0.151804	0.154743	0.152890
vxreg1	-13.840280	-13.347920	-14.001290	-13.417940
vxreg2	2.104370	2.072655	2.103003	2.067217
skew		0.945136		0.941444
shape	48.76753*	45.41837*	1.797324	1.782964
AIC	-8.0966	-8.0973	-8.0982	-8.0993
BIC	-8.0713	-8.0695	-8.0729	-8.0714
MSE;IN	0.00263072	0.00260657	0.00263778	0.00260964
MSE;OUT	0.00144031	0.00447137	0.00107366	0.00199498

ARMA-GJRGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000412	0.000165	0.000269	0.000174
ar1	-0.473621*	-0.628454	0.854971	-0.860245
ma1	0.495595*	0.618121	-0.871855	0.865067
omega	0.000000*	0.000001	0.000000*	0.000000
alpha1	0.000081*	0.000209*	0.022109	0.000080
beta1	0.906142	0.871198	0.905159	0.898639
gamma1	0.156433	0.159422	0.086361	0.160409
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.000010	0.000015	0.000013	0.000009
skew		0.914723		0.890249
shape	7.886802	11.036260	1.557421	1.461407
AIC	-8.0097	-8.0345	-8.0275	-8.0187
BIC	-7.9844	-8.0066	-8.0022	-7.9908
MSE;IN	0.00284962	0.00280514	0.00278708	0.00284784
MSE;OUT	NA	NA	NA	NA

ARMA-TGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000256	0.000119*	0.000273	0.000201
ar1	0.406625	-0.841803	0.486212	0.566025
ma1	-0.421466	0.847032	-0.505669	-0.594397
omega	0.000134	0.000154	0.000127	0.000115
alpha1	0.062509	0.064845	0.059923	0.057753
beta1	0.918962	0.914853	0.922236	0.927476
eta11	0.834602	0.999999	0.814436	0.802553
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.002195	0.002676	0.002210	0.002162
skew		0.908168		0.906064
shape	16.969280	14.227030	1.647856	1.640537
AIC	-8.0515	-8.0634	-8.0551	-8.0597
BIC	-8.0262	-8.0355	-8.0297	-8.0318
MSE;IN	0.00265722	0.00261497	0.00265839	0.00265504
MSE;OUT	0.00033619	0.00034383	0.00034034	0.00031637

5th Period

ARMA-GARCH;(1,1)				
	STD	SSTD	GED	SGED
mu				
ar1	0.778249*	0.385741*	0.820041	0.968680
ma1	-0.725381*	-0.299076*	-0.790700	-1.000000
omega	0.000005*	0.000006	0.000005*	0.000004*
alpha1	0.253309	0.176464	0.284539	0.272276
beta1	0.745691	0.822536	0.714461	0.726724
gamma1				
vxreg1				
vxreg2				
skew		0.750229		0.856041
shape	3.341334	2.839255	0.946340	0.861567
AIC	-6.3555	-6.3983	-6.3655	-6.4265
BIC	-6.1806	-6.1943	-6.1906	-6.2225
MSE;IN	0.20492090	0.19850470	0.20938070	0.21000630
MSE;OUT	0.02082369	0.02123710	0.02095818	0.02104951

ARMA-EGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000183*	0.000233	0.000112*	0.000112*
ar1	-0.810249	0.286460	-0.790651*	-0.216401*
ma1	0.855054	-0.325601	0.841186*	0.256763
omega	-0.811208*	-0.172275	-0.624045*	-0.648838
alpha1	0.039472*	0.912491	-0.002859*	-0.16317*
beta1	0.928259	1.000000	0.947018	0.941407
gamma1	0.609718	-0.726948	0.618478	0.660119
vxreg1	-36.53459*	-39.49994*	-31.12022*	-20.43285*
vxreg2	4.315082	4.704880	4.066457	3.849547
skew		0.840449		0.807888
shape	5.082723*	2.119725	1.428149	1.235493
AIC	-6.5765	-6.4626	-6.5752	-6.5620
BIC	-6.2851	-6.3220	-6.2838	-6.2414
MSE;IN	0.24472330	8.12010200	0.21407740	0.20738340
MSE;OUT	0.05410338	1.17420500	0.04731256	0.03533190

ARMA-GJRGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	-0.000112*	-0.000997	-0.000050	-0.001036
ar1	-0.716997	0.846804	-0.648849	-0.010282
ma1	0.760069	-0.761250	0.827242	0.098284
omega	0.000007*	0.000000*	0.000000*	0.000005*
alpha1	0.000022*	0.000004*	0.196721	0.000046*
beta1	0.832541	0.954899	0.584996	0.869641
gamma1	0.999941	0.287905	0.929114	0.225801
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.000233	0.000338	0.000016	0.000048
skew		0.643122		0.684382
shape	2.335014	2.258964	0.940049	0.749112
AIC	-6.5436	-6.6786	-6.4762	-6.4258
BIC	-6.2522	-6.3580	-6.1848	-6.1052
MSE;IN	1.02360500	0.36711440	0.50647330	0.19357730
MSE;OUT	0.03433664	0.27421730	0.02036504	0.02062643

ARMA-TGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000577	0.000598	0.001333	-0.000537
ar1	-0.645672	0.992050	0.089960	0.543412
ma1	0.702812	-0.906261	-0.033050	-0.444114
omega	0.000025	0.000000*	0.000000*	0.000000*
alpha1	0.048182	0.094001	0.067995	0.046535
beta1	0.975137	0.962671	0.925353	0.968351
eta11	0.999999	1.000000	1.000000	1.000000
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.020375	0.020616	0.010102	0.007711
skew		0.716380		0.549875
shape	2.274780	2.268669	1.715937	0.824627
AIC	-6.7869	-6.8222	-6.8046	-6.5632
BIC	-6.4955	-6.5016	-6.5132	-6.2426
MSE;IN	0.31887840	0.42375030	0.15807170	0.16065390
MSE;OUT	0.10875070	0.13120610	0.01746755	0.02011174

Note 1: contrary to the text for simplification * means not significant at 5% level
Note 2: $^{1, 2,3, 4}$ represent 150, 200, 250 and 300 observations used in window size

