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A comparative evaluation of VaR models using Monte Carlo simulations

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Master in Finance

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Department of Finance

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I hope that I am making you proud, “Avô Nelson”

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Resumo

Os modelos de regressão de quantis surgem como uma metodologia *Value-at-Risk* (VaR) alternativa que não requer nenhum pressuposto específico quanto à distribuição dos retornos. Esta dissertação descreve e testa um modelo recente, proposto por Zheng et al. (2018), para estimação do VaR através da regressão de quantis e introduz uma transformação não trivial que permite o uso de modelos *Generalized Autoregressive Conditional Heteroskedasticity* (GARCH). O estudo desenvolvido por este investigador apresenta resultados promissores relativamente ao uso desta abordagem de estimação do quantil condicional para um modelo GARCH(1,1). Testamos este novo modelo comparando-o com um grupo de *benchmarks* compostos por metodologias VaR tradicionais e outros modelos VaR de regressão de quantis. De modo a avaliar o desempenho deste novo modelo VaR, geramos retornos através de simulações de Monte Carlo que seguem um processo GARCH(1,1) idêntico ao que foi utilizado por Zheng et al. (2018). Depois, mudamos os parâmetros do processo gerador de retornos para, na nossa opinião, suposições mais realistas quanto à volatilidade diária no longo prazo. Confirmamos a superioridade do desempenho deste novo modelo quando os parâmetros do processo gerador de retornos é o mesmo do que o que foi definido por Zheng et al. (2018), no entanto, o mesmo não acontece quando utilizamos parâmetros mais realistas. Os novos resultados mostram que a parametrização de Zheng et al. (2018) penaliza bastante o desempenho dos *benchmarks*.

Palavras-chave: Regressão de Quantis, Value-at-Risk, GARCH, Simulações de Monte Carlo

Classificação JEL: C32, G32.

Abstract

Quantile regression models emerges as an alternative Value-at-Risk (VaR) methodology that does not require any specific distribution assumption. This dissertation describes and tests a recent quantile regression VaR model that introduces a nontrivial transformation enabling the use of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) volatility models, proposed by Zheng et al. (2018). His study has shown that this approach to the conditional quantile estimation for a GARCH(1,1) model provides promising results. We test this new model against a group of benchmarks composed by traditional VaR methods and other quantile regression VaR models. In order to evaluate the performance of this new VaR model, we generate returns by Monte Carlo simulations following a GARCH(1,1) process similar to what was carried out by Zheng et al. (2018). After, we change the returns process parameters to, in our opinion, more realistic assumptions on the daily persistent volatility. We confirm the superior performance of the new proposed model when the return generating process is simulated with Zheng et al. (2018) parameters, however, the same does not happen when a more reasonable parametrization is simulated. New results show that the benchmark group is heavily penalized by Zheng et al. (2018) parametrization.

Keywords: Quantile Regression, Value-at-Risk, GARCH, Monte Carlo Simulations

JEL Classification: C32, G32.

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List of Abbreviations

ARCH – Autoregressive Conditional Heteroskedasticity;
CAViaR – Conditional Autoregressive Value-at-Risk;
EWMA – Exponentially Weighted Moving Average;
GARCH – Generalized Autoregressive Conditional Heteroskedasticity;
LARCH – Linear Autoregressive Conditional Heteroskedasticity;
LGARCH – Linear Generalized Autoregressive Conditional Heteroskedasticity;
MSE – Mean Squared Error;
NRMSD – Normalized Root Mean Squared Deviation;
NoR – No Reestimation;
QMLE – Quasi-maximum Likelihood Estimate;
QAR – Quantile Autoregression;
SGT – Skewed Generalized T Student;
VaR – Value-at-Risk.

1 Introduction

Value-at-Risk (VaR) tells us how much the loss in a portfolio can be, over a given time horizon, for a given confidence level. It is a well established tool that allows us to simplify a very complex concept, the risk of a portfolio, into a monetary amount. This is one of the reasons that made VaR so popular. VaR has been widely used by financial institutions and regulators for many years and during that time several VaR models were developed. There is not a single model that can be defined as being the state-of-the-art since all VaR models have their strengths and weaknesses. Traditional VaR models can be classified into three main groups: Parametric VaR, Historical VaR and Monte Carlo VaR models. Parametric VaR models assume that portfolio returns follow a given distribution, usually the Normal or the Student's-T distribution, which is quite a strong assumption to make. Historical VaR models makes the weaker assumption that the empirical distribution observed in a given sample, whatever it may be, will remain the same in the future. Monte Carlo VaR models work with a chosen functional form for the stochastic process that generates returns and, with that, it is possible to estimate VaR based on simulations of series of portfolio returns. This can become computationally intense and requires expertise to correctly define the returns' generating process. Thus, the demand for a robust model that is able to produce accurate forecasts not only in normal market conditions but, specially, on turbulent times like the massive volatility increase that we experienced with the novel coronavirus world pandemic crisis, motivates the study of new types of models.

In the recent years quantile regression VaR models have gained popularity for their flexibility. These models estimate the VaR by running a quantile regression of the portfolio returns against some explanatory variables, which we are completely free to choose. Quantile regression VaR models have the advantage of not making any explicit distributional assumption regarding portfolio returns. There is still a lot of research to be done, but recent studies show that these models tend to perform better than the traditional VaR models (Guo, 2013). Motivated by the promising results, the lack of studies around this topic and the importance of a robust model to assess market risk, our main purpose will be to evaluate the performance of a new quantile regression VaR model proposed by Zheng et al. (2018), comparing it with other quantile regression specifications and with traditional VaR models. The model proposed by Zheng et al. (2018) estimates the conditional quantile of the portfolio returns with an hybrid quantile regression estimator for a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) process. The author introduces a transformation that allows the use of the Bollerslev (1986) GARCH(1,1) model, but which can be extended to any other model of the conditional variance. This transformation overcomes linearity technical difficulties on quantile regression estimation of conditional heteroskedastic models. One of his experiments have concluded that this model is superior to the well known RiskMetrics model.

Similar to Zheng et al. (2018) methodology, we simulate returns using Monte Carlo simulations with the author original GARCH simulated parameters and also with a different set of parameters that we believe to be more adjusted to model the daily market returns volatility. Additionally, because we can not be sure that financial returns follow heteroskedastic processes throughout all the time, we also simulate returns with a Skewed Generalized T Student (SGT) distribution with 4 different parametrizations. These parametrizations represent a Normal distribution, a symmetric distribution with heavy tails, a distribution with heavy tails and negative skewness and finally a distribution with heavy tails and positive skewness. We simulate 4500 daily returns and define a test window of 10 years of 1-day VaR estimations. VaRs are estimated at the 99%, 97.5% and 95% confidence levels. We run 1000 simulations for each return generating process. We evaluate the performance of all models by comparing the estimated VaR with the true VaR that can be obtained from the knowledge of the return generating process. With this procedure we avoid the use of backtesting methods that have some limitations and would introduce noise into the analysis. Hence, we compare the models accuracy across all simulations with the normalized root mean squared deviation (NRMSD).

We define two quantile regression models as benchmarks. The first uses only a volatility estimate as explanatory variable whereas the second adds a constant term to the former. Two different volatility estimation methods are considered, the exponentially weighted moving average (EWMA) and the equally weighted method over a rolling sample. For all these, two versions are also tested, one with reestimation of the quantile regression parameters every 20 days and another that does not reestimate parameters. Other benchmarks are considered from the traditional VaR models such as the Historical VaR, the Historical Volatility Adjusted VaR, the RiskMetrics VaR model (Parametric Normal with EWMA volatility estimates), the Parametric Normal with equally weighted volatility estimates over a rolling sample, the Parametric SGT EWMA and Parametric SGT with equally weighted volatility models. Finally, a different version of the GARCH quantile regression model is created by substituting the variance estimate by the true simulated variance. This way we can better understand if knowing the true volatility is a major improvement to the VaR estimates or even with that advantage the quantile regression model struggles to efficiently predict the conditional quantile.

The analysis of the NRMSD results from the simulation of the GARCH return generating process with the parametrization of Zheng et al. (2018) confirms the superiority of the quantile regression VaR model proposed by Zheng et al. (2018). However, using a more reasonable GARCH parametrization for the return generating process shows that this is no longer the case. As it turns out, the performance of the quantile regression VaR model proposed by Zheng et al. (2018) is weaker than that of several benchmark models under reasonable GARCH parametrizations for the return generating process, but is more robust to different parametrizations. The parametrization chosen by Zheng et al. (2018)

just happens to heavily penalize the performance of the benchmark models.

From the simulations of the SGT return generating process, the NRMSD results show that the quantile regression VaR model proposed by Zheng et al. (2018) has, once more, a robust performance but it is still not superior to the other quantile regression benchmarks tested. Regarding the benchmark group, the quantile regression VaR model with a constant and an EWMA estimate seems to perform similar to the quantile regression VaR model proposed by Zheng et al. (2018), with slightly superior results. Also, the quantile regression VaR model with the equally weighted volatility estimation method is robust to different simulations of the SGT return generating process. It is important to note that, in this homoskedastic return generating process, adding a constant term to the quantile regression VaR model with equally weighted volatility does not improve the performance of this model. On the contrary, adding this constant term to the quantile regression specification with an EWMA volatility has a significant performance improvement, when compared with the quantile regression VaR model with only an EWMA volatility estimate as explanatory variable. Overall, the analysis of the NRMSD results show that the reestimation of parameters has significant improvement on reducing the NRMSD results dispersion, specially when we consider the results from the simulation of the SGT return generating process.

The remainder of this dissertation is organized as follows: in Section 2 we summarize the existing literature regarding VaR models, including the Zheng et al. (2018) model; Section 3 we present the details regarding the simulations carried, the evaluation criteria and the different models tested; Section 4 we analyze the main NRMSD results statistics; Section 5 concludes.

2 Literature Review

We can find some debate about VaR's accuracy, the necessary probabilistic assumptions and the large variety of models (Beder, 1995). Despite that, VaR has been used by most of the financial institutions to measure market risk. We can find different examples of its use to assess the portfolio's financial exposure and, additionally, as a tool to establish minimum capital requirements (Federal Deposit Insurance Corporation, 1999; Basel Committee, 2004; European Commission, 2014). In recent years, the Basel Accords on banking regulation have been trying to improve VaR's accuracy (Chen, 2013).

The first application of Value-at-Risk was in the early 1990s, mainly due to the work of J.P. Morgan's internal risk management department, where it was being developed and used since the late 1980s (Holton, 2002). The h -day $Var_{h,\alpha}$, for a confidence level of $(100 - \alpha)\%$, where $\mathcal{F}_{t-1}$ represents the information available at time $t - 1$, can be mathematically defined as:

$$\Pr(r_t < -Var_{h,\alpha} | \mathcal{F}_{t-1}), \quad (1)$$

The RiskMetrics VaR model (Morgan J P and Reuters TM, 1996) assumes that returns are normally distributed. Additionally, the forecast for the conditional variance is made by using an exponentially decaying weighted average. This method is called EWMA (Exponentially Weighted Moving Average) and its goal is to more accurately capture the current market conditions in terms of return volatility. Towards that end, instead of giving an equal weight to all return observations, it gives more weight to the most recent ones. The major downside of this model is that it makes a distributional assumption (normality), and also sets a simplifying assumption by defining the decay factor as a predefined number, that may not be suitable for all portfolios. We find a broad number of scientific evidence that financial returns do not follow a normal distribution (Guo, 2013). It is well documented that equity returns often exhibit a leptokurtic distribution, hence with heavier tails than a normal distribution with the same variance. Also, the distribution of financial returns is usually asymmetric with negative skew. This is fundamentally explained by the fact that we can typically observe large extreme events often that what it would be expected in a normal distribution and, also, the probability is more concentrated on the left tail.

The RiskMetrics VaR model was the building block for other parametric methods. The main developments were made on the volatility specification, the use of alternative distribution assumptions that can capture skewness and kurtosis of financial returns and on higher-order conditional moments that vary over time (Abad, Benito, and López, 2014).

Another class of models are the non-parametric. The main one is the Historical VaR model that describes the future returns distribution as an approximation of the past empirical sample's distribution. Var_{α} is estimated as the α quantile. It relies a lot on

the sample chosen, particularly on the sample size that has to be large enough to allow statistical inference but cannot go outside of the volatility cluster (Dowd, 2007). Later, using the same principles with a different approach, the Historical VaR with Volatility Adjustment was proposed by Hull and White (1998) in an attempt to overcome the major difficulties of the previous model. The series of returns is refined with a technical adjustment that reflects the current market volatility and the α quantile is then estimated based on that series of volatility-adjusted returns.

Monte Carlo VaR models are very flexible models that allow us to simulate future returns based on past returns distribution, on predefined distributions or based on stochastic processes. Then, we can estimate VaR_α empirically, by using simulated distributions. It is specially useful for non-linear portfolios (Ammann and Reich, 2001). This flexibility has some drawbacks as it can be computationally intensive and we can be setting the wrong choices of the simulation process (Abad, Benito, and López, 2014).

Quantile regression VaR models were introduced by Koenker and Bassett (1978) and have attracted a lot of attention by researchers. If we consider the following linear model:

$$r_t = \beta'x_t + u_t, \quad (2)$$

Where u_t are i.i.d. innovations with mean 0, x_t is a vector of regressors and β is a vector of unknown parameters, then the τ^{th} quantile of r_t can be estimated as:

$$Q_\tau(r_t|x_t) = x_t'\hat{\beta}_\tau, \quad (3)$$

where

$$\hat{\beta}_\tau = \arg \min_{\beta} \sum_t (r_t - x_t'\beta) (\tau - I_{r_t - x_t'\beta < 0}) \quad (4)$$

and I_z is an indicator function of event, assuming a value of 1 if z is true and 0 otherwise. The process above consists on minimizing the tilted deviations.