Annex B – S&P 500

All Sample

ARMA-GARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000327	0.000240	0.000328	0.000211
ar1	0.745247	0.732523	0.735701	0.719811
ma1	-0.798333	-0.806905	-0.787686	-0.793836
omega	0.000000*	0.000000*	0.000000*	0.000000*
alpha1	0.112771	0.106240	0.115700	0.105844
beta1	0.886188	0.890014	0.878453	0.886951
gamma1				
vxreg1				
vxreg2				
skew		0.859804		0.866961
shape	6.217360	7.075376	1.310326	1.360318
AIC	-8.1476	-8.1584	-8.1499	-8.1616
BIC	-8.1393	-8.1490	-8.1416	-8.1522
MSE;IN	0.00822631	0.00819554	0.00818027	0.00817969
MSE;OUT	0.00356996	0.00325567	0.00363845	0.00347738

ARMA-EGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000248	0.000217	0.000249	0.000209
ar1	0.394566	0.431980	0.389114	0.432455
ma1	-0.463994	-0.509739	-0.457928	-0.509802
omega	-0.134961	-0.144027	-0.138646	-0.147367
alpha1	-0.100312	-0.101661	-0.098773	-0.100610
beta1	0.988141	0.987242	0.987859	0.986964
gamma1	0.113600	0.115344	0.112615	0.114771
vxreg1	-5.803835*	-6.098567*	-5.894071*	-6.164847*
vxreg2	2.235946	2.116189	2.203189	2.085951
skew		0.877423		0.883333
shape	17.161601	20.796647	1.674679	1.714252
AIC	-8.2658	-8.2733	-8.2668	-8.2744
BIC	-8.2539	-8.2603	-8.2550	-8.2614
MSE;IN	0.00755123	0.00749611	0.00752669	0.00748200
MSE;OUT	0.01952199	0.37281640	0.02006465	0.01112817

ARMA-GJRGARCH;(1,1)				
	STD[4]	SSTD[4]	GED	SGED
mu	0.000254	0.000203	0.000209	0.000128
ar1	0.323811	0.347040	0.481452	0.483418
ma1	-0.379850	-0.421783	-0.525616	-0.550034
omega	0.000000*	0.000000*	0.000000*	0.000000*
alpha1	0.000544	0.000176*	0.000103	0.000336
beta1	0.878700	0.892816	0.893331	0.902882
gamma1	0.203916	0.168940	0.176351	0.156298
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.000007	0.000008	0.000004	0.000004
skew		0.872562		0.857503
shape	9.644120	12.029860	1.442798	1.497595
AIC	-8.2088	-8.2232	-8.2003	-8.2131
BIC	-8.1970	-8.2102	-8.1884	-8.2002
MSE;IN	0.00778938	0.00775209	0.00782510	0.00782233
MSE;OUT	0.00373384	0.00291734	0.00601412	0.00579296

ARMA-TGARCH;(1,1)				
	STD[2]	SSTD	GED	SGED
mu	0.000238	0.000106	0.000168	0.000083
ar1	0.123010	0.282144	0.173633	0.254422
ma1	-0.183776	-0.355280	-0.233317	-0.327373
omega	0.000070	0.000096	0.000104	0.000103
alpha1	0.063887	0.073546	0.075524	0.074192
beta1	0.931802	0.920283	0.915619	0.918352
eta11	1.000000	0.999999	0.999980	0.999984
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.001968	0.001442	0.001387	0.001404
skew		0.849814		0.854138
shape	11.448300	11.992280	1.531106	1.581651
AIC	-8.2333	-8.2384	-8.2261	-8.2402
BIC	-8.2215	-8.2254	-8.2143	-8.2272
MSE;IN	0.00771948	0.00762364	0.00764472	0.00763177
MSE;OUT	0.00417532	NA	0.00436015	0.00305380

1st Period

ARMA-GARCH;(1,1)				
	STD	SSTD	GED	SGED
mu				
ar1	0.747443	0.757053	0.748823	0.76321
ma1	-0.774072	-0.798324	-0.776134	-0.807034
omega	0.000001*	0.000001*	0.000001*	0.000001*
alpha1	0.082191*	0.084548*	0.085938	0.083169*
beta1	0.888868	0.889663	0.883128	0.890445
gamma1				
vxreg1				
vxreg2				
skew		0.905008		0.889705
shape	10.393590	10.940090	1.576950	1.600420
AIC	-7.5535	-7.5261	-7.5207	-7.5241
BIC	-7.4987	-7.4971	-7.4958	-7.4950
MSE;IN	0.00430186	0.00429719	0.00429982	0.00429482
MSE;OUT	NA	0.00113971	0.00113121	0.00101246

ARMA-EGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.00002*	0.000012*	0.000009*	0.000007*
ar1	0.684911	0.672770	0.693696	0.679808
ma1	-0.717029	-0.710146	-0.724339	-0.716322
omega	-0.239917	-0.232083	-0.244340	-0.235457
alpha1	-0.093803	-0.092844	-0.093871	-0.093081
beta1	0.977356	0.978081	0.976894	0.977727
gamma1	0.079624	0.081048	0.078528	0.080099
vxreg1	0.131647*	0.14758*	0.042464*	0.066129*
vxreg2	1.770020	1.716531	1.732061	1.674467
skew		0.934150		0.932679
shape	43.12374*	46.7135*	1.946278	1.956782
AIC	-7.6256	-7.6258	-7.6247	-7.6250
BIC	-7.5842	-7.5802	-7.5833	-7.5794
MSE;IN	0.00375354	0.00375553	0.00375858	0.00376047
MSE;OUT	0.01655656	0.36889500	0.01688814	0.00853793

ARMA-GJRGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	-0.000026*	-0.000063*	-0.000092	-0.000007
ar1	0.788237	0.799527	-0.554635	0.420248
ma1	-0.810646	-0.814353	0.541561	-0.414543
omega	0.000001*	0.000001*	0.000001	0.000001*
alpha1	0.000031*	0.000014*	0.000015*	0.000014*
beta1	0.903136	0.888165	0.880803	0.878997
gamma1	0.136609	0.148169	0.169651	0.186400
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.000019	0.000019	0.000019	0.000019
skew		0.934377		0.918347
shape	38.47022*	30.42257*	1.905193	1.983963
AIC	-7.6041	-7.6054	-7.6036	-7.5400
BIC	-7.5627	-7.5598	-7.5622	-7.5598
MSE;IN	0.00397779	0.00398810	0.00398693	0.00399610
MSE;OUT	NA	NA	0.00108396	0.00097585