This allowed a robust approach to estimate models for conditional quantiles likewise linear regression methods for conditional mean. Portnoy (1991) studied the asymptotic properties that allowed non-stationary models and Koul and Saleh (1995) extended the work of Koenker and Bassett (1978) to autoregression models. Furthermore, an important extension to ARCH models appeared in Koenker and Zhao (1996) and more recently an extension to GARCH models in Xiao and Koenker (2009). Based on the fact that Value-at-Risk is a quantile of future portfolio returns conditional on the information known today, Robert F Engle and Manganelli (1999) proposed a new type of models that do not require strong assumptions (e.g. normality and i.i.d. returns). The Conditional Autoregressive Value-at-Risk (CAViaR) is based on the fact that volatility tends to cluster over time and, following the same reasoning of Robert F Engle and Manganelli (1999), it means

that the distribution of returns tend to be autocorrelated, hence the VaR should exhibit a similar behavior. A Quantile Autoregression (QAR) model was studied by Koenker and Xiao (2006). They considered a linear quantile autoregression model whose autoregressive parameters may vary depending on the quantile chosen.

One of the most important tasks when estimating VaR is to forecast the volatility. Modeling volatility on financial markets have been an extensive field explored by researchers (see Assaf, 2017). Two general approaches are usually taken, the implied volatility methods that use options to extract the implied volatility and historical data. The first approach has been widely studied, for example, by using volatility indices. Blair, Poon, and Taylor (2001) have studied the accuracy of VIX forecasting the volatility of S&P 100, that proven to be the most accurate in the out-of-sample analysis. Regarding the second approach, the most important models belong to the ARCH/GARCH family.

Robert F. Engle (1982) was the first to introduce time series models with conditional heteroskedasticity, the autoregressive conditional heteroskedasticity (ARCH) model where the recent past could be used to forecast the future variance. The $ARCH_q$ model can be written as:

$$\begin{aligned} r_t &= x_t\beta + u_t, \\ u_t &= \sigma_t\epsilon_t, \end{aligned} \tag{5}$$

$$\sigma_t^2 = a_0 + \sum_{i=1}^q \alpha_i u_{t-i}^2, \tag{6}$$

where r_t is the dependent variable, x_t is a vector with explanatory variables, β is a vector with unknown parameters and ϵ_t is a sequence of i.i.d. innovations with mean zero and variance one. Also, to guarantee that unconditional and conditional variances (σ_t^2) are both positive, the restrictions $\alpha_0 > 0$ and $\alpha_i \geq 0$, ($i = 1, 2, \dots, q$) are needed. Furthermore, a more parsimonious specification appeared, the generalized autoregressive conditional heteroskedasticity (GARCH) model (Bollerslev, 1986). The $GARCH_q$ model can be represented as:

$$\begin{aligned} r_t &= x_t\beta + u_t, \\ u_t &= \sigma_t\epsilon_t, \\ \sigma_t^2 &= a_0 + \sum_{i=1}^q \alpha_i u_{t-i}^2 + \sum_{i=1}^p \delta_i \sigma_{t-i}^2. \end{aligned} \tag{7}$$

These models are frequently used in asset pricing and financial risk management. However, the conditional quantile of r_t depends on a linear relationship with the volatility, as:

$$Q_\tau(r_t|F_{t-1}) = \sigma_{t-j}Q_{\tau,\epsilon} \quad (8)$$

and traditional GARCH models provide a linear equation to the conditional variance, which means that the standard deviation (being the square root of the variance) will not have a linear relationship with the explanatory variables. Hence, quantile regression for GARCH models faces some technical difficulties. To tackle this problem, Koenker and Zhao (1996) considered the $LARCH_q$ model. A linear ARCH model, where:

$$\sigma_t = a_0 + \sum_{i=1}^q \alpha_i |u_{t-i}|. \quad (9)$$

The conditional quantile is estimated through a linear quantile regression.

Additionally, Xiao and Koenker (2009) proposed a two-step approach for quantile regression with linear GARCH time series (LGARCH). The conditional τ^{th} quantile can be represented as:

$$\begin{aligned} r_t &= \sigma_t \epsilon_t \\ \sigma_t &= a_0 + \sum_{i=1}^q \alpha_i |r_{t-i}| + \sum_{i=1}^p \delta_i \sigma_{t-i} \end{aligned}$$

$$Q_\tau(r_t|\mathcal{F}_{t-1}) = \sigma_t Q_{\tau,\epsilon} \quad (10)$$

with ϵ_t being a sequence of i.i.d. innovations with mean zero and variance 1 and $Q_{\tau,\epsilon}$ the τ^{th} quantile of ϵ_t . The first step is to set some initial estimates for unobservable σ_{t-i} 's and finally estimate the linear quantile regression. Recently, Guo (2013) estimated VaRs of five international equity indices (U.S. S&P Composite Index, Japanese Nikkei 225 Index, U.K. FTSE 100 Index and the Hong Kong Hang Sheng Index). This study used weekly returns from September 1976 to August 1999 and the quantile regression method approach of Koenker and Bassett (1978) with AR-ARCH models. Similarly, they defined:

$$r_t = \alpha' x_t + u_t,$$

$$u_t = \sigma_t \epsilon_t,$$

$$\sigma_t = \gamma_0 + \sum_{i=1}^q \gamma_i |u_{t-i}|, \quad (11)$$

with $(\gamma_0 \dots \gamma_q) > 0$. Assuming that ϵ_t have a general distribution D , including commonly used distributions and $r_t = \alpha_0 + \sum_{i=1}^s \alpha_i r_{t-i} + u_t$ where s is the maximum lag used. So, VaR_t at τ -percent level is:

$$-VaR_{t,\tau} = x_t \beta + \gamma'_\tau Z_t, \quad (12)$$

where $Z_t = (1, |u_{t-1}|, \dots, |u_{t-q}|)'$ and $(\gamma_0 \dots \gamma_q) D_\tau^{-1}$. Also, γ_τ is estimated by the following estimator:

$$\hat{\gamma}_\tau = \arg \min_{\gamma} \sum_t (u_t - Z'_t \gamma) (\tau - I_{u_t - Z'_t \gamma < 0}). \quad (13)$$

Sequential tests are conducted to choose the optimal lags for the AR-ARCH processes. Then, a performance comparison with VaR by RiskMetrics and VaR by GARCH Normality Assumption (asymmetric GARCH with the conditionally normal return distribution assumption) reveals that the VaR by Quantile Regression is more robust than RiskMetrics. Also, for the 5% VaR of the S&P 500 Index the quantile regression approach seems to produce higher VaRs than the asymmetric GARCH, during low volatility periods. On high volatility periods, the asymmetric GARCH method produced much higher VaRs. The authors considered that the quantile regression approach generated more reasonable VaRs at that high volatility period.

There is accumulated evidence that GARCH models can capture the persistent influence of long past shocks better than ARCH models (Xiao and Koenker, 2009). Motivated by that, an hybrid quantile regression estimation for time series with conditional heteroskedasticity was introduced by Zheng et al. (2018). The main advance is related to a transformation $T : \mathbb{R} \rightarrow \mathbb{R}$ for the conditional quantile:

$$T(x) = x^2 \text{sgn}(x) \quad (14)$$

where $\text{sgn}(x)$ is the sign function

$$\text{sgn}(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \\ 0 & \text{otherwise} \end{cases}$$

and it works as the inverse of the square-root function as it is also continuous and non decreasing on $\mathbb{R}$. This transformation allows the use of a large variety of GARCH models which are linear models for the conditional variance, rather than to be stuck with linear models for the conditional standard deviation. The drawback of this transformation is that the conditional mean must be 0.

The hybrid conditional quantile estimation procedure starts with the estimation of

GARCH parameters via Gaussian quasi-maximum likelihood estimator (QMLE), using a sample average to compute the initial variance estimates. After that, they first estimate $Q_\tau(r_t|\mathcal{F}_{t-1})$ by estimating $Q_\tau(T(r_t)|\mathcal{F}_{t-1})$:

$$Q_\tau(T(r_t)|\mathcal{F}_{t-1}) = \left(a_0 + \sum_{i=1}^q \alpha_i |r_{t-i}^2| + \sum_{j=1}^p \delta_j \sigma_{t-j}^2 \right) T(Q_{\tau,\epsilon}) = \theta'_\tau x_t, \quad (15)$$

$$x_t = \begin{bmatrix} 1 & r_{t-1}^2 & \dots & r_{t-q}^2 & \sigma_{t-1}^2 & \dots & \sigma_{t-p}^2 \end{bmatrix}'$$

$$\theta_\tau = T(Q_{\tau,\epsilon}) \begin{bmatrix} a_0 & a_1 & \dots & a_q & \delta_1 & \dots & \delta_p \end{bmatrix}.$$

Zheng et al. (2018) also proposes a more efficient technique to estimate θ_τ , a method to minimize the sum of the weighted tilted deviations. So, if $T(r_t) = y_t$ we have:

$$\hat{\theta}_\tau = \arg \min_{\theta} \sum_{t=1}^n \frac{1}{\hat{\sigma}_t^2} (y_t - \theta' x_t) (\tau - I_{y_t - \theta' x_t < 0}), \quad (16)$$

Then, relying on the monotonicity of the transformation, by applying the inverse transformation $T^{-1}(x) = \sqrt{|x|} \text{sgn}(x)$ to $Q_\tau(y_t|\mathcal{F}_{t-1})$, the author estimates the τ conditional quantile of r_t . Finally, in order to evaluate the performance of this method, two different time series were generated by a GARCH(1,1) model with Gaussian and Student's-t innovations, using Monte Carlo simulations. The author uses two processes defined by: $(a_0, a_1, \delta_1) = (0.1, 0.8, 0.15)$ and $(a_0, a_1, \delta_1) = (0.1, 0.15, 0.8)$. Models on test include a group of three models that use the estimation process described above, with two of them with different initial estimation guesses for the variance, competing with a CAViaR model and the well known RiskMetrics model. A total of 1000 simulations are carried for three different sample sizes (200, 500 and 1000). The performance of all models are examined on the biases and the mean squared errors (MSEs) forecasts for the 5% conditional quantile. MSE results show that, for the first set of parameters, the model with the hybrid estimator as described here is the best method, for the second set of parameters, the CAViaR shows a better performance.

3 Methodology

We conduct Monte Carlo simulations in Matlab, with different return generating processes, in order to examine the conditional quantile estimator method proposed by Zheng et al. (2018). The advantage of using Monte Carlo simulations instead of real portfolio returns is not having to rely on a battery of backtesting methods to analyze the performance of the VaR models, which would necessarily introduce noise in the analysis. Similar to the studies proposed by Zheng et al. (2018), we decided to define and control the playground where the VaR models are tested.

So, we start with returns being generated by the same GARCH process that is assumed for the Zheng et al. (2018) model. This process can be generated as:

$$r_t = \mu_t + \epsilon_t, \epsilon_t \sim N(0, \sigma_t^2) \quad (17)$$

$$\sigma_t^2 = \omega + \gamma\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2. \quad (18)$$

with $(\omega, \gamma, \beta) > 0$ and $\gamma + \beta > 1$ to ensure σ_t remains positive. By our choice, we will set $\mu_t = 0, \forall t$ and for the heteroskedastic process, two sets of parameters are used:

1. $(\omega, \gamma, \beta) = (0.1, 0.15, 0.8)$ — this is the same set of parameters used by Zheng et al. (2018);
2. $(\omega, \gamma, \beta) = (3.125 \times 10^{-7}, 0.05, 0.9)$ — a new set of parameters chosen by us;

The first set serves as a direct comparison to the original paper, since we use the same set of one of the experiments carried by Zheng et al. (2018). To initiate the process, we assumed the steady state variance given by $\frac{\omega}{1-\beta-\gamma}$, this will be our σ_1^2 . Note that this set of parameters used by Zheng et al. (2018) has a large implied steady state variance: $\frac{0.1}{1-0.15-0.8} = 2$. This means a daily volatility of 141%, quite an outrageous value. Based on this, a second set of more reasonable parameters is considered with $\omega = 0.0025^2(1 - \gamma - \beta) = 3.125 \times 10^{-7}$, consequently with a steady state daily volatility of 0.25%.

The true VaR at the α quantile level can be easily computed as:

$$VaR_{t,\alpha} = \Phi_N^{-1}(1 - \alpha)\sigma_t - \mu_t, \quad (19)$$

where Φ_N^{-1} is the inverse of the standard normal cumulative distribution function, computed with the Matlab function `norminv(x)`.

Although the heteroskedasticity of financial data is the main reason for the developments and use of GARCH models, there is no certainty that it is always present. Thus, we also conduct simulations with an homoskedastic process, in order to evaluate the performance of the model discussed by Zheng in capturing the homoskedasticity. With this in

mind, we perform Monte Carlo simulations using the Skewed Generalized T Distribution (SGT), studied by Theodossiou (1998).

The SGT distribution can be defined by 5 parameters, namely: mean (μ), standard deviation (σ), skewness (s) and kurtosis (p and q) with $(\sigma, p, q) > 0$ and $-1 < s < 1$. To work with this distribution on Matlab we need the “Flexible distributions toolbox”, a toolbox that includes all the main functions used, developed by Ahmed BenSaïda, sets a minimum bound of $q > 2$. As p and q increases, the function becomes more platykurtic and vice-versa, when p and q decreases it becomes more leptokurtic (kurtosis increases). This flexible distribution grant us the possibility of controlling tails and skewness at the same time. High flexibility plays a huge role here giving us the possibility of studying special cases like the Normal distribution with $SGT(\mu, \sigma, p = 2, q = \infty, s = 0)$ and the Skewed T Distribution with $SGT(\mu, \sigma, p = 2, q, s)$, besides many others that are not going to be the focus of our work but were studied by Hansen, McDonald, and Newey (2010). The common presence of heavy tails and skewness in financial returns and the possibility of fitting both properties, at the same time, is of great importance, specially due to well known difficulties and limitations of more used distributions such as the Normal distribution on capturing these behaviors.