ARMA-TGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000011*	-0.000012*	-0.000032*	-0.000018*
ar1	0.648953	0.636903	-0.876535	0.637727
ma1	-0.669779	-0.663882	0.865588	-0.663542
omega	0.000157	0.000154	0.000186	0.000162
alpha1	0.055593	0.055406	0.063019	0.056730
beta1	0.926404	0.927445	0.915793	0.925060
eta11	1.000000	1.000000	1.000000	1.000000
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.002268	0.002193	0.002186	0.002162
skew		0.928075		0.925429
shape	34.93712*	41.48733*	1.921716	1.917709
AIC	-7.6098	-7.6104	-7.6090	-7.6098
BIC	-7.5684	-7.5648	-7.5675	-7.5642
MSE;IN	0.00388097	0.00388156	0.00387808	0.00388171
MSE;OUT	0.00091886	0.00093208	0.00095729	0.00091032

Note 1: contrary to the text for simplification * means not significant at 5% level
Note 2: [1, 2,3, 4] represent 150, 200, 250 and 300 observations used in window size

2nd Period

ARMA-GARCH;(1,1)				
	STD	SSTD	GED	SGED
mu				
ar1	0.359678*	0.591863	0.259947*	0.521109
ma1	-0.421256*	-0.658115	-0.323901*	-0.589984
omega	0.000000*	0.000000*	0.000000*	0.000000*
alpha1	0.044824	0.042863	0.043187	0.042211
beta1	0.942561	0.946705	0.944670	0.948410
gamma1				
vxreg1				
vxreg2				
skew		0.911894		0.908178
shape	20.32646*	14.194310	1.669793	1.573656
AIC	-8.6311	-8.6327	-8.6347	-8.6394
BIC	-8.6014	-8.5980	-8.6050	-8.6047
MSE;IN	0.00043466	0.00043442	0.00043477	0.00043450
MSE;OUT	0.00001890	0.00001922	0.00001900	0.00002053

ARMA-EGARCH;(1,1)				
	STD[2]	SSTD[3]	GED[4]	SGED[4]
mu	0.000139*	0.000130	0.00015*	0.000140
ar1	0.245860	0.235675	0.250774	0.255551*
ma1	-0.343186	-0.340753	-0.347124	-0.357699*
omega	-0.040318	-0.015039	-0.025298	-0.012188
alpha1	-0.049012	-0.050264	-0.048198	-0.049884
beta1	0.996569	0.998782	0.997945	0.999074
gamma1	0.056327	0.052110	0.053465	0.051463
vxreg1	-0.449036*	-99.66814*	-99.98711*	-99.99773*
vxreg2	1.996638	2.055066	2.028072	2.051029
skew		0.899555		0.895290
shape	57.981848*	42.78352*	1.849003	1.769865
AIC	-8.7089	-8.7136	-8.7106	-8.7154
BIC	-8.6593	-8.6591	-8.6610	-8.6608
MSE;IN	0.00039827	0.00039634	0.00039780	0.00039640
MSE;OUT	0.00001686	0.00001599	0.00002082	0.00001812

ARMA-GJRGARCH;(1,1)				
	STD	SSTD[2]	GED	SGED
mu		0.000226	0.000226	0.000176
ar1		0.648339	0.478111	0.379735
ma1		-0.701249	-0.547621	-0.454179
omega		0.000000*	0.000000*	0.000000*
alpha1		0.006747	0.010782*	0.006470*
beta1		0.959856	0.946669	0.949487
gamma1		0.056599	0.064143	0.070084*
vxreg1		0.000000*	0.000000*	0.000000*
vxreg2		0.000009	0.000000*	0.000000*
skew		0.932944		0.919714
shape		24.000040	1.631630	1.598855
AIC		-8.6956	-8.6498	-8.6526
BIC		-8.6460	-8.6002	-8.5980
MSE;IN		0.00042220	0.00043076	0.00043094
MSE;OUT		0.00002209	0.00002038	0.00002259

ARMA-TGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000131*	0.000104*	0.000170	0.000131*
ar1	0.247037*	0.253685	0.303042	0.294449
ma1	-0.333186*	-0.346590	-0.386305	-0.383552
omega	0.000063	0.000056	0.000063	0.000055
alpha1	0.049623	0.048337	0.050108	0.048947
beta1	0.941353	0.944915	0.940575	0.944293
eta11	0.559712	0.594980	0.522920	0.558594
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.001245	0.001258	0.001217	0.001230
skew		0.912232		0.911773
shape	16.852600	14.527120	1.632414	1.569674
AIC	-8.6619	-8.6643	-8.6625	-8.6656
BIC	-8.6123	-8.6098	-8.6129	-8.6111
MSE;IN	0.00042817	0.00042690	0.00042864	0.00042723
MSE;OUT	0.00001968	0.00001947	0.00001869	0.00002002

3rd Period

ARMA-GARCH;(1,1)				
	STD	SSTD	GED	SGED
mu				
ar1	0.096189*	0.531357	-0.161053	0.481440
ma1	-0.189542*	-0.644615	0.086623*	-0.582056
omega	0.000000*	0.000000*	0.000001*	0.000001*
alpha1	0.110485	0.109556	0.105496	0.106165
beta1	0.888515	0.889443	0.888299	0.888048
gamma1				
vxreg1				
vxreg2				
skew		0.820145		0.841498
shape	5.750625	6.141023	1.237568	1.241508
AIC	-7.4630	-7.4849	-7.4778	-7.5027
BIC	-7.4333	-7.4502	-7.4481	-7.4680
MSE;IN	0.02235323	0.02203023	0.02238816	0.02208453
MSE;OUT	0.00012572	0.00012396	0.00012279	0.00011752

ARMA-EGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000389	0.000300	0.000425	0.000314
ar1	0.156405	0.522326	0.063167*	0.530836
ma1	-0.273983	-0.635904	-0.171313*	-0.635357
omega	-0.267191	-0.308159	-0.264879	-0.316167
alpha1	-0.10182	-0.102763	-0.102324	-0.105304
beta1	0.976654	0.973000	0.977072	0.972495
gamma1	0.092085	0.081372	0.092462	0.081630
vxreg1	-59.20075	-69.851100	-60.040650	-74.138200
vxreg2	2.314154	2.101775	2.297543	2.125080
skew		0.834217		0.852194
shape	18.97565*	56.461340*	1.585721	1.678268
AIC	-7.6182	-7.6303	-7.6235	-7.6345
BIC	-7.5686	-7.5758	-7.5739	-7.5799
MSE;IN	0.02043998	0.01996707	0.02040720	0.02000931
MSE;OUT	0.00017865	0.00020381	0.00010718	0.00111935

ARMA-GJRGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000376	0.000242	0.000417	0.000235
ar1	0.381402*	0.424372	0.261737	0.411688
ma1	-0.494637*	-0.553188	-0.363227	-0.527792
omega	0.000000*	0.000000*	0.000000*	0.000000
alpha1	0.000087*	0.000005*	0.000093*	0.000151
beta1	0.900372*	0.903706	0.900566	0.901477
gamma1	0.162944	0.153975	0.160683	0.155992
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.000018	0.000018	0.000018	0.000018
skew		0.828752		0.858107
shape	8.610744	12.0291*	1.388050	1.472522
AIC	-7.5458	-7.5616	-7.5575	-7.5717
BIC	-7.4962	-7.5071	-7.5079	-7.5172
MSE;IN	0.02071655	0.02062743	0.02080394	0.02067400
MSE;OUT	NA	NA	NA	NA

ARMA-TGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000309	0.000174*	0.000377	0.000181*
ar1	-0.001056*	0.115353	-0.045919*	0.087704*
ma1	-0.106110*	-0.234194	-0.050246*	-0.196138
omega	0.000086	0.000094	0.000082	0.000095
alpha1	0.062075	0.062210	0.061849	0.062989
beta1	0.934188	0.933523	0.934260	0.932641
eta11	1.000000	1.000000	1.000000	1.000000
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.002264	0.002130	0.002254	0.002130
skew		0.832552		0.853661
shape	11.210770	16.02778*	1.446016	1.522194
AIC	-7.5719	-7.5868	-7.5821	-7.5955
BIC	-7.5224	-7.5323	-7.5325	-7.5410
MSE;IN	0.02027955	0.02027055	0.02032390	0.02029090
MSE;OUT	0.00010949	0.00016406	0.00016810	0.00016362

Note 1: contrary to the text for simplification * means not significant at 5% level
Note 2: [1, 2,3, 4] represent 150, 200, 250 and 300 observations used in window size

4^{th} Period

ARMA-GARCH;(1,1)				
	STD	SSTD	GED	SGED
mu				
ar1	-0.907445		-0.909081	
ma1	0.893967		0.893972	
omega	0.000000*		0.000001*	
alpha1	0.171359		0.172798	
beta1	0.811361		0.794626	
gamma1				
vxreg1				
vxreg2				
skew				
shape	5.193121		1.215050	
AIC	-8.5853		-8.5921	
BIC	-8.8700		-8.5768	
MSE;IN	0.00140521		0.00139405	
MSE;OUT	0.00003090		0.00003028	

ARMA-EGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000279	0.000240	0.000273	0.000231
ar1	0.167108*	0.249761	0.149539	0.223442
ma1	-0.233531*	-0.320585	-0.214319	-0.293901
omega	-0.285215	-0.304625	-0.300921	-0.320699
alpha1	-0.173403	-0.176306	-0.173275	-0.175232
beta1	0.975331	0.973443	0.974026	0.972079
gamma1	0.142840	0.144067	0.143555	0.145198
vxreg1	9.36138*	8.823766*	9.056418*	8.412821*
vxreg2	2.324766	2.170590	2.310323	2.166033
skew		0.873273		0.889178
shape	14.856539	17.060404	1.619321	1.679785
AIC	-8.7328	-8.7403	-8.7357	-8.7416
BIC	-8.7073	-8.7123	-8.7102	-8.7136
MSE;IN	0.00137729	0.00127633	0.00135758	0.00126780
MSE;OUT	0.00003314	0.00003126	0.00004990	31675.92000000

ARMA-GJRGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000300	0.000222	0.000281	0.000207
ar1	0.617761	0.483533	0.141822	0.196665
ma1	-0.651102	-0.539503	-0.189867	-0.270544
omega	0.000001*	0.000001*	0.000000	0.000000
alpha1	0.000157*	0.000137*	0.000142*	0.000193*
beta1	0.813536	0.818312	0.829892	0.825576
gamma1	0.256840	0.268455	0.244088	0.260416
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.000008	0.000007	0.000007	0.000007
skew		0.852357		0.864469
shape	7.376061	7.517666	1.432291	1.495273
AIC	-8.6883	-8.6975	-8.6871	-8.6980
BIC	-8.6628	-8.6695	-8.6616	-8.6699
MSE;IN	0.00127525	0.00128812	0.00128069	0.00128721
MSE;OUT	NA	NA	NA	NA

ARMA-TGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000262	0.000207	0.000252	0.000187
ar1	0.299783	0.402628	0.223184*	0.300012
ma1	-0.356485	-0.469109	-0.279748*	-0.371583
omega	0.000130	0.000131	0.000136	0.000137
alpha1	0.097101	0.095816	0.096214	0.096351
beta1	0.884333	0.886241	0.882782	0.883933
eta11	1.000000	1.000000	1.000000	1.000000
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.001749	0.001657	0.001752	0.001653
skew		0.842961		0.856847
shape	9.223821	11.157720	1.492302	1.576511
AIC	-8.7065	-8.7193	-8.7093	-8.7207
BIC	-8.6810	-8.6913	-8.6839	-8.6927
MSE;IN	0.00123254	0.00123460	0.00123537	0.00123545
MSE;OUT	0.00002649	0.00002473	0.00002571	0.00002350

5^{th} Period

ARMA-GARCH;(1,1)				
	STD	SSTD	GED	SGED
mu				
ar1	-0.470588*	-0.444573*	-0.538390*	-0.484553
ma1	0.282649*	0.205812*	0.292484*	0.212539
omega	0.000003*	0.000002*	0.000003*	0.000003*
alpha1	0.412271	0.359162	0.423400	0.385026
beta1	0.586729	0.639838	0.575600	0.613975
gamma1				
vxreg1				
vxreg2				
skew		0.779737		0.814525
shape	6.06489*	5.855935*	1.236740	1.188510
AIC	-6.3696	-6.3880	-6.3852	-6.4127
BIC	-6.1935	-6.1825	-6.2091	-6.2072
MSE;IN	0.14875860	0.14833450	0.14508360	0.14609310
MSE;OUT	0.00357372	0.00361229	0.00369898	0.00348629