Returns (r) are generated using the `sgtrnd(x)` function from the “Flexible distributions toolbox”. This process can be described as:

$$r_t \sim SGT(\mu, \sigma, p, q, s) \quad (20)$$

Regarding the simulations of SGT returns, we define 4 main sets of parameters:

1. $SGT(\mu = 0.02\%, \sigma = 1.25\%, p = 2, q = \infty, s = 0)$ — this is the Normal distribution special case, equivalent to $N(0.02\%, 1.25\%)$, it has neither skew or heavy tails;
2. $SGT(\mu = 0.02\%, \sigma = 1.25\%, p = 2, q = 3, s = 0)$ — keeping the distribution symmetric, we study the heavy tails, equivalent to a Student-t distribution with 6 degrees of freedom;
3. $SGT(\mu = 0.02\%, \sigma = 1.25\%, p = 2, q = 3, s = -0.3)$ — this scenario has heavy tails and negative skewness ($s < 0$), similar to financial market returns distribution in general;
4. $SGT(\mu = 0.02\%, \sigma = 1.25\%, p = 2, q = 3, s = 0.3)$ — in this scenario the returns distribution has heavy tails but with positive skewness ($s > 0$) instead.

Again, since we know the distribution of returns, estimating the real VaR is trivial. The true VaR for each scenario is given by the inverse SGT cumulative distribution to each specified quantile, using the same parameters of the generating process. In Matlab we can just use the function `sgtinva( $\alpha, [p, q, s], \mu = 0.02\%, \sigma = 1.25\%$ )`.

For each of all simulations presented so far, we will generate 4500 returns, that represent 4500 market days. The last 2500 returns will be part of the test window (nearly 10 years of calendar days) and each scenario will be simulated 1000 times, the same number of simulations used by Zheng et al. (2018) in his experiments. We will estimate VaR at the 1%, 2.5% and 5% significance levels (α). However, our analysis will focus on the $\alpha = 1\%$, since it is the official significance level recommended by the Basel Committee (2004).

In each simulation, for each testing day, we will compare how far models estimations ($\hat{VaR}_t$) were from the true VaR (VaR_t) using the Normalized Root Mean Squared Deviation (NRMSD):

$$NRMSD = \frac{\sqrt{\frac{\sum_{n=1}^n (\hat{VaR}_t - VaR_t)^2}{n}}}{\frac{\sum_{n=1}^n VaR_t}{n}}, \quad (21)$$

This measure is similar to the mean squared error (MSE) used by the original paper, however, since we will examine more than one quantile level it is important to normalize the results in order to guarantee that they can be compared. For the sake of model performance appraisal we will compare the mean of the NRMSD 1000 simulation's results, also, for understanding the results dispersion, we will analyze several statistics such as: standard deviation, minimum, maximum, 5% and 95% quantile. The consistency of results is important because we do not want to favor a model that just gets lucky in one simulation and the next one he completely fails. So, the other figures will also be important if mean results are very similar. The logic behind this is: we want that the average normalized deviation to be as low as possible but we also want to have the most consistent results across all simulations.

Ideally we would like to simulate 10000 times or more. However, every model that we need to estimate parameters (either the quantile regression models or the ones that have to fit a distribution) will have two versions. One where we will reestimate the parameters every 20 days and other that only estimates the parameters once and keep them until the end of the test window (the ones named “([...]-NoR”). Hence, we will be studying a reasonable amount of models and scenarios for 3 different significance levels, many of those models need to frequently fit distributions or GARCH parameters, that can become computationally intensive. Despite that, since the major time consumption is due to models that require parameters reestimation, we carried additional 10000 simulations to all remaining models. Results confirmed that 1000 simulations appears to be enough, since the increased number of simulations did not have a meaningful impact in our analysis. The complete results from 10000 simulations are provided in the Appendix A.2.

All models are presented in the next subsections.

3.1 Quantile regression with GARCH(1,1)

The model we want to examine was first proposed by Zheng et al. (2018) and tries to improve previous quantile regression models by enabling the use of volatility models that explain the conditional variance instead of the conditional standard deviation. Hence, the author extends quantile regression to GARCH(1,1) models. Researchers have studied and proposed a large number of GARCH models (symmetric and asymmetric). This is the building-block model of the GARCH family but tends to work well against other GARCH(p,q) variations, as described by Namugaya, Weke, and Charles (2014).

To estimate the GARCH parameters we use the *garch*(x) function from Matlab with Gaussian distributed innovations and an initial guess for the $\hat{\sigma}_1^2 = \frac{1}{n} \sum_{i=1}^n r_{t-i}^2$ with $n = 500$. Hence, a total of 4000 variance estimates will be carried. Returns are $T(x)$ transformed as described in (14), in order to get $y_t = T(r_t)$. We estimate the quantile of y_t by minimizing the weighted tilted deviation as proposed by Zheng et al. (2018). Similar to what was described before, for an α quantile we have:

$$\hat{\theta}_\alpha = \arg \min_{\theta} \sum_{t=1}^n \frac{1}{\hat{\sigma}_t^2} (y_t - \theta' \hat{x}_t) (\alpha - I_{y_t - \theta' \hat{x}_t < 0}), \quad (22)$$

with $\hat{x}_t = \begin{bmatrix} 1 & r_{t-1}^2 & \hat{\sigma}_{t-1}^2 \end{bmatrix}'$ and $\theta = \begin{bmatrix} \theta_0 & \theta_1 & \theta_2 \end{bmatrix}'$. The α quantile estimate for the transformed return y_t is then given by

$$Q_\tau(y_t | \mathcal{F}_{t-1}) = \hat{\theta}'_\alpha \hat{x}_t = \hat{\theta}_0 + \hat{\theta}_1 r_{t-1}^2 + \hat{\theta}_2 \hat{\sigma}_{t-1}^2$$

Transforming the quantile of y_t into a quantile for the return r_t using

$$Q_\tau(x_t | \mathcal{F}_{t-1}) = T^{-1} [(Q_\tau(y_t | \mathcal{F}_{t-1}))] \quad (23)$$

we finally obtain the VaR estimate:

$$VaR_t = -\sqrt{\left| \hat{\theta}_0 + \hat{\theta}_1 r_{t-1}^2 + \hat{\theta}_2 \hat{\sigma}_{t-1}^2 \right|} \text{sgn}(\hat{\theta}_0 + \hat{\theta}_1 r_{t-1}^2 + \hat{\theta}_2 \hat{\sigma}_{t-1}^2). \quad (24)$$

All parameters, including the necessary GARCH volatility estimates, will be reestimated every 20 days over the 2500 days of the test window. The sum of the weighted tilted deviations will be calculated with our function `SumWTTD(x)`¹. The minimum sum of the weighted tilted deviations will be calculated using the `fminsearch(x)` function, considering a sample size of the last 1000 observations. Additionally, we also present results, on Appendix A.1.2 and Appendix A.2.2, where 500 observations were used to understand the sample size sensitivity. Also, we will create a version of this model with the only difference being that it does not reestimate the parameters along the test period.

¹All the Matlab code is available on request.

With the aim of understanding upcoming difficulties in estimating the coefficients correctly, we will test a model that is exactly as the Zheng's model and that makes use of the true variance from the simulated returns. So, instead of estimating a $GARCH(1, 1)$ model that fits a sample of returns and then estimate the quantile regression parameters, we will only have to estimate the former. Obviously, this would not be possible in a real context where we do not know and control the return generating process. Here, we can study how much would the knowledge of the true variance help to estimate the quantile regression model.

3.2 Benchmarks

In order to benchmark Zheng's model, we select several models as benchmarks from three different VaR methodologies: Historical VaR models, Parametric VaR models and other Quantile Regression VaR models. Regarding the Parametric and Quantile Regression VaR models, we will test two different volatility estimation approaches: equally weighted with a rolling sample and EWMA.

The equally weighted method is the simplest method and is described as:

$$\hat{\sigma}_t = \sqrt{\frac{1}{n} \sum_{i=1}^n r_{t-i}^2}. \quad (25)$$

Each return observation from the sample n observations will have the same weight in volatility estimation. The choice of the sample size is important because it has to balance having enough data to achieve statistical accuracy but not too much to avoid a distant past to influence the volatility estimate. We find $n = 500$ to be a good balance of this two forces. Implicitly we are taking into account the last 2 calendar years of daily returns to predict what will be the next day volatility.

To obtain a time series of volatility estimates we will use the standard deviation of the first 500 returns, generated by the Monte Carlo simulation, to estimate our $\hat{\sigma}_1$. For the following days we will use a rolling window with a fixed sample size of $n = 500$. Hence, every time that we move to the next day estimation, the last 500th return used on the last estimation will be disregarded and the most recent past return will be considered. Since we have 4500 returns, we are able to estimate a time series of 4000 volatility estimates.

An alternative method to estimate volatility is the EWMA. This method uses a decay factor λ that can be set between 0 and 1:

$$\hat{\sigma}_t^2 = (1 - \lambda) \sum_{i=1}^{\infty} \lambda^{i-1} r_{t-i}^2, \quad (26)$$

The recursive version of this formula is:

$$\hat{\sigma}_t^2 = \lambda \hat{\sigma}_{t-1}^2 + (1 - \lambda) r_{t-1}^2, \quad (27)$$

The choice of λ is quite arbitrary, a lower λ makes our estimation more reactive to recent observations and a higher λ gives less weight to recent observations. We are going to define the same as proposed by Morgan J P and Reuters TM (1996) for daily returns: 0.94.

We will generate a time series of 4500 data points, with a warm-up period for the initial estimate of $\hat{\sigma}_1^2 = r_1^2$. In theory all sample observations are used, but the λ factor makes past observations become less and less important until they become negligible. Also, generating 4500 data points stabilizes the variance estimation and makes the initial guess even more negligible, taking into consideration that our models will not directly use the first 1000 observations.

3.2.1 Quantile Regression VaR models

Quantile regression models are very flexible, as they allow us to define our own explanatory variables. In our study we will start with two specifications to estimate the VaR_t , one with only a volatility estimate for day t as explanatory variable and another that besides the volatility estimate, adds a constant term. Close to what we did with the Zheng's model, we duplicate all models to study if reestimating the quantile regression parameters every 20 days in a rolling window mechanism versus not reestimating it at all, have any meaningful impact on results. All models will also be estimated with a sample size of 1000 and 500 observations.

We start with the simplest structure possible to estimate the VaR_t via quantile regression, a model with just one explanatory variable, in this case, a volatility estimate:

$$VaR_t = c_1 \hat{\sigma}_t \quad (28)$$

The coefficient c_1 is estimated, for each α level, adapting the Koenker and Bassett (1978) method, as described in (4). With this we are able to estimate VaR without knowing or having to assume a distribution. The coefficient c_1 that minimizes that sum, is estimated with the *fminsearch*(x) Matlab function. The conditional quantile will be explained by a linear function that depends on σ_t . In order to have a version of this model without reestimation, we do not reestimate this parameter again, the VaR series will be estimated using the same c_1 that we estimate once at the beginning of the testing period.

We can have the same quantile regression structure described in (28), but making a different decision regarding the volatility estimation process. Now, instead of using the equally weighted method we use the Exponential Weighted Moving Average (EWMA) method.

So far, we are implicitly assuming that VaR_t can be fully explained by the estimated

volatility of day t in a linear relationship without a intercept. Now, we can study if a constant term increases the estimation power. So, using the equally weighted method to estimate the volatility we have a new specification:

$$VaR_t = c_0 + c_1 * \hat{\sigma}_t \quad (29)$$

The parameters c_0 and c_1 are estimated at the same time for each quantile with a similar process as described in (4). Again, we also want to understand how the volatility estimation method changes the performance of our models. So, we test a VaR model that includes a constant term and an EWMA volatility estimate.

3.2.2 Parametric SGT

Parametric models assume that portfolio returns and/or innovations of returns follow a standardized distribution. We can assume that returns follow the SGT distribution explained by the 5 parameters that we already discussed. So, our main task will be to fit the distribution of returns (in the real world we don't know what are these parameters) with our sample and, based on the estimated parameters, predict what is the Value-at-Risk.

The first 3 parameters that define the skewness (s) and tails (p and q) can be estimated using the `sgtfit(x)` function as described by BenSaïda and Slim (2016). For calculation purposes it is important to note that, as stated by the author on his function description and with PSI being our q , "although there is no theoretical upper limit to the degree-of-freedom parameter 'PSI' (...) the upper limit of the SGT PSI parameter is arbitrarily set to 200". Although the author has that in mind, this function actually estimates the parameters by maximum likelihood without this upper bound limit (the author uses infinity as the upper limit). We decided to use 150 and set it before the maximum likelihood estimation. Doing this, we help preventing it to enter into these anomalous results. For fitting purposes, a sample of the last 1000 returns was also considered.

Since in the previous models with reestimation we would update the parameters every 20 days (that gives us more than 100 fitting processes per simulation), for this model we will do the same. Additionally, for distributions with heavy tails like the $SGT(\mu, \sigma, 2, 3, s)$, the fitting function often stays very far away from the real distribution. This provides absurd VaR estimates and the reason behind is that the $\hat{q}$ parameter fitted is equal to the lower bound (probably due to the sample size used being too small). To prevent that, for the no reestimation version, before we compute the VaR_t , we check if $\hat{q}$ rounded at two decimal cases is equal to 2 (we have to round it because a tolerance of 1E-5 for all bounds is used in the calculations of the `sgtfit(x)` so the real bound is defined by $2 + tolerance$). If the result is 2, we will stop the simulation and move on to the next simulation. For the model with reestimation, since we fit the distribution many times for every simulation,

and interrupting the simulation every time that one fitting process went wrong would be too much time consuming, if any moment the function gets stuck in this bound, we will use the last fit until a maximum of 3 consecutive times. If the fitting fails for 3 consecutive times the simulation is also interrupted and discarded, and we move to a new simulation until we have 1000 simulations completed.

After estimating p , q and s , we assume $\mu = 0$. The reason behind it is that we are dealing with discounted daily returns. Hence, in a real context if markets are efficient we would not expect that daily market changes over the risk free rate being much different from 0. Furthermore, we estimate σ with the equally weighted method and the EWMA estimate, as we did before. The VaR_t will be estimated using the inverse cumulative SGT distribution Matlab function `sgtinvs(x)`, employing the estimated parameters.

3.2.3 Parametric Normal VaR

Assuming the Normal distribution, where r_t is the portfolio's return at time t , ϵ_t are random variables with mean 0:

$$r_t = \epsilon_t \sim N(0, \sigma_t^2), \quad (30)$$

The Parametric Normal VaR focus on two parameters: mean and standard deviation. The single asset portfolio VaR, in percentage, to day t for a $(1 - \alpha)$ confidence level is computed as:

$$VaR_{t,\alpha} = \Phi_N^{-1}(1 - \alpha)\sigma_t - \mu_t \quad (31)$$

As we discussed before, we can disregard the last part of this equation. The volatility will be estimated with the equally weighted method previously described.

The Parametric Normal VaR methodology offers simplicity and, naturally, it is well known for the RiskMetrics VaR model approach. For these reasons, it is widely used on a daily basis for financial institutions to report VaR estimates.

In order to improve volatility estimation, RiskMetrics uses the EWMA method as (26) and already described for other models. As previously discussed, RiskMetrics model uses λ as 0.94 for daily data. Hence, we will also consider a VaR model that is using a method that increases the weight of recent observations and, so, we hope to improve the response of the Parametric Normal model to current volatility.

3.2.4 Historical VaR

As previously stated, empirical evidence shows that returns are not normally distributed (see Sheikh and Qiao, 2009). By using Parametric Normal models, we might be undervaluing the size of actual losses if the real distribution is negatively skewed and presents heavier tails and we expect that these weaknesses are revealed in our simulated scenarios.

Hence, rather than defining a distribution of portfolio's returns we can simply use the distribution (whatever it is) from the data that we selected. This method is also simple to implement and basically we start by defining the sample size (n), we will keep 1000 observations and, as we stated earlier, the $VaR_{h,\alpha}$ is just the α quantile of the empirical distribution of h day financial returns (for $\alpha = 0.01$ we need at least 100 observations). We use the $\text{quantile}(x)$ function of Matlab. The sample size could be the major decision of estimating VaR from Historical VaR model. If we define n large enough we can fit the portfolio returns empirical distribution much better. However, it is always a trade-off between having enough information from the past without going to far.

A few refinements to some features can avoid a large event in the far past to have a great impact on our VaR estimate. Here, we can refine this model so that it reflects current market conditions. We do that by adjusting portfolio returns as proposed by Hull and White (1998):

$$\hat{r}_t = \frac{r_t}{\sigma_t} \sigma_T \quad (32)$$

where σ_T is the most recent daily volatility estimate (EWMA) made at the end of $T - 1$, and σ_t is the past volatility on day t , with $T > t$. We assume that the probability distribution of $\frac{r_t}{\sigma_t}$ is stationary. Therefore, we can replace r_t by $\hat{r}_t$ and instead of estimating VaR from a series of historical percentage returns, we use historical changes that are adjusted by a ratio of the current volatility to the past volatility at time t . Bearing in mind that the set of data (1000 adjusted returns) that we choose will still be a main driver of our estimate (Abad, Benito, and López, 2014).

4 Results analysis

4.1 GARCH simulations with Zheng et al. (2018)'s parametrization

Table 4.1: This table shows the summary Normalized Root Mean Squared Deviation (NRMSD) statistics for 1000 simulations of GARCH returns, generated with $(\omega, \gamma, \beta) = (0.1, 0.15, 0.8)$ as Zheng et al. (2018). Each simulation tests 2500 daily 1% VaR estimates.

GARCH(1,1) returns, $\alpha = 1\%$							
	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	7.9%	7.8%	2.0%	2.7%	5.0%	11.6%	17.8%
GARCHvolknownNoR	7.2%	6.7%	3.6%	1.6%	2.4%	14.0%	23.7%
GARCHQR	10.3%	10.1%	2.3%	5.3%	7.2%	14.5%	23.4%
GARCHQRNoR	8.8%	8.3%	3.9%	1.3%	3.6%	16.2%	27.4%
voltEqW	38.9%	36.0%	11.9%	22.6%	27.0%	60.4%	147.8%
voltEqWNoR	36.9%	34.3%	11.2%	22.3%	26.4%	55.7%	174.2%
voltEWMA	18.0%	17.2%	3.6%	12.6%	14.2%	24.3%	55.7%
voltEWMANoR	17.4%	16.7%	3.4%	11.8%	13.7%	23.6%	58.5%
c+voltEqW	41.2%	36.4%	17.0%	21.7%	27.3%	70.0%	194.7%
c+voltEqWNoR	39.8%	34.9%	24.6%	22.3%	26.8%	68.2%	596.7%
c+voltEWMA	18.1%	17.2%	3.6%	12.6%	14.2%	24.3%	55.7%
c+voltEWMANoR	17.4%	16.7%	3.4%	11.8%	13.7%	23.6%	58.5%
ParamSGT	34.6%	33.2%	7.4%	22.3%	25.8%	48.1%	86.9%
ParamSGTNoR	34.1%	32.8%	6.9%	22.2%	26.1%	46.7%	89.8%
ParamSGTEWMA	18.6%	18.0%	3.7%	11.9%	14.0%	25.5%	39.2%
ParamSGTEWMANoR	18.1%	17.6%	3.2%	12.0%	14.0%	23.9%	39.8%
NormalEqW	32.0%	30.8%	6.1%	21.5%	25.1%	42.2%	85.2%
RiskMetrics	15.8%	15.4%	2.3%	11.8%	13.3%	20.0%	40.7%
Historical	36.0%	33.8%	10.4%	22.1%	26.4%	52.8%	209.5%
HistVolAdj	18.3%	17.6%	3.5%	12.5%	14.1%	24.6%	51.6%

Starting with GARCH returns simulated with $\omega = 0.1$, $\gamma = 0.15$ and $\beta = 0.8$ (the exact same set of parameters used by Zheng et al., 2018), whose results are presented on Table 4.1 and Table A.1 in the Appendix, we confirm that the GARCH hybrid quantile estimator is the best model with an average NRMSD of 8.8% for the no reestimation version and 10.3% for the one that reestimates parameters every 20 days, at 1% significance level, and with a similar performance for the remainder quantiles. Obviously, we are not taking into consideration the GARCH quantile regression model that makes use of information impossible to know by the time of the estimation (GARCHvolknown). Previously knowing the true variance, before estimating quantile regression parameters, improved the NRMSD average by around 2 percentage points across all quantiles as observed on Table 4.1 and Table A.1. These superior performances compare with much higher deviations from the benchmarks, with the closest one being the RiskMetrics VaR method with 15.8% and

the worst being the quantile regression with equally weighted volatility and a constant term (average NRMSD of 41.2%). It is worth to remind that Zheng et al. (2018) had already shown that their GARCH quantile regression model surpassed the RiskMetrics model with $(\omega, \gamma, \beta) = (0.1, 0.15, 0.8)$, and our results confirm this.

We also highlight the strong performance from the group of models that make use of an EWMA volatility estimate. All of them have very similar and well-behaved performances (NRMSD averages between 17.4% and 18.6% at 1% significance level). This technique consistently captured heteroskedasticity from returns distribution, regardless the VaR methodology tested (historical, parametric or quantile regression).

It is interesting to notice that, regarding the quantile regression benchmark models, adding a constant to the regression did not make much of a difference. Both models have almost the same results, probably due to the constant term being almost irrelevant since returns are simulated based on a process with mean zero. Another noteworthy detail is that the RiskMetrics model has the same NRMSD at every quantile, and the same is true for the Parametric Normal model with equally weighted volatility. This happens because, here, returns follow a GARCH(1,1) process with GARCH innovations being normal distributed, deviations will be caused by the mean and volatility estimates, hence, the difference between the true VaR and the estimated VaR will be the same.

Surprisingly, the periodic reestimation of parameters may slightly increase the average NRMSD results for all models at 1% significance level. The same happens to almost all models at the remaining quantiles, the exceptions are the Parametric SGT, Parametric SGT with EWMA volatility estimate and the quantile regression VaR model with a constant term at 2.5% and 5% significance levels. For this returns process, it seems that it is not worth to do it. For some models, like the GARCH(1,1) quantile regression model it seems to mitigate the dispersion of the results. Since it does not happen to all models, we can not say whether the reestimation of parameters every 20 days is meaningful or not.

Overall, we note that the performances of all models are similar in all quantiles and invite the reader to check the full results presented on Table A.1 in the Appendix A.1 and that the main conclusions remain the same for a sample size of 500 observations as shown by Table A.7, Table A.8 and Table A.9.

4.2 GARCH simulations with a realistic parametrization

Table 4.2: This table shows the summary NRMSD statistics for 1000 simulations of GARCH returns, generated with $(\omega, \gamma, \beta) = (3.125 \times 10^{-7}, 0.05, 0.9)$. Each simulation tests 2500 daily 1% VaR estimates.

GARCH(1,1) returns, $\alpha = 1\%$							
	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	9.0%	8.9%	1.5%	5.1%	6.8%	11.8%	16.7%
GARCHvolknownNoR	8.7%	8.3%	3.5%	1.3%	3.2%	14.8%	31.7%
GARCHQR	10.3%	9.9%	3.4%	6.6%	7.8%	13.2%	84.1%
GARCHQRNoR	9.2%	9.1%	3.5%	1.0%	3.8%	15.8%	23.2%
voltEqW	13.4%	13.1%	2.2%	8.8%	10.6%	17.5%	22.1%
voltEqWNoR	12.9%	12.3%	2.5%	8.9%	10.1%	18.2%	27.3%
voltEWMA	10.2%	9.9%	1.6%	6.3%	8.0%	13.4%	18.6%
voltEWMANoR	9.8%	9.3%	2.2%	6.0%	7.4%	14.5%	19.8%
c+voltEqW	15.4%	14.9%	2.8%	10.2%	11.9%	20.7%	29.1%
c+voltEqWNoR	16.9%	14.2%	7.4%	8.6%	10.4%	33.0%	62.8%
c+voltEWMA	8.2%	8.0%	1.8%	4.3%	5.6%	11.1%	18.4%
c+voltEWMANoR	7.7%	7.1%	3.6%	2.4%	3.3%	15.1%	29.7%
ParamSGT	11.9%	11.7%	1.5%	8.7%	9.8%	14.8%	19.5%
ParamSGTNoR	11.9%	11.6%	1.5%	8.7%	9.9%	14.8%	18.0%
ParamSGTEWMA	8.9%	8.8%	1.1%	6.1%	7.3%	10.8%	14.8%
ParamSGTEWMANoR	8.8%	8.7%	1.1%	5.9%	7.3%	10.9%	13.6%
NormalEqW	11.4%	11.3%	1.2%	8.7%	9.6%	13.8%	17.1%
RiskMetrics	8.3%	8.3%	0.8%	5.6%	7.0%	9.7%	12.7%
Historical	12.5%	12.2%	2.0%	8.6%	10.1%	16.1%	38.6%
HistVolAdj	10.8%	10.4%	1.9%	6.6%	8.2%	14.5%	19.3%

As we can see from Table 4.2 when we move to GARCH returns with a more reasonable parametrization, things get really surprising. Models that in the previous simulations underperformed badly, are now better than the quantile regression GARCH model from Zheng et al. (2018). Although the author's model performance (measured by the NRMSD mean) worsens when the parameters are not reestimated and when the true volatility is used (in terms of NRMSD standard deviations, all these versions get worse), their performance remains basically constant. The greatest change happens on benchmark models. In an opposite direction, we clearly see some huge improvements from all benchmarks, with approximately 50% decrease on the average NRMSD. Also, we note that all quantile regression models with less explanatory variables are performing better than the new GARCH(1,1) quantile regression estimator of Zheng et al. (2018), presenting a lower standard deviation at all quantiles and overall lower mean NRMSD (few exceptions on the quantile regression with the EWMA volatility as explanatory variable at 2.5% quantile).

RiskMetrics can be seen as a good example of the difference between the two GARCH return simulations. This model that previously underperformed on Zheng et al. (2018) pa-

per and in our simulations using the same parameters, now goes from an average NRMSD of 15.8% to 8.3% and the NRMSD results dispersion also drops from a standard deviation of 2.3% to 0.8%. The largest reduction was achieved by the Historical (no adjustment) with mean results that were all above 31.5% at 1% quantile, on the previous experiment, and now drops to an average NRMSD of 12.5% at the same significance level. It is also worth to mention that now the reestimation of parameters, at 1% quantile always minimize the results dispersion but it still increases the average NRMSD. For the 2.5% and 5% significance levels, this is not true for the GARCH quantile regression model since it has an increased volatility with reestimation of parameters.

The overall NRMSD results from the benchmarks have decreased about 50% at all quantiles. The only VaR model that has higher standard deviation than the (Zheng et al., 2018) quantile regression GARCH model, at all quantiles, is the quantile regression model with a constant term equally weighted volatility and no reestimation of parameters. Also, at the 1% significance level, the quantile regression VaR model with a constant term and EWMA volatility has slightly higher standard deviation than the (Zheng et al., 2018) VaR model.

So far, it seems that the original paper cherry picking GARCH parameters for the returns simulation have a deeply impact on the benchmarks performance, making the tested model (the quantile regression with a GARCH(1,1) specification) look much better. We can also confirm this idea for a sample size of 500 observations and at all quantiles, comparing the full results presented on Table A.7, Table A.8 and Table A.9 versus the results presented on Table A.10, Table A.11 and Table A.12 in the Appendix. Also, as expected, using 500 observations instead of 1000 produce generally worse results and same will happen in other return generating processes.