ARMA-EGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000782	-0.000061	0.000928	0.000793
ar1	-0.359473*	-0.733801	-0.296765*	-0.532507
ma1	0.162307*	0.474489	0.137876*	0.301529
omega	0.085881*	0.050911	0.09107*	-0.226603
alpha1	-0.407780*	-0.697684	-0.404266*	-0.385751
beta1	1.000000	0.998691	1.000000	0.980700
gamma1	0.860608	0.376776	0.882071	0.687411
vxreg1	59.380083*	43.15457	60.724364*	13.638647
vxreg2	3.373975	2.200247	3.283523	2.039898
skew		0.038742		0.011263
shape	99.999962*	59.704940	2.605109	3.165210
AIC	-6.5735	-6.6992	-6.5799	-6.6633
BIC	-6.2800	-6.3763	-6.2864	-6.3405
MSE;IN	0.22090630	0.20616440	0.19958660	0.14394140
MSE;OUT	0.03343034	0.03020727	0.02143546	0.01010663

ARMA-GJRGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu		0.000000*	0.000220	0.000033
ar1		-0.506481	-0.548307	-0.547493
ma1		0.410446	0.449266	0.399417
omega		0.000001*	0.000001	0.000001
alpha1		0.109998	0.136568	0.069476
beta1		0.546429	0.479581	0.586107
gamma1		0.999858	0.999815	0.999924
vxreg1		0.000000*	0.000000*	0.000000*
vxreg2		0.000034	0.000030	0.000029
skew		0.869991		0.818690
shape		12.34336*	2.297130	2.186770
AIC		-6.5530	-6.5073	-6.5004
BIC		-6.2301	-6.2138	-6.1775
MSE;IN		0.30186970	0.28056080	0.28264690
MSE;OUT		NA	0.00617808	0.00579295

ARMA-TGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu		0.000378	0.000185*	0.000895
ar1		-0.281660	-0.666232	0.063049
ma1		-0.014065	0.618175	-0.371621
omega		0.000000*	0.000165	0.000000*
alpha1		0.178603	0.182375	0.181667
beta1		0.865784	0.827328	0.843749
eta11		0.999999	0.599453*	0.171206
vxreg1		0.000000*	0.000000*	0.000000*
vxreg2		0.010185	0.014068*	0.016717
skew		0.026620		0.033020
shape		48.727890	49.99999*	4.791580
AIC		-6.7421	-6.7728	-6.7626
BIC		-6.4192	-6.4793	-6.4398
MSE;IN		0.24488640	0.15675290	0.14598850
MSE;OUT		0.00341016	0.00713580	0.00458788

Note 1: contrary to the text for simplification * means not significant at 5% level
Note 2: [1, 2,3, 4] represent 150, 200, 250 and 300 observations used in window size

Annex C – Nikkei 225

All Sample

ARMA-GARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000269	0.000186	0.000244	
ar1	0.842922	0.817534	-0.017698*	
ma1	-0.857430	-0.840619	-0.005249*	
omega	0.000001*	0.000001*	0.000001*	
alpha1	0.090367	0.088873	0.097991	
beta1	0.899349	0.899968	0.889247	
gamma1				
vxreg1				
vxreg2				
skew		0.915364		
shape	7.552878	8.046767	1.401526	
AIC	-7.5065	-7.5101	-7.5075	
BIC	-7.4980	-7.5004	-7.4990	
MSE;IN	0.01166827	0.01165305	0.01164267	
MSE;OUT	0.00337807	0.00340371	0.00326825	

ARMA-EGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000044*	0.000003*	0.000048*	-0.000005*
ar1	0.200868	0.334299	0.248954	0.346051
ma1	-0.254660	-0.396457	-0.298756	-0.406964
omega	-0.197398	-0.208145	-0.197509	-0.208910
alpha1	-0.091606	-0.091907	-0.090365	-0.091291
beta1	0.981189	0.980006	0.981162	0.979891
gamma1	0.133181	0.135343	0.135194	0.136454
vxreg1	-0.081927*	0.339163*	0.453610*	0.900487*
vxreg2	1.960568	1.968228	1.914865	1.935114
skew		0.879013		0.881182
shape	17.816831	20.538020	1.703783	1.737922
AIC	-7.6080	-7.6155	-7.6079	-7.6157
BIC	-7.5959	-7.6022	-7.5959	-7.6025
MSE;IN	0.01078091	0.01078186	0.01078914	0.01079135
MSE;OUT	0.00694721	0.00536293	0.00640491	0.00634504

ARMA-GJRGARCH;(1,1)				
	STD[2]	SSTD[2]	GED[1]	SGED[1]
mu	0.000067	0.000031*	0.000097	-0.000009
ar1	-0.489463	0.145786	0.195183	-0.076605
ma1	0.483540	-0.183743	-0.229041	0.039197
omega	0.000001	0.000001	0.000001	0.000001
alpha1	0.051530	0.033277	0.038817	0.039573
beta1	0.856536	0.896126	0.888151	0.887048
gamma1	0.135262	0.099611	0.091295	0.096575
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.000024	0.000024	0.000024	0.000025
skew		0.909978		0.901504
shape	11.797490	12.112850	1.586741	1.648032
AIC	-7.5651	-7.5742	-7.5687	-7.5757
BIC	-7.5530	-7.5609	-7.5566	-7.5624
MSE;IN	0.01107152	0.01111046	0.01109601	0.01109550
MSE;OUT	0.00318706	0.00317187	0.00314899	0.00313255

ARMA-TGARCH;(1,1)				
	STD[1]	SSTD[1]	GED	SGED
mu	0.000064*	-0.000008*	0.000069*	-0.000006*
ar1	-0.077750*	0.035896*	-0.106910	-0.019876*
ma1	0.037576*	-0.086225*	0.067056*	-0.026698*
omega	0.000138	0.000150	0.000140	0.000153
alpha1	0.079562	0.080661	0.080641	0.081629
beta1	0.913152	0.918481	0.911973	0.909289
eta11	0.611212	0.682021	0.588290	0.603540
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.003437	0.003579	0.003338	0.003322
skew		0.885812		0.894592
shape	13.551550	15.218260	1.634382	1.663745
AIC	-7.5882	-7.5966	-7.5883	-7.5946
BIC	-7.5761	-7.5833	-7.5762	-7.5813
MSE;IN	0.01099551	0.01096750	0.01101953	0.01099435
MSE;OUT	0.00255923	0.00252046	0.00254875	0.00255212

1st Period

ARMA-GARCH;(1,1)				
	STD	SSTD	GED	SGED
mu				
ar1	0.834471	0.829529	0.844273	0.839202
ma1	-0.859650	-0.854012	-0.868446	-0.863118
omega	0.000002*	0.000002*	0.000002*	0.000002*
alpha1	0.065155	0.064210	0.067868*	0.067207
beta1	0.892666	0.894360	0.887094	0.886938
gamma1				
vxreg1				
vxreg2				
skew		1.027608		1.031270
shape	9.641180	9.725265	1.521453	1.516007
AIC	-7.1673	-7.1660	-7.1654	-7.1643
BIC	-7.1421	-7.1366	-7.1401	-7.1348
MSE;IN	0.00769623	0.00769664	0.00769576	0.00769527
MSE;OUT	NA	NA	0.00169459	0.00170035

ARMA-EGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	-0.000333	-0.000354	-0.000346	-0.000374
ar1	0.642101	0.635600	0.661654	0.648514*
ma1	-0.700264	-0.695850	-0.715594	-0.705360*
omega	-0.256712	-0.254619	-0.300870	-0.283713
alpha1	-0.049518	-0.050014	-0.048839	-0.049332
beta1	0.975290	0.975441	0.971062	0.972637
gamma1	0.002709*	0.002706*	0.010399	0.007290
vxreg1	-9.432042	-8.918602	-11.659915	-10.30846
vxreg2	1.335728	1.360345	1.312293	1.341351
skew		0.962766		0.957226
shape	16.979856	17.436662	1.720645	1.728274
AIC	-7.2255	-7.2245	-7.2231	-7.2224
BIC	-7.1834	-7.1782	-7.1810	-7.1761
MSE;IN	0.00721543	0.00721548	0.00721899	0.00721769
MSE;OUT	0.00636080	0.00474489	0.00543065	0.00573592

ARMA-GJRGARCH;(1,1)				
	STD	SSTD	GED[3]	SGED[3]
mu	-0.000286*	-0.000292*	-0.000292	-0.000331
ar1	0.808141	0.803242	0.826099	0.778730
ma1	-0.841182	-0.836702	-0.857733	-0.814369
omega	0.000003	0.000003	0.000003	0.000002
alpha1	0.020852	0.020911	0.021711	0.010557
beta1	0.879607	0.879371	0.875253	0.903132
gamma1	0.079928	0.079991	0.081656	0.081382
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.000029	0.000029	0.000030	0.000033
skew		0.992901		0.986040
shape	14.281410	14.317770	1.650946	1.658021
AIC	-7.1996	-7.1980	-7.1987	-7.2011
BIC	-7.1575	-7.1517	-7.1566	-7.1548
MSE;IN	0.00748049	0.00748043	0.00748007	0.00745888
MSE;OUT	NA	NA	0.00156828	0.00155924

ARMA-TGARCH;(1,1)				
	STD[1]	SSTD[1]	GED[1]	SGED[2]
mu	-0.000312	-0.000322*	-0.000320	-0.000335
ar1	0.718907	0.714294	0.744586	0.734888
ma1	-0.758789	-0.754646	-0.780765	-0.771730
omega	0.000210	0.000213	0.000224	0.000224
alpha1	0.034499	0.034556	0.036503	0.035729
beta1	0.941801	0.941379	0.938201	0.938954
eta11	0.999994	0.999985	0.978621	0.997189
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.003369	0.003385	0.003338	0.003359
skew		0.984324		0.981134
shape	15.111230	15.101420	1.677619	1.678375
AIC	-7.2120	-7.2105	-7.2103	-7.2089
BIC	-7.1699	-7.1642	-7.1682	-7.1626
MSE;IN	0.00733828	0.00733788	0.00734041	0.00733954
MSE;OUT	0.00121411	0.00120061	0.00121990	0.00123813

2^{nd} Period

ARMA-GARCH;(1,1)				
	STD	SSTD	GED	SGED
mu				
ar1	0.246123*	0.756824	0.29532*	0.749406
ma1	-0.248390*	-0.772777	-0.302754*	-0.763962
omega	0.000000*	0.000000*	0.000000*	0.000000*
alpha1	0.059924	0.059746	0.061374	0.061237
beta1	0.932156	0.932336	0.930097	0.930292
gamma1				
vxreg1				
vxreg2				
skew		0.893081		0.899786
shape	13.831050	16.123230	1.601839	1.659251
AIC	-7.7576	-7.7617	-7.7593	-7.7630
BIC	-7.7273	-7.7263	-7.7289	-7.7276
MSE;IN	0.00208691	0.00208622	0.00208689	0.00208642
MSE;OUT	NA	0.00012650	0.00012866	0.00012887

ARMA-EGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000238	0.000202	0.000222*	0.000189
ar1	0.645005	0.663468	0.562825	0.636670
ma1	-0.663267	-0.695520	-0.585410	-0.668459
omega	-0.158382	-0.169100	-0.152352	-0.169067
alpha1	-0.068080	-0.059510	-0.068644	-0.062854
beta1	0.984036	0.983097	0.984619	0.983099
gamma1	0.159448	0.157625	0.159749	0.158338
vxreg1	14.059407*	11.432422*	14.747098*	12.368189
vxreg2	1.087870	1.144313	1.114746	1.159257
skew		0.870019		0.880910
shape	23.689159	58.394945*	1.710566	1.809791
AIC	-7.7886	-7.7949	-7.7909	-7.7962
BIC	-7.7380	-7.7393	-7.7404	-7.7407
MSE;IN	0.00202163	0.00202646	0.00202304	0.00202623
MSE;OUT	0.00011346	0.00012292	0.00010128	106.46690000