4.3 Normal simulations

From Table 4.3 we check the performance of the GARCH quantile regression VaR model in a SGT(2,∞,0) distribution which is equivalent to the normal distribution. It is important to note that we are now in an homoskedastic return generating process. Here, the 1% VaR estimated by the GARCH(1,1) model has a very close performance to the quantile regression model with a constant and EWMA volatility, either with reestimation of parameters (7.3% of NRMSD mean for the GARCH(1,1) versus 7.2% for the benchmark) and without reestimating parameters (5.6% versus 6.5%).

All models with equally weighted volatility stand out. Since our returns are generated by homoskedastic volatility, models with constant volatility are superior to the correspondent versions with the EWMA method that has some noisy estimates. That helps to explain our top three models in this scenario, at all significance levels as we can see from Table A.3 in the Appendix. The Normal equally weighted model is a natural winner

Table 4.3: This table shows the summary NRMSD statistics for 1000 simulations of the $SGT(0.02\%, 1.25\%, 2, \infty, 0)$ returns. Each simulation tests 2500 daily 1% VaR estimates.

SGT($2, \infty, 0$) returns, $\alpha = 1\%$							
	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	6.2%	6.1%	1.7%	2.4%	3.7%	9.2%	12.5%
GARCHvolknownNoR	5.3%	4.8%	3.6%	0.0%	0.5%	12.5%	18.1%
GARCH2G	7.3%	7.1%	2.1%	2.6%	4.2%	10.9%	18.7%
GARCH2GNoR	5.6%	5.1%	3.5%	0.0%	0.9%	11.9%	21.1%
voltEqW	5.8%	5.7%	1.6%	2.0%	3.3%	8.6%	12.9%
voltEqWNoR	5.1%	4.4%	2.6%	1.4%	2.3%	10.4%	18.4%
voltEWMA	13.9%	13.7%	1.5%	10.7%	11.9%	16.5%	20.5%
voltEWMANoR	13.7%	13.3%	2.1%	10.4%	11.5%	17.8%	29.7%
c+voltEqW	8.4%	8.2%	2.2%	3.3%	5.1%	12.2%	18.4%
c+voltEqWNoR	10.3%	8.0%	8.4%	0.4%	2.0%	26.0%	104.1%
c+voltEWMA	7.2%	7.1%	1.7%	2.2%	4.7%	10.3%	13.2%
c+voltEWMANoR	6.5%	6.1%	3.5%	0.4%	1.7%	12.8%	24.4%
ParamSGT	4.0%	3.9%	1.1%	1.6%	2.4%	6.0%	8.4%
ParamSGTNoR	3.9%	3.7%	1.4%	1.1%	2.1%	6.7%	10.3%
ParamSGTEWMA	12.6%	12.6%	0.8%	10.3%	11.3%	14.1%	15.4%
ParamSGTEWMANoR	12.6%	12.5%	0.9%	10.2%	11.3%	14.2%	16.2%
NormalEqW	3.2%	3.1%	0.8%	1.5%	2.0%	4.5%	6.6%
RiskMetrics	12.3%	12.3%	0.7%	9.9%	11.2%	13.6%	14.9%
Historical	4.8%	4.6%	1.6%	1.5%	2.6%	7.6%	13.5%
HistVolAdj	14.7%	14.4%	1.7%	10.9%	12.3%	17.9%	25.0%

(only 3.2% of average NRMSD at 1% quantile). Results are as expected, since it assumes a normal distribution combined with a equally weighted method for volatility estimation, fits very well our returns generating process. It is followed by the Parametric SGT with equally weighted volatility (around 4% NRMSD mean for both versions), that seems to be quite good on fitting the normal distribution. Obviously, the equally weighted method makes a huge difference, the corresponding version with the EWMA method gets more than three times higher mean NRMSD for all quantiles. Finally, the Historical VaR with no adjustment stays behind these two with an average NRMSD of 4.8%.

Looking at other benchmarks, RiskMetrics and the Historical Volatility Adjusted are hurt by the use of the EWMA method. RiskMetrics, however, has the lowest standard deviation of all models in all quantiles (0.7%). It should be pointed out that the Normal equally weighted VaR model and the RiskMetrics, each, have the same results in every quantile for the same reasons explained when the returns were generated by a GARCH with Gaussian innovations process, now returns are normally distributed.

It is also important to notice that the reestimation of parameters did not make much of a difference if we look at the average of NRMSD results. Nevertheless, it helped to lower the standard deviation of all models (Parametric SGT and quantile regression models),

shorting the range between minimum and maximum NRMSD values. As an example, we can look at the quantile regression model with a constant and EWMA volatility without reestimating parameters, at 1% quantile, it has a NRMSD standard deviation of 3.5% with mean 6.5% while his counterpart version with reestimation has 1.7% NRMSD standard deviation but 7.2% of average deviation. Same happens in the other two quantiles.

The quantile regression models, in general, had a good performance in all quantiles (this performance gets worse as α decreases). The highest mean NRMSD goes to the EWMA which defeated the Historical VaR model with volatility adjustment by mean and dispersion of NRMSD results. The quantile regression model with only an equally weighted volatility as an explanatory variable had the best performance in this group of models, with the no reestimation version also having the lowest mean but a higher dispersion of results, as we can confirm by the standard deviation that almost doubled (from 1.6% to 2.6%). It is interesting to notice that, if we add a constant to the quantile regression model with equally weighted volatility, the estimating power decreases. However, for the quantile regression model with the EWMA method, this constant term was the key factor to mitigate the EWMA volatility estimate variability. This last model outperformed its variant without the constant term and even the model with equally weighted volatility and a constant. The intercept seems to help balancing the benefits from having a current market volatility provided by the EWMA smoothing factor technique and also a constant term that helps to minimize the impact of the EWMA variability. Quantile regression coefficients estimates could overweight this intercept instead of the EWMA volatility term in order to best capture the homoscedasticity of returns.

4.4 SGT simulations

Now if we look at the results presented in Table 4.4 from a $SGT(0.02\%, 1.25\%, 2, 3, 0)$ return process, we can clearly observe that, overall, if the distribution has heavy tails, all models have more difficulty in predicting the VaR. It is clear that the average deviation and, also, the level of dispersion of NRMSD result have a considerable increase by all metrics, for all quantiles as we can see from the results presented in Table A.4.

The GARCH quantile regression model has an average NRMSD of 17.6% with reestimation of parameters and 13.7% without it. Again, it stays very close to the other quantile regression models with less explanatory variables if we do not consider the EWMA volatility quantile regression model and the quantile regression model with equally weighted volatility and a constant, that have higher NRMSD results at 1% significance level. The additional constant on the equally weighted volatility seems to add extra noise to the coefficients estimation at 1% level (this is only verified at this specific quantile) with the average NRMSD going from 17.6% to 20.8%. Apart from the quantile regression VaR model and EWMA volatility estimate with and without reestimation of parameters,

Table 4.4: This table shows the summary NRMSD statistics for 1000 simulations of the $SGT(0.02\%, 1.25\%, 2, 3, 0)$ returns process. Each simulation tests 2500 daily 1% VaR estimates.

SGT(2, 3, 0) returns, $\alpha = 1\%$							
	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	15.1%	13.8%	7.0%	4.8%	8.2%	26.0%	107.1%
GARCHvolknownNoR	13.0%	10.9%	10.0%	0.1%	2.4%	31.0%	115.3%
GARCH2G	17.6%	16.4%	6.8%	4.7%	9.8%	29.5%	63.3%
GARCH2GNoR	13.7%	11.3%	11.3%	0.1%	2.5%	34.1%	150.2%
voltEqW	17.6%	15.7%	9.0%	4.8%	9.0%	31.7%	100.3%
voltEqWNoR	17.1%	14.2%	13.1%	3.2%	6.8%	36.8%	201.7%
voltEWMA	38.8%	35.0%	15.5%	20.2%	25.5%	63.2%	183.6%
voltEWMANoR	37.6%	33.2%	17.3%	19.9%	23.2%	64.4%	296.8%
c+voltEqW	20.8%	19.4%	8.1%	7.7%	12.0%	34.5%	117.0%
c+voltEqWNoR	26.9%	17.7%	41.1%	0.2%	3.5%	76.8%	860.4%
c+voltEWMA	17.6%	16.2%	7.4%	5.6%	10.3%	28.6%	95.3%
c+voltEWMANoR	15.5%	13.4%	11.1%	0.4%	3.6%	34.6%	127.6%
ParamSGT	11.6%	9.8%	6.5%	3.5%	5.8%	25.0%	62.1%
ParamSGTNoR	13.2%	10.2%	11.8%	3.4%	5.8%	28.7%	234.9%
ParamSGTEWMA	32.5%	29.5%	11.2%	20.0%	22.9%	50.8%	203.4%
ParamSGTEWMANoR	33.2%	29.6%	13.6%	19.9%	22.9%	54.4%	262.6%
NormalEqW	18.4%	17.0%	8.7%	7.4%	12.2%	27.1%	103.7%
RiskMetrics	33.8%	30.9%	10.5%	25.6%	27.2%	48.5%	148.6%
Historical	10.9%	10.5%	3.9%	2.6%	5.6%	17.9%	31.2%
HistVolAdj	43.6%	39.6%	17.7%	21.3%	26.8%	71.2%	201.3%

where the average NRMSD is over 35% in all significance levels, the quantile regression models have a solid performance around 17% NRMSD mean. Generally increasing with the α , they beat the RiskMetrics and the Historical with volatility adjustment by a 50% distance in the average NRMSD at all quantiles. The quantile regression model with a constant and an EWMA estimate beats all the other quantile regression benchmarks, in all quantiles and for both versions (with and without reestimation). Adding a constant term seems to help predicting the VaR in this scenario of extreme observations, in both volatility estimation methods, except for the quantile regression model and equally weight volatility at 1% significance level.

Models assuming the normality of returns, such as the RiskMetrics and the Parametric Normal equally weighted underestimate the VaR and that helps to explain how their average NRMSD increases more than four times and two times, correspondingly .

Also, regarding the volatility estimation method, EWMA models keep to find huge difficulties in modeling homoskedastic returns (with mean NRMSD being higher than 30% at all quantiles, exception for the model with a constant and EWMA estimate), compared to the equally weighted versions (with the quantile regression model with a constant and equally weighted volatility being the worst in this category but only achieving 26.9% of

an average NRMSD) at 1% significance level. In general, comparing the two volatility estimation methods, the EWMA models produce NRMSD results that more than double the ones from equally weighted versions.

The Historical VaR (no adjustment) has the best performance, achieving the lowest mean in every quantile (10.9% at the 1% significance level) and dispersion (3.9% also at the 1% VaR series). In the presence of homoskedastic simulated returns, there is no need for the volatility adjustment done to the Historical VaR, in this case the variability of estimates introduces too much noise that ruins the VaR estimates. The Parametric SGT keeps its good performance, being second, specially for the 1% and 2.5% 1-day VaR not far from the previous model with 11.6% average NRMSD at 1% VaR. Regarding the frequency of reestimation, we can observe an improvement on the Parametric SGT models, by NRMSD mean and dispersion. Despite on that, looking at the quantile regression models it is clear that it helps to decrease the standard deviation (NoR models are responsible for the highest maximum deviation but also the lowest minimum). Hence, the reestimation of parameters remains to have a positive impact on consistency but it is not clear that it shortens the NRMSD results. It is interesting to note that the largest reductions on standard deviation happens on the structures with equally weighted volatility estimation, for instance, at 1% significance level the quantile regression model with a constant and a equally weighted volatility estimate as explanatory variables has a standard deviation of 41.1% that drops to 8.1% with reestimation of parameters (in the EWMA models the decrease is nearly 2 percentage points).

From Table 4.5 we analyze the effects of negative skewness combined with the heavy tails. The performance of all models got worse than on the previous scenarios. It is important to be aware that this scenario could be the one that better represents financial assets returns behavior as previously explained.

As usual, Historical (no adjustment) is, once more, the winner for all quantiles. Simplicity, so far, pays-off. At the 1% quantile its average NRMSD is 11.8% and looking at all quantiles the NRMSD mean gap for the second best model tested stays around 5 percentage points as we can observe from the results presented on Table A.5 in the Appendix.

At 1% significance level, the GARCH quantile estimator tested has a very similar performance to the one explanatory quantile regression model with equally weighted volatility (19.1% and 19.3% correspondingly). Also, at all quantiles we can observe that both models have similar performances to the quantile regression model with a constant and a volatility estimate (EWMA and equally weighted with reestimated parameters) and the Parametric SGT equally weighted volatility. At the 1% quantile these performances are between an average NRMSD around 19% and 22%. The quantile regression model with the constant term and equally weighted volatility, the Historical volatility adjusted model, the Parametric Normal (both RiskMetrics and equally weighted VaR models) and the

Table 4.5: This table shows the summary NRMSD statistics for 1000 simulations of the $SGT(0.02\%, 1.25\%, 2, 3, -0.3)$ returns process. Each simulation tests 2500 daily 1% VaR estimates.