ARMA-GJRGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000302	0.000257	0.000263	0.000219*
ar1	0.853057	0.778619	-0.734645	-0.795336
ma1	-0.867711	-0.806284	0.742769	0.804484
omega	0.000000*	0.000000*	0.000000	0.000000*
alpha1	0.046713	0.046698	0.041488	0.041502
beta1	0.932351	0.935444	0.930973	0.931085
gamma1	0.022071*	0.017534	0.033771	0.032331
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.000014	0.000014	0.000013	0.000014
skew		0.878910		0.897119
shape	18.65461*	27.09627*	1.686173	1.773813
AIC	-7.7768	-7.7820	-7.7779	-7.7818
BIC	-7.7263	-7.7265	-7.7274	-7.7262
MSE;IN	0.00204447	0.00204430	0.00204419	0.00204335
MSE;OUT	NA	NA	0.00017790	0.00017689

ARMA-TGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000260	0.000232	0.000245*	0.000218*
ar1	0.811592	0.766388	0.787368	0.752296
ma1	-0.834018	-0.800705	-0.810988	-0.786129
omega	0.000062*	0.000060*	0.000063	0.000062
alpha1	0.072872	0.070613	0.073454	0.071199
beta1	0.930329	0.932417	0.929825	0.931767
eta11	0.201878*	0.172782*	0.209536*	0.185655*
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.002213	0.002297	0.002259	0.002320
skew		0.868985		0.878801
shape	23.71653*	50.61331*	1.707654	1.817088
AIC	-7.7833	-7.7899	-7.7856	-7.9010
BIC	-7.7328	-7.7343	-7.7350	-7.7355
MSE;IN	0.00202427	0.00202491	0.00202360	0.00202411
MSE;OUT	0.00010684	0.00011323	0.00012868	0.00012131

3^{rd} Period

ARMA-GARCH;(1,1)				
	STD	SSTD	GED	SGED
mu				
ar1	-0.918600	0.727893	-0.915414	-0.249342*
ma1	0.885351	-0.767648	0.881690	0.190124*
omega	0.000001*	0.000001*	0.000001*	0.000001*
alpha1	0.119698	0.117331	0.123961	0.123001
beta1	0.864872	0.868232	0.859568	0.861453
gamma1				
vxreg1				
vxreg2				
skew		0.849930		0.865338
shape	13.486250	19.24261*	1.551033	1.621052
AIC	-7.1321	-7.1385	-7.1380	-7.1441
BIC	-7.1017	-7.1029	-7.1075	-7.1085
MSE;IN	0.03305767	0.03248524	0.03298266	0.03274978
MSE;OUT	NA	NA	0.00034897	0.00036432

ARMA-EGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	-0.000174*	-0.000214*	-0.000165*	-0.000217*
ar1	-0.074524*	-0.145471	-0.077622*	-0.139982
ma1	-0.011106*	0.056122*	-0.006952*	0.050459*
omega	-0.214550	-0.188740	-0.216007	-0.188620
alpha1	-0.110042	-0.127684	-0.109504	-0.127913
beta1	0.979268	0.980985	0.979193	0.980964
gamma1	0.153364	0.152853	0.154376	0.154105
vxreg1	-3.247077*	2.085984*	-3.362396*	2.356546*
vxreg2	1.841461	1.709720	1.846224	1.709189
skew		0.840975		0.841122
shape	99.939461*	59.995484*	1.920580	1.948068
AIC	-7.2052	-7.2161	-7.2060	-7.2176
BIC	-7.1545	-7.1603	-7.1552	-7.1617
MSE;IN	0.03110237	0.03096622	0.03111725	0.03097760
MSE;OUT	0.00056215	0.00058850	0.00055728	0.00046916

ARMA-GJRGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu			-0.000079*	-0.000168
ar1			-0.906845	-0.934781
ma1			0.870139	0.896676
omega			0.000001	0.000001
alpha1			0.029398	0.030163
beta1			0.887145	0.884561
gamma1			0.133104	0.141248
vxreg1			0.000000*	0.000000*
vxreg2			0.000033	0.000033
skew				0.825762
shape			1.752579	1.831904
AIC			-7.1837	-7.1994
BIC			-7.1329	-7.1435
MSE;IN			0.03160615	0.03163360
MSE;OUT			0.00028809	0.00029123

ARMA-TGARCH;(1,1)				
	STD[4]	SSTD[4]	GED	SGED
mu	-0.000208*	-0.000242*	-0.000152*	-0.000248*
ar1	-0.896752	-0.919447	-0.913520	-0.918084
ma1	0.857196	0.875203	0.881584	0.873998
omega	0.000142	0.000155	0.000160	0.000166
alpha1	0.078154	0.081676	0.074436	0.084703
beta1	0.917263	0.912950	0.918800	0.912679
eta11	0.774154	0.801327	0.899245	0.817095
vxreg1	0.000000*	0.000000*	0.000000*	0.016906*
vxreg2	0.004669	0.004680	0.000000*	0.004652
skew		0.821178		
shape	75.94142*	59.99875*	1.695823	1.905082
AIC	-7.2073	-7.2228	-7.1610	-7.2241
BIC	-7.1565	-7.1669	-7.1102	-7.1682
MSE;IN	0.03157507	0.03130710	0.03236097	0.03125009
MSE;OUT	0.00039609	0.00039608	0.00061144	0.00068129

Note 1: contrary to the text for simplification * means not significant at 5% level
Note 2: [1, 2,3, 4] represent 150, 200, 250 and 300 observations used in window size

4th Period

ARMA-GARCH;(1,1)				
	STD	SSTD	GED	SGED
mu				
ar1	-0.316280*		-0.378375	0.764286
ma1	0.303397*		0.356969	-0.789633
omega	0.000001*		0.000001*	0.000001*
alpha1	0.119379		0.124869	0.121964*
beta1	0.850507		0.841569	0.846555
gamma1				
vxreg1				
vxreg2				
skew				0.877242
shape	5.703139		1.285328	1.308304
AIC	-7.7458		-7.7461	-7.7571
BIC	-7.7302		-7.7305	-7.7389
MSE;IN	0.00799739		0.00798584	0.00797348
MSE;OUT	0.00068050		0.00068144	0.00069710

ARMA-EGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	0.000142*	0.000113*	0.000139*	0.00011*
ar1	-0.134821	0.058492*	-0.136034	0.071759*
ma1	0.070949*	-0.131841	0.071238*	-0.146777*
omega	-0.562053	-0.543820	-0.542971	-0.530610
alpha1	-0.127357	-0.126956	-0.120552	-0.121556
beta1	0.949645	0.951035	0.951420	0.952276
gamma1	0.184661	0.186077	0.183072	0.184674
vxreg1	-16.155240	-15.103280	-16.192629*	-15.12206*
vxreg2	3.059586	3.041037	3.044794	3.032931
skew		0.861979		0.864142
shape	20.444970	23.684510	1.788353	1.807708
AIC	-7.9360	-7.9448	-7.9351	-7.9444
BIC	-7.9100	-7.9162	-7.9091	-7.9158
MSE;IN	0.00556549	0.00558337	0.00557422	0.00558933
MSE;OUT	0.00012358	0.00084732	48382.94000000	0.00017031

ARMA-GJRGARCH;(1,1)				
	STD	SSTD	GED[1]	SGED
mu	0.000254	0.000126	0.000203	0.000012*
ar1	-0.962732	-0.541672	-0.162668	0.144971
ma1	0.974059	0.511392	0.123011	-0.189349
omega	0.000002	0.000001	0.000001	0.000002
alpha1	0.003699	0.025698	0.026883	0.030768
beta1	0.795811	0.847708	0.840690	0.805135
gamma1	0.219156	0.120294	0.130288	0.175653
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.000028	0.000030	0.000032	0.000033
skew		0.872838		0.853659
shape	9.459147	8.555528	1.567371	1.614721
AIC	-7.8585	-7.8701	-7.8654	-7.8764
BIC	-7.8325	-7.8414	-7.8393	-7.8478
MSE;IN	0.00748232	0.00750263	0.00747656	0.00748694
MSE;OUT	0.00068503	NA	0.00068473	0.00068727

ARMA-TGARCH;(1,1)				
	STD	SSTD	GED	SGED[1]
mu	0.000153*	-0.000299	0.000085*	-0.000354*
ar1	-0.220324	0.993125	-0.321844	0.992808
ma1	0.179821	-0.982896	0.271668	-0.981643
omega	0.000290	0.000258	0.000410	0.000246
alpha1	0.094796	0.092117	0.118456	0.089367
beta1	0.865991	0.879082	0.824533	0.883755
eta11	0.802972	0.999997	0.678286	0.999998
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.004957	0.005097	0.005831	0.005096
skew		0.831087		0.831962
shape	15.393320	21.870840	1.731681	1.798812
AIC	-7.9094	-7.9241	-7.9110	-7.9231
BIC	-7.8833	-7.8955	-7.8849	-7.8945
MSE;IN	0.00696695	0.00689187	0.00685373	0.00690081
MSE;OUT	0.00035224	0.00032255	0.00029837	0.00034519

5th Period

ARMA-GARCH;(1,1)				
	STD	SSTD	GED	SGED
mu				
ar1	0.627027*	0.641303*	0.57608*	0.664856
ma1	-0.533218*	-0.549339*	-0.482395*	-0.582294
omega	0.000005*	0.000006*	0.000005*	0.000008*
alpha1	0.249584*	0.258606	0.242062	0.266287
beta1	0.725173	0.711583	0.724131	0.690593
gamma1				
vxreg1				
vxreg2				
skew		1.073501		1.168291
shape	9.883278*	9.036933*	1.656492	1.434292
AIC	-6.4617	-6.4377	-6.4600	-6.4412
BIC	-6.2790	-6.2246	-6.2773	-6.2281
MSE;IN	0.03538710	0.03529474	0.03517470	0.03500673
MSE;OUT	0.00318558	0.00320912	0.00318508	0.00322642

ARMA-EGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	-0.001312	-0.000826	-0.000899	-0.000787
ar1	-0.814137	0.936386	0.939009	0.940765
ma1	0.777123	-0.979378	-0.993569	-0.999281
omega	-0.024633	-0.058702	-0.051132	-0.037276
alpha1	-0.283945	-0.208011	-0.213519	-0.21036
beta1	0.999922	0.999607	0.998691	0.999957
gamma1	-0.292391	-0.351857	-0.281924	-0.144276
vxreg1	1.21267	-1.160724	2.209508	1.082589
vxreg2	3.118438	2.998701	3.262722	2.977659
skew		1.207783		0.079789
shape	18.108966	23.340585	3.500456	4.735443
AIC	-6.6917	-6.6907	-6.7861	-6.8220
BIC	-6.3873	-6.3559	-6.4817	-6.4871
MSE;IN	0.14755700	0.08856869	0.07721327	0.05577769
MSE;OUT	0.00386048	0.01302616	0.00269071	0.00467736

ARMA-GJRGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	-0.00081	-0.000306	-0.000943	-0.001634
ar1	-0.943604	-0.944440	-0.938319	-0.139228
ma1	0.863061	0.872802	0.844510	0.207823
omega	0.000001*	0.000002*	0.000001*	0.000000*
alpha1	0.000023*	0.000033*	0.000024*	0.000051*
beta1	0.844100	0.862756	0.858012	0.917349
gamma1	0.316436	0.210360	0.276016	0.192434
vxreg1	0.000000*	0.000000*	0.000000*	0.000000*
vxreg2	0.000074	0.000087	0.000078	0.000096
skew		1.393636		2.336434
shape	51.57282*	12.776004*	2.834993	5.478152
AIC	-6.5846	-6.5925	-6.5978	-6.5829
BIC	-6.2802	-6.2577	-6.2934	-6.2480
MSE;IN	0.03633069	0.03411793	0.03502890	0.03294562
MSE;OUT	NA	NA	NA	NA

ARMA-TGARCH;(1,1)				
	STD	SSTD	GED	SGED
mu	-0.000663		-0.000618	-0.000639
ar1	-0.973286		-0.969553	-0.971341
ma1	0.859678		0.859713	0.858706
omega	0.000039		0.000004*	0.000017
alpha1	0.082349		0.067595	0.075861
beta1	0.927085		0.936901	0.927267
eta11	0.999999		0.999999	0.999998
vxreg1	0.000000*		0.000000*	0.000000*
vxreg2	0.009691		0.009509	0.009256
skew				1.409316
shape	24.35209*		3.661645	3.873971
AIC	-6.8252		-6.9918	-6.9517
BIC	-6.5208		-6.6874	-6.6169
MSE;IN	0.03696421		0.03352957	0.03440573
MSE;OUT	NA		0.00404101	NA

Note 1: contrary to the text for simplification * means not significant at 5% level
Note 2: [1, 2,3, 4] represent 150, 200, 250 and 300 observations used in window size