SGT(2, 3, -0.3) returns, $\alpha = 1\%$							
	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	16.1%	14.8%	7.1%	5.2%	9.0%	27.0%	121.5%
GARCHvolknownNoR	14.4%	12.2%	11.5%	0.1%	2.5%	33.5%	173.1%
GARCH2G	19.1%	17.9%	7.1%	5.8%	10.6%	31.1%	65.2%
GARCH2GNoR	15.1%	12.5%	13.3%	0.1%	2.7%	33.4%	212.5%
voltEqW	19.3%	17.2%	9.9%	6.1%	10.0%	36.0%	152.5%
voltEqWNoR	19.3%	15.5%	15.4%	3.8%	7.6%	40.2%	202.3%
voltEWMA	40.4%	36.4%	16.9%	20.7%	25.2%	67.4%	240.1%
voltEWMANoR	41.2%	36.1%	19.7%	21.3%	25.0%	72.8%	279.7%
c+voltEqW	23.3%	21.7%	9.3%	8.7%	13.1%	38.4%	126.5%
c+voltEqWNoR	36.5%	21.7%	47.7%	0.8%	5.0%	119.3%	564.3%
c+voltEWMA	18.3%	17.2%	6.7%	6.4%	10.5%	29.3%	69.5%
c+voltEWMANoR	17.8%	15.3%	12.3%	0.4%	4.5%	37.8%	155.4%
ParamSGT	21.7%	21.6%	3.6%	12.1%	16.0%	27.3%	43.4%
ParamSGTNoR	22.2%	21.7%	5.0%	11.9%	16.0%	28.7%	59.8%
ParamSGTEWMA	36.6%	34.3%	7.4%	28.6%	30.6%	51.1%	89.8%
ParamSGTEWMANoR	36.9%	34.3%	8.6%	28.4%	30.6%	53.2%	132.4%
NormalEqW	30.6%	30.7%	4.4%	17.5%	25.1%	35.3%	109.5%
RiskMetrics	41.4%	39.9%	7.4%	33.1%	37.1%	50.3%	156.5%
Historical	11.8%	11.3%	4.3%	2.7%	5.9%	19.5%	32.8%
HistVolAdj	46.3%	42.1%	19.9%	20.0%	27.2%	77.7%	285.0%

Parametric SGT EWMA, has average NRMSD means between 35% and 46% looking at all quantiles.

Most models benefit from a reestimation of parameters (quantile regression models and Parametric SGT), with both mean and dispersion reduction. The exceptions are the quantile regression model with a constant and an EWMA estimate that has lower standard deviation but higher mean than the no reestimation version (at all quantiles), similar to what also happens to the GARCH models. For instance, looking at the GARCH(1,1) quantile regression VaR model, not reestimating parameters produces NRMSD results for the 1% VaR results with a variability of 13.3% versus 7.1% and NRMSD mean of 15.1% versus 19.1%. The same logic applies to the GARCH model with known volatility.

EWMA models continue to lose by far to the equally weighted models in all quantiles. Once again, exception to the VaR model with a constant term and EWMA estimate (with and without reestimated parameters) that actually beats all the other quantile regression models benchmarks in mean (18.3% at 1% significance level) and dispersion figures.

Quantile regression VaR models with the constant term, including the GARCH models, are superior to the ones without it. However, exclusively at 1% quantile, that is not the case for the VaR model with a constant term and an equally weighted volatility estimate

(with and without reestimated parameters). This model is not able to win against his matching version of only one explanatory variable.

The Parametric SGT VaR model with the equally weighted volatility estimate has a huge increase on the NRMSD mean results, from the previous scenarios. It seems to struggle to fit the true parameters, specially at low significance levels. Since we are now generating returns with negative skewness and heavy tails, this could mean that it could be underestimating the VaR. The difference for the Historical VaR (no adjustment) as been very close, with a small advantage for the Historical VaR but this time this difference is huge. Since we are already generating returns from a SGT distribution, the main driver of the Parametric SGT VaR model estimates is the set of the fitted parameters. Hence, we were expecting it to perform much better.

Table 4.6: This table shows the summary NRMSD statistics for 1000 simulations of the $SGT(0.02\%, 1.25\%, 2, 3, 0.3)$ returns process. Each simulation tests 2500 daily 1% VaR estimates.

SGT(2, 3, 0.3) returns, $\alpha = 1\%$							
	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	13.3%	12.1%	5.9%	3.7%	7.2%	22.8%	59.9%
GARCHvolknownNoR	11.5%	9.8%	9.1%	0.1%	1.6%	27.0%	74.7%
GARCH2G	15.9%	14.3%	7.3%	4.6%	8.5%	27.3%	80.1%
GARCH2GNoR	12.2%	9.9%	11.4%	0.1%	2.1%	27.8%	191.5%
voltEqW	17.1%	14.7%	11.3%	4.8%	8.5%	30.8%	193.9%
voltEqWNoR	18.6%	15.0%	15.4%	4.4%	6.9%	41.7%	296.6%
voltEWMA	41.9%	37.5%	18.4%	22.1%	26.7%	71.3%	347.0%
voltEWMANoR	41.9%	37.2%	21.2%	21.2%	24.9%	74.3%	434.2%
c+voltEqW	16.9%	15.9%	5.9%	5.7%	9.8%	27.9%	65.4%
c+voltEqWNoR	20.4%	14.5%	22.0%	0.2%	3.3%	59.9%	357.3%
c+voltEWMA	14.8%	13.7%	6.3%	5.4%	8.4%	24.0%	90.6%
c+voltEWMANoR	13.3%	11.3%	9.3%	0.4%	3.0%	31.5%	111.9%
ParamSGT	43.1%	41.2%	12.0%	17.8%	28.7%	65.2%	122.5%
ParamSGTNoR	44.3%	40.7%	21.0%	10.5%	28.3%	72.1%	457.9%
ParamSGTEWMA	57.7%	53.9%	19.5%	31.6%	39.1%	89.5%	349.5%
ParamSGTEWMANoR	58.2%	53.4%	24.2%	26.6%	38.0%	93.9%	485.8%
NormalEqW	27.1%	23.9%	17.7%	6.4%	13.3%	49.3%	402.8%
RiskMetrics	44.9%	40.1%	21.3%	22.4%	27.9%	76.8%	415.2%
Historical	9.0%	8.6%	3.3%	2.1%	4.6%	15.5%	24.8%
HistVolAdj	46.5%	41.9%	20.8%	22.9%	28.7%	78.0%	381.6%

Moving on to the last scenario, where returns have heavy tails but positive skewness. From the results in Table 4.6 and in Table A.6, models that does not use the quantile regression methodology or the Historical VaR with no adjustment, are still getting worse results in all quantiles, specially for the Parametric SGT models and, also, the Parametric Normal equally weighted volatility. We are not fully aware of the reasons behind these huge increases (as an example, the Parametric SGT model with reestimation of parameters

and equally weighted volatility in the $SGT(0.02\%, 1.25\%, 2, 3, 0)$, at 1% quantile has 11.6% of average NRMSD and now has 43.1%). A possible explanation could have to be that the distribution shift to the right causes an increased number of high positive returns that will strike high volatility estimates (none of our models/benchmarks have asymmetric volatility estimation) and that will push the VaR estimates to be too high. This strange effect is not so evident on the 5% quantile, for instance, the Parametric SGT equally weighted (both versions) recovers more than 20 percentage points of the average NRMSD from the 2.5% VaR, finishing with more reasonable results when we compare them with the other SGT simulations.

Looking at all significance levels on Table 4.6 and on Table A.6 in the Appendix, quantile regression models show a lot of flexibility and, in general, have increased their performance comparing to the negative skewed scenario. The two GARCH models with no reestimation parameters seem to have the lowest average NRMSD on this group (11.5% for the true volatility model and 12.2% for the estimated volatility version) but, as usual, results are less consistent when comparing to the periodic reestimation of parameters that has higher average results but lower variability of the outcome. Quantile regression VaR models with two parameters performed better than in all $SGT(0.02\%, 1.25\%, 2, 3, \lambda)$ returns distributions. The best quantile regression benchmark model is the one with a constant term and EWMA volatility specification, either the reestimation version (average of 14.8%) and the NoR model (13.3%). The single explanatory model with EWMA volatility is still the worst quantile regression model, what reinforces the conclusion that the homoskedasticity of simulated returns penalizes the EWMA estimates. Although the EWMA method could theoretically be able to capture this behavior in the long run, the variability of estimates adds too much noise to the VaR estimate. Nevertheless, results show that adding a constant term, it helps minimizing all this noise and just be the leading explanatory variable of the quantile regression structure, smoothing the estimates.

NoR models are, in general, slightly worse on average and, all of them, in dispersion of results. Again we invite the reader to read the full results from the additional simulations carried to all models that does not reestimate parameters on Appendix A.2.

5 Conclusion

Our main goal with this dissertation was to evaluate the performance of the new quantile regression model proposed by Zheng et al. (2018), on estimating the 1-day Value-at-Risk, against more traditional methods and other quantile regression VaR models. Although this approach does not belong to the regular models class, largely used by financial institutions, we believe that the advantage of not making any particular distributional assumption and the huge flexibility make it a very useful model to be considered. Naturally, estimating conditional quantiles can be computationally intensive, depending on the number of explanatory variables used, and the huge flexibility can bite decision makers back since it brings the same size of accountability.

The GARCH(1,1) quantile regression model overcomes the problem of estimating the conditional quantile based on a linear relationship with a variance estimate of GARCH models, introducing a new transformation (14) that works as the inverse of the square root. The quantile regression parameters are estimated as proposed by Koenker and Bassett (1978). Results from Zheng et al. (2018) show that this new model is superior to the well known RiskMetrics model. Also, this same model is tested but making use of the previously defined volatility. We want to understand how much difficult it is to estimate the quantile regression parameters even with a crystal ball that gives us the volatility generated in the returns' process.

In order to examine this model performance, we first generate returns using the Monte Carlo simulation process, with returns following a GARCH(1,1) process. Two sets of parameters for the generating process are used. First we use the same parameters $(\omega, \gamma, \beta) = (0.1, 0.15, 0.8)$ as Zheng et al. (2018), that, in our opinion, are not suited for daily returns since the implicit long run daily standard deviation is unreasonable (141%). We then ran our simulations with more realistic parameters: $(\omega, \gamma, \beta) = (3.125 \times 10^{-7}, 0.05, 0.9)$. Additionally, since in the real world we do not know if returns exhibits always and every time heteroskedastic processes, we simulate a Skewed Generalized T distribution (SGT). This distribution allow us to define the tail behavior and the presence of skewness. Hence, we test 4 main scenarios for the returns distribution: Normal, heavy tails without skewness, heavy tails with negative skewness and heavy tails with positive skewness. Finally, for each simulation, a total of 4500 returns are generated. A test window with the last 2500 is defined so that we have the same number of daily VaR estimates for all models. We simulate this process 1000 times and for each simulation we compare the estimated VaR with the true VaR given by the parameters set to each scenario, for significance levels of 1%, 2.5% and 5%. Since 1000 times could not be enough to avoid simulation bias, and it was not feasible for time restrictions to increase the number of simulations, we also run 10000 simulations for the group of models that does not need to reestimate parameters (main time consuming component). For evaluation purposes we compute and record the

NRMSD for each simulation. Naturally, the best model in every scenario will be the one with the lowest average NRMSD. Also we have to take into consideration other metrics that reveal results dispersion like the NRMSD standard deviation, minimum, maximum, 5% quantile and 95% quantile. We seek for efficiency with consistency.

As benchmarks, we start by selecting two simple quantile regression structures to predict the VaR_t . So, from the quantile regression methodologies we start with a model that has only the volatility estimate for day t as a parameter. The second has the volatility estimate for t and also a constant term. From each of these two structures we can have another two versions, one that uses a volatility estimate with a equally weighted method based on the last 500 returns and other that uses an EWMA estimate with $\lambda = 0.94$. Our aim is to check if the GARCH(1,1) quantile regression model is really worth it when compared with specifications with fewer parameters, hence with less computational time. We also compare this model performance with more traditional models like the Parametric Normal model (RiskMetrics model and Parametric Normal with equally weighted volatility estimation process), the Parametric SGT with equally weighted method and the Parametric SGT with EWMA method, the Historical VaR with EWMA volatility adjustment and, finally, the Historical VaR (no adjustment made). For all applicable models, we test if the reestimation of parameters makes the difference. This is done by estimating the Value-at-Risk from all these structures with reestimation of parameters every 20 days and, additionally, without any reestimation of parameters. All models are defined with a sample size of 1000 observations.

The results show that by using the same GARCH returns parameters as Zheng et al. (2018), the benchmark models struggle a lot more and make the GARCH quantile estimator shine as the best model by far. If we rather use more reasonable parameters, it is game changing and the GARCH(1,1) quantile regression model is defeated not only by the RiskMetrics model but also by the quantile regression models tested (except at the 2.5% significance level where the quantile regression model with a constant term and an EWMA estimate slightly unperformed). Overall, as expected, models with EWMA estimation are better than the equally weighted ones in both returns processes.

Regarding the SGT simulations, results on the four scenarios show that the Historical (no adjustment) is the best performing model in the presence of heavy tails, the simplicity pays-off. Also, the homoskedasticity of returns can not be efficiently captured by the EWMA models pushing well known VaR models like the RiskMetrics model and the Historical VaR with volatility adjustment to the bottom. The only exception is the quantile regression model with EWMA estimate and an intercept, that is able to avoid this problem by having the constant term in the quantile regression structure that does all the job. GARCH(1,1) quantile regression model has a robust performance at all scenarios but it is not superior to the quantile regression model with just an intercept and an EWMA estimate in any scenario.

Overall, the reestimation of parameters does not seem to make much of a difference in terms of the average NRMSD result (slightly lower mean) but it certainly helps to increase the consistency of results, mainly with lower variability. The Parametric SGT equally weighted appears to perform reasonably well in all scenarios (except for the positive skewness scenario), even in the Normal returns scenario he does not stay far away from the best model (Parametric Normal VaR with equally weighted volatility). Also on GARCH returns, with reasonable parameters it does not find problems with heteroskedasticity. These results are important because if we do not know the true distribution of returns we would rather choose the Parametric SGT VaR model over the Parametric Normal VaR model for its consistency in different scenarios. Regarding the quantile regression models, overall they are very flexible models working solid in all scenarios, specially the equally weighted models, without going too far away from the true VaR although neither of these models won in any scenario and quantile. All quantile regression models are pretty robust in both environments (GARCH and SGT returns). They were very consistent at the top of the picking order in any scenario without exception. Results show that including a constant term dramatically increased the explanation power of the EWMA volatility model. In the equally weighted model it was only effective on the 2.5% and 5% quantiles. Nevertheless, combining the results of all the experiments, if we had to pick one model from the quantile regression models, the constant term with EWMA volatility model seems to be the most consistent. It combines an EWMA estimate that improves the volatility estimate with the current market conditions and, also, has a constant term that helps to balance in the presence of homoskedasticity. Hence, the GARCH(1,1) quantile regression VaR model proposed by Zheng et al. (2018) seems to not improve the already established quantile regression specifications.

For further research we believe that it would be interesting to extend the examination of this recent quantile regression model using other returns generating processes and different parameters reestimation frequency. Then, it could be interesting to test with financial market data, despite the necessary cumbersome backtesting procedures. One exciting experiment would be to evaluate if we can improve these models with exogenous variables. Instead of only using models that describe volatility with variables within the model, we would have the conditional standard deviation depending on implied volatilities extracted from volatility indices.

Bibliography

- Abad, Pilar, Sonia Benito, and Carmen López (2014). “A comprehensive review of Value at Risk methodologies”. In: *Spanish Review of Financial Economics* 12.1, pp. 15–32. ISSN: 21731268. DOI: 10.1016/j.srfe.2013.06.001.
- Ammann, Manuel and Christian Reich (2001). “VaR for nonlinear financial instruments – linear approximation or full Monte Carlo?” In: *Financial Markets and Portfolio Management* 15.3, pp. 363–378. ISSN: 1934-4554. DOI: 10.1007/s11408-001-0306-9.
- Assaf, Ata (2017). “The Stochastic Volatility Model, Regime Switching and Value-at-Risk (VaR) in International Equity Markets”. In: *Journal of Mathematical Finance* 07.02, pp. 491–512. ISSN: 2162-2434. DOI: 10.4236/jmf.2017.72026.
- Basel Comittee (2004). *Basel II - International Convergence of Capital Measurement and Capital Standards: A Revised Framework*. Tech. rep. June, p. 251. URL: <https://www.bis.org/publ/bcbs107.htm>.
- Beder, Tanya Styblo (1995). “VAR: Seductive but Dangerous”. In: *Financial Analysts Journal* 51.5, pp. 12–24. ISSN: 0015-198X. DOI: 10.2469/faj.v51.n5.1932.
- BenSaïda, Ahmed and Skander Slim (2016). “Highly flexible distributions to fit multiple frequency financial returns”. In: *Physica A: Statistical Mechanics and its Applications* 442, pp. 203–213. ISSN: 0378-4371. DOI: <https://doi.org/10.1016/j.physa.2015.09.021>. URL: <http://www.sciencedirect.com/science/article/pii/S0378437115007487>.
- Blair, Bevan J, Ser-Huang Poon, and Stephen J Taylor (2001). “Forecasting S&P 100 volatility: the incremental information content of implied volatilities and high-frequency index returns”. In: *Journal of Econometrics* 105.1, pp. 5–26. ISSN: 0304-4076. DOI: [https://doi.org/10.1016/S0304-4076\(01\)00068-9](https://doi.org/10.1016/S0304-4076(01)00068-9). URL: <http://www.sciencedirect.com/science/article/pii/S0304407601000689>.
- Bollerslev, Tim (1986). “Generalized autoregressive conditional heteroskedasticity”. In: *Journal of Econometrics* 31.3, pp. 307–327.
- Chen, James Ming (2013). “Measuring Market Risk Under Basel II, 2.5, and III: VAR, Stressed VAR, and Expected Shortfall”. In: *SSRN Electronic Journal* 6891.502, pp. 184–201. DOI: 10.2139/ssrn.2252463.
- Dowd, Kevin (2007). *Measuring Market Risk (2nd Edition)*, p. 412. ISBN: 9780470013038.
- Engle, Robert F and Simone Manganelli (1999). “CAViaR: Conditional Value at Risk by Quantile Regression”. In: *National Bureau of Economic Research Working Paper*

- Series No. 7341. September. DOI: 10.3386/w7341. URL: <http://www.nber.org/papers/w7341>. URL: <http://www.nber.org/papers/w7341.pdf>.
- Engle, Robert F. (1982). *Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation*.
- European Commission (2014). "COMMISSION DELEGATED REGULATION (EU) No 604/2014". In: *Official Journal of the European Union*, pp. 48–119.
- Federal Deposit Insurance Corporation (1999). *1999 Annual Report*. Tech. rep.
- Guo, Hongtao (2013). "Risk analysis using regression quantiles: evidence from international equity markets". In: *The International journal of Business and Finance Research* 7.2, pp. 1–16.
- Hansen, Christian, James B. McDonald, and Whitney K. Newey (2010). "Instrumental variables estimation with flexible distributions". In: *Journal of Business and Economic Statistics*. ISSN: 07350015. DOI: 10.1198/jbes.2009.06161.
- Holton, Glyn A (2002). "History of Value-at-Risk:1922-1998". In: *unpublished*.
- Hull, John and Alan White (1998). "Incorporating volatility updating into the historical simulation method for value-at-risk". In: *The Journal of Risk* 1.1, pp. 5–19. ISSN: 14651211. DOI: 10.21314/jor.1998.001.
- Koenker, Roger and Gilbert Bassett (1978). "Regression Quantiles". In: *Econometrica* 46.1, p. 33. ISSN: 00129682. DOI: 10.2307/1913643.
- Koenker, Roger and Zhijie Xiao (2006). "Quantile autoregression". In: *Journal of the American Statistical Association* 101.475, pp. 980–990. ISSN: 01621459. DOI: 10.1198/016214506000000672.
- Koenker, Roger and Quanshui Zhao (1996). *Conditional Quantile Estimation and Inference for Arch Models*. DOI: 10.1017/s0266466600007167.
- Koul, Hira and E. Saleh (1995). "Autoregression Quantiles and Related Rank-Scores Processes". In: *Statistics* 12.2, pp. 670–689. ISSN: 00905364. DOI: 10.1214/aos/1176348654. URL: <http://projecteuclid.org/euclid.aos/1176348654>.
- Morgan J P and Reuters TM (1996). "RiskMetrics- Technical Report". In: URL: <http://www.jpmorgan.com/RiskManagement/RiskMetrics/RiskMetrics.html>.
- Namugaya, Jalira, Patrick G. O. Weke, and W. M. Charles (2014). "Modelling volatility of stock returns: Is GARCH(1,1) enough?" In: *International Journal of Sciences: Basic and Applied Research* 16.2, pp. 216–223. ISSN: 2307-4531.

- Portnoy, Stephen (1991). “Asymptotic behavior of regression quantiles in non-stationary, dependent cases”. In: *Journal of Multivariate Analysis*. ISSN: 10957243. DOI: 10.1016/0047-259X(91)90034-Y.
- Sheikh, Abdullah Z. and Hongtao Qiao (2009). “Non-normality of Market Returns”. In: *The Journal of Alternative Investments* 12.3, pp. 8–35. DOI: 10.3905/JAI.2010.12.3.008.
- Theodossiou, Peter (1998). “Financial Data and the Skewed Generalized T Distribution”. In: *SSRN Electronic Journal*. DOI: 10.2139/ssrn.65037.
- Xiao, Zhijie and Roger Koenker (2009). “Conditional quantile estimation for garch models”. In: *Evaluation*, pp. 1–44.
- Zheng, Yao et al. (2018). “Hybrid quantile regression estimation for time series models with conditional heteroscedasticity”. In: *Journal of the Royal Statistical Society. Series B: Statistical Methodology* 80.5, pp. 975–993. ISSN: 14679868. DOI: 10.1111/rssb.12277.

A Appendix

A.1 Results Summary, 1000 simulations

A.1.1 NRMSD results with 1000 sample size

Table A.1: This table shows the summary NRMSE statistics of VaR estimates at the 2.5% and 5% significance levels for the 1000 simulations of GARCH returns, generated with $(\omega, \gamma, \beta) = (0.1, 0.15, 0.8)$ as Zheng et al. (2018). All models estimate daily VaR using a sample size of 1000 observations, including to estimate all parameters, except for the volatility estimate with equally weighted method that uses a sample size of 500. The summary statistics for the 1% significance level were presented on Table 4.1.

GARCH(1,1) returns, Zheng et al. (2018)'s parametrization, $n = 1000$														
$\alpha = 2.5\%$														
Model	Mean	Median	StDev	Min	5%	95%	Max	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	6.9%	6.7%	1.7%	3.1%	4.7%	10.0%	12.9%	6.6%	6.5%	1.6%	3.1%	4.3%	9.4%	13.7%
GARCHvolknownNoR	6.4%	5.9%	3.2%	1.5%	2.2%	12.8%	21.4%	6.2%	5.8%	3.0%	1.4%	2.2%	11.6%	19.9%
GARCHQR	9.4%	9.1%	1.9%	4.8%	6.7%	12.9%	19.5%	9.1%	8.8%	1.8%	4.4%	6.5%	12.2%	17.6%
GARCHQRNoR	8.1%	7.6%	3.6%	0.8%	3.3%	14.3%	31.8%	7.9%	7.5%	3.3%	1.2%	3.1%	14.1%	21.6%
volEqW	34.7%	32.9%	7.8%	22.1%	26.1%	48.0%	95.1%	33.2%	31.8%	6.4%	21.6%	25.9%	44.0%	78.2%
volEqWNoR	33.8%	32.1%	7.6%	22.2%	25.6%	46.0%	99.8%	32.8%	31.3%	6.4%	22.5%	25.9%	43.7%	82.9%
volEWMA	17.3%	16.6%	3.0%	12.1%	14.0%	22.8%	48.9%	17.0%	16.4%	2.8%	12.0%	13.9%	21.8%	41.1%
volEWMANoR	16.8%	16.3%	2.9%	11.7%	13.6%	21.6%	50.5%	16.5%	16.0%	2.6%	11.7%	13.6%	21.0%	43.5%
c+volEqW	36.2%	33.5%	11.0%	21.7%	26.6%	53.7%	129.1%	34.0%	32.3%	7.8%	21.3%	26.3%	46.5%	97.8%
c+volEqWNoR	36.5%	33.3%	13.8%	22.2%	26.4%	57.6%	194.7%	35.2%	32.4%	11.5%	22.2%	26.1%	52.7%	190.6%
c+volEWMA	17.3%	16.6%	3.0%	12.2%	14.0%	22.8%	48.8%	17.1%	16.4%	2.8%	12.3%	14.0%	22.0%	40.6%
c+volEWMANoR	16.8%	16.3%	2.9%	11.7%	13.6%	21.8%	50.5%	16.5%	16.0%	2.6%	11.7%	13.6%	21.2%	43.4%
ParamsGT	32.3%	31.2%	5.7%	21.9%	25.2%	42.1%	73.3%	31.5%	30.5%	5.2%	21.7%	25.0%	40.1%	70.2%
ParamsGTNoR	32.3%	31.1%	6.1%	21.5%	25.2%	42.5%	86.5%	31.9%	30.7%	5.9%	21.7%	25.2%	41.6%	84.5%
ParamsGTEWMA	16.0%	15.8%	1.9%	12.0%	13.5%	19.5%	27.8%	15.8%	15.6%	1.8%	12.2%	13.6%	19.0%	31.7%
ParamsGTEWMANoR	16.1%	15.7%	2.4%	12.0%	13.5%	19.9%	45.8%	16.2%	15.8%	2.6%	12.3%	13.7%	20.2%	60.6%
NormalEqW	32.0%	30.8%	6.1%	21.5%	25.1%	42.2%	85.2%	32.0%	30.8%	6.1%	21.5%	25.1%	42.2%	85.2%
RiskMetrics	15.8%	15.4%	2.3%	11.8%	13.3%	20.0%	40.7%	15.8%	15.4%	2.3%	11.8%	13.3%	20.0%	40.7%
Historical	32.7%	31.2%	6.5%	21.9%	25.5%	44.1%	95.3%	31.7%	30.4%	5.6%	21.3%	25.2%	42.8%	75.2%
HistVolAdj	17.5%	16.7%	3.1%	12.3%	13.8%	23.0%	48.6%	17.0%	16.4%	2.8%	11.8%	13.8%	22.0%	43.9%

Table A.2: This table shows the summary NRMDS statistics of VaR estimates at the 2.5% and 5% significance levels for the 1000 simulations of GARCH returns, generated with $(\omega, \gamma, \beta) = (3.125 \times 10^{-7}, 0.05, 0.9)$. All models estimate daily VaR using a sample size of 1000 observations, including to estimate all parameters, except for the volatility estimate with equally weighted method that uses a sample size of 500. The summary statistics for the 1% significance level were presented on Table 4.2.

GARCH(1,1) returns, realistic parametrization, $n = 1000$														
Model	$\alpha = 2.5\%$						$\alpha = 5\%$							
	Mean	Median	StDev	Min	5%	95%	Max	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	6.6%	6.5%	1.4%	3.2%	4.6%	9.0%	13.4%	7.0%	6.9%	1.4%	3.6%	4.9%	9.3%	12.7%
GARCHvolknownNoR	6.1%	6.0%	3.1%	0.0%	1.6%	11.9%	20.4%	6.7%	6.3%	3.0%	0.8%	2.8%	12.4%	21.6%
GARCHQR	8.6%	8.1%	5.4%	4.3%	6.0%	11.3%	100.9%	9.1%	8.4%	5.4%	4.9%	6.5%	11.7%	113.4%
GARCHQRNoR	6.9%	6.8%	3.2%	0.0%	2.1%	12.8%	18.6%	7.5%	7.2%	3.2%	1.9%	3.0%	13.0%	21.6%
voltEqW	12.8%	12.5%	1.8%	9.1%	10.4%	16.1%	19.8%	12.7%	12.5%	1.6%	9.0%	10.5%	15.5%	19.2%
voltEqWNoR	12.5%	12.2%	1.9%	8.7%	10.1%	16.1%	26.4%	12.3%	11.9%	1.8%	8.7%	9.9%	15.7%	25.6%
voltEWMA	9.6%	9.5%	1.3%	6.6%	7.9%	12.0%	17.3%	9.4%	9.3%	1.2%	6.8%	7.9%	11.6%	17.7%
voltEWMANoR	9.3%	9.0%	1.6%	6.5%	7.5%	12.6%	16.7%	9.1%	8.8%	1.4%	6.5%	7.4%	11.8%	15.6%
c+voltEqW	14.2%	13.8%	2.1%	9.8%	11.4%	18.1%	28.1%	13.7%	13.5%	1.8%	9.3%	11.2%	17.2%	24.2%
c+voltEqWNoR	15.7%	13.4%	6.5%	8.8%	10.2%	28.8%	70.7%	15.1%	13.2%	6.0%	8.4%	10.1%	26.0%	62.2%
c+voltEWMA	7.2%	7.0%	1.5%	4.0%	5.0%	9.9%	19.1%	6.8%	6.6%	1.4%	4.0%	4.9%	9.3%	16.6%
c+voltEWMANoR	6.5%	5.9%	2.8%	2.5%	3.2%	12.1%	20.4%	6.4%	5.7%	2.8%	2.6%	3.0%	11.6%	19.0%
ParamSGT	11.6%	11.4%	1.3%	8.7%	9.8%	14.0%	17.8%	11.5%	11.3%	1.2%	8.7%	9.8%	13.7%	16.9%
ParamSGTNoR	11.6%	11.5%	1.3%	8.7%	9.8%	14.1%	17.2%	11.5%	11.4%	1.2%	8.7%	9.8%	13.8%	17.0%
ParamSGTEWMA	8.5%	8.5%	0.9%	6.1%	7.1%	10.0%	12.9%	8.4%	8.4%	0.8%	5.9%	7.1%	9.8%	11.5%
ParamSGTEWMANoR	8.5%	8.5%	0.9%	5.8%	7.1%	10.2%	11.5%	8.4%	8.4%	0.8%	6.0%	7.1%	9.9%	10.8%
NormalEqW	11.4%	11.3%	1.2%	8.7%	9.6%	13.8%	17.1%	11.4%	11.3%	1.2%	8.7%	9.6%	13.8%	17.1%
RiskMetrics	8.3%	8.3%	0.8%	5.6%	7.0%	9.7%	12.7%	8.3%	8.3%	0.8%	5.6%	7.0%	9.7%	12.7%
Historical	12.0%	11.8%	1.5%	8.7%	9.9%	14.8%	25.0%	11.8%	11.6%	1.3%	8.6%	9.9%	14.2%	20.3%
Hist VolAdj	10.1%	9.9%	1.5%	6.6%	8.1%	12.9%	17.7%	9.8%	9.6%	1.4%	6.7%	7.9%	12.5%	19.6%

Table A.3: This table shows the summary NRMSE statistics of VaR estimates at the 2.5% and 5% significance levels for the 1000 simulations of $SGT(0.02\%, 1.25\%, 2, \infty, 0)$ returns. All models estimate daily VaR using a sample size of 1000 observations, including to estimate all parameters, except for the volatility estimate with equally weighted method that uses a sample size of 500. The summary statistics for the 1% significance level were presented on Table 4.3.

Model	$\alpha = 2.5\%$										$\alpha = 5\%$					
	Mean	Median	StDev	Min	5%	95%	Max	Mean	Median	StDev	Min	5%	95%	Max		
GARCHvolknown	5.4%	5.3%	1.4%	2.5%	3.4%	8.1%	10.0%	5.5%	5.3%	1.3%	2.2%	3.5%	7.6%	11.7%		
GARCHvolknownNoR	4.7%	4.2%	3.1%	0.0%	0.6%	10.6%	16.5%	5.0%	4.7%	2.7%	0.2%	1.2%	9.9%	15.6%		
GARCHQR	6.3%	6.1%	1.8%	2.4%	3.9%	9.7%	14.9%	6.8%	6.5%	1.8%	1.9%	4.2%	10.1%	15.3%		
GARCHQRNoR	4.9%	4.4%	3.1%	0.1%	0.8%	10.7%	24.1%	5.3%	4.9%	2.8%	0.1%	1.5%	10.1%	27.8%		
volEqW	5.3%	5.1%	1.5%	1.7%	3.2%	7.9%	12.2%	5.2%	5.1%	1.4%	1.9%	3.2%	7.6%	11.6%		
volEqWNoR	4.8%	4.2%	2.1%	1.3%	2.2%	8.9%	15.6%	4.6%	4.1%	2.1%	1.5%	2.2%	8.7%	16.2%		
volEWMA	13.3%	13.2%	1.1%	10.6%	11.7%	15.4%	18.7%	13.1%	13.0%	1.0%	10.6%	11.6%	14.9%	17.1%		
volEWMANoR	13.1%	12.8%	1.4%	10.3%	11.4%	15.9%	19.9%	12.9%	12.7%	1.2%	10.3%	11.3%	15.0%	18.9%		
c+volEqW	7.3%	7.2%	1.9%	3.0%	4.5%	10.7%	16.4%	7.0%	6.7%	1.8%	2.7%	4.4%	10.2%	15.1%		
c+volEqWNoR	8.8%	6.8%	7.4%	0.2%	1.5%	22.0%	53.6%	8.4%	6.4%	7.3%	0.1%	1.5%	21.8%	59.0%		
c+volEWMA	6.1%	5.9%	1.4%	2.6%	4.1%	8.6%	12.2%	5.8%	5.7%	1.4%	2.9%	3.8%	8.2%	10.8%		
c+volEWMANoR	5.7%	5.3%	3.0%	0.1%	1.6%	11.1%	19.3%	5.4%	5.1%	2.9%	0.2%	1.3%	10.7%	16.9%		
ParamSGT	3.6%	3.6%	0.9%	1.5%	2.2%	5.3%	7.8%	3.4%	3.3%	0.9%	1.3%	2.1%	5.0%	7.4%		
ParamSGTNoR	3.6%	3.4%	1.1%	1.1%	2.0%	5.7%	8.7%	3.4%	3.3%	1.0%	1.1%	2.0%	5.2%	8.5%		
ParamSGTEWMA	12.5%	12.4%	0.8%	10.2%	11.2%	13.8%	15.1%	12.4%	12.4%	0.7%	10.2%	11.2%	13.7%	15.0%		
ParamSGTEWMANoR	12.5%	12.4%	0.8%	10.2%	11.2%	13.9%	15.5%	12.4%	12.4%	0.8%	10.1%	11.1%	13.8%	15.4%		
NormalEqW	3.2%	3.1%	0.8%	1.5%	2.0%	4.6%	6.7%	3.3%	3.2%	0.8%	1.5%	2.1%	4.7%	6.8%		
RiskMetrics	12.4%	12.3%	0.7%	9.9%	11.2%	13.6%	14.9%	12.4%	12.3%	0.7%	9.9%	11.2%	13.7%	14.9%		
Historical	4.1%	4.0%	1.4%	1.4%	2.2%	6.7%	9.8%	4.0%	3.8%	1.3%	1.2%	2.0%	6.3%	10.3%		
HistVolAdj	13.9%	13.8%	1.3%	10.5%	12.0%	16.2%	18.9%	13.5%	13.4%	1.2%	10.6%	11.8%	15.7%	17.7%		

Table A.4: This table shows the summary NRMSE statistics of VaR estimates at the 2.5% and 5% significance levels for the 1000 simulations of $SGT(0.02\%, 1.25\%, 2, 3, 0)$ returns. All models estimate daily VaR using a sample size of 1000 observations, including to estimate all parameters, except for the volatility estimate with equally weighted method that uses a sample size of 500. The summary statistics for the 1% significance level were presented on Table 4.4.

$SGT(0.02\%, 1.25\%, 2, 3, 0)$ returns, $n = 1000$												
Model	$\alpha = 2.5\%$						$\alpha = 5\%$					
	Mean	Median	StDev	Min	5%	95%	Max	Mean	Median	StDev	Min	5%
GARCHvolknown	11.0%	10.2%	4.5%	3.3%	5.9%	18.3%	51.5%	9.4%	8.7%	4.3%	2.8%	4.9%
GARCHvolknownNoR	9.6%	8.4%	7.3%	0.1%	1.2%	23.1%	65.4%	8.4%	7.5%	6.4%	0.0%	1.2%
GARCHQR	13.4%	12.4%	5.5%	3.0%	7.2%	23.0%	63.6%	12.7%	11.6%	5.5%	3.8%	7.0%
GARCHQRNoR	9.9%	8.4%	8.1%	0.1%	1.5%	23.5%	130.7%	9.0%	7.8%	6.2%	0.1%	1.7%
voltEqW	15.5%	13.5%	8.6%	5.8%	8.0%	28.4%	108.8%	14.4%	12.5%	8.3%	4.2%	7.1%
voltEqWNoR	16.2%	13.0%	12.7%	3.8%	6.5%	35.6%	198.7%	16.0%	13.0%	12.6%	4.0%	6.3%
voltEWMA	37.2%	33.4%	15.1%	19.1%	24.7%	60.2%	174.2%	38.8%	35.0%	15.5%	20.2%	25.5%
voltEWMANoR	36.2%	32.0%	16.4%	19.7%	23.1%	60.6%	271.1%	35.5%	31.0%	16.1%	19.7%	23.0%
c+voltEqW	14.1%	13.5%	4.3%	5.4%	8.5%	21.9%	54.5%	11.3%	10.8%	3.5%	4.6%	6.7%
c+voltEqWNoR	19.2%	12.9%	27.9%	0.2%	2.7%	54.1%	597.3%	15.1%	10.4%	23.0%	0.1%	2.3%
c+voltEWMA	12.5%	11.6%	5.0%	5.0%	7.5%	19.5%	97.5%	10.0%	9.5%	3.3%	4.1%	6.1%
c+voltEWMANoR	11.5%	10.2%	7.5%	0.4%	2.4%	25.7%	70.4%	9.4%	8.3%	6.2%	0.2%	2.2%
ParamSGT	10.0%	8.7%	5.0%	3.4%	5.2%	19.6%	53.1%	9.8%	8.7%	4.4%	3.5%	5.3%
ParamSGTNoR	13.7%	10.7%	11.9%	3.9%	5.9%	31.5%	233.0%	14.7%	11.5%	12.1%	3.4%	6.3%
ParamSGTEWMA	32.0%	29.4%	10.5%	19.5%	22.8%	49.3%	200.0%	31.9%	29.4%	10.1%	19.4%	22.9%
ParamSGTEWMANoR	33.5%	29.9%	13.5%	19.5%	23.1%	53.7%	260.8%	33.8%	30.5%	13.4%	19.4%	23.2%
NormalEqW	14.9%	10.8%	14.2%	3.4%	5.6%	37.4%	133.0%	25.8%	21.8%	16.3%	6.6%	12.8%
RiskMetrics	34.6%	30.7%	14.8%	18.9%	23.0%	57.8%	181.2%	41.7%	37.6%	17.3%	19.5%	26.9%
Historical	7.8%	7.4%	2.7%	2.3%	4.1%	12.8%	20.1%	6.2%	6.0%	2.2%	2.0%	3.2%
HistVolAdj	41.1%	37.1%	16.9%	19.3%	26.7%	66.3%	197.0%	39.8%	35.5%	16.7%	19.8%	25.9%

Table A.5: This table shows the summary NRMSE statistics of VaR estimates at the 2.5% and 5% significance levels for the 1000 simulations of $SGT(0.02\%, 1.25\%, 2, 3, -0.3)$ returns. All models estimate daily VaR using a sample size of 1000 observations, including to estimate all parameters, except for the volatility estimate with equally weighted method that uses a sample size of 500. The summary statistics for the 1% significance level were presented on Table 4.5.

Model	$SGT(0.02\%, 1.25\%, 2, 3, -0.3)$ returns, $n = 1000$									
	$\alpha = 2.5\%$					$\alpha = 5\%$				
	Mean	Median	StDev	Min	5%	95%	Max	Mean	Median	StDev
GARCHvolknown	11.9%	11.1%	4.6%	4.7%	7.0%	19.4%	64.1%	10.7%	9.8%	4.5%
GARCHvolknownNoR	10.8%	9.4%	7.1%	0.1%	2.0%	24.3%	53.5%	9.7%	8.3%	6.8%
GARCHQR	15.6%	14.6%	6.8%	4.3%	8.5%	26.0%	116.9%	14.4%	13.3%	6.6%
GARCHQRNoR	11.6%	9.8%	8.7%	0.2%	2.0%	27.0%	107.5%	10.4%	9.1%	7.4%
volEqW	16.7%	14.8%	9.0%	5.4%	8.4%	31.0%	138.4%	15.6%	13.9%	8.4%
volEqWNoR	18.5%	14.8%	14.5%	4.3%	7.4%	42.9%	192.4%	18.1%	14.4%	14.0%
volEWM	38.7%	35.1%	16.3%	19.0%	24.9%	65.6%	244.9%	37.6%	34.3%	15.7%
volEWMANoR	40.2%	35.0%	19.1%	21.1%	24.7%	73.0%	253.9%	39.3%	34.8%	18.0%
c+volEqW	16.2%	15.3%	5.4%	5.1%	9.6%	24.4%	73.0%	13.3%	12.7%	4.2%
c+volEqWNoR	25.3%	15.3%	34.1%	0.4%	2.9%	79.1%	308.2%	19.2%	12.2%	24.1%
c+volEWM	13.8%	12.9%	4.9%	5.4%	8.1%	22.0%	56.8%	11.7%	11.0%	4.4%
c+volEWMANoR	12.8%	11.3%	9.1%	0.6%	2.6%	27.7%	142.8%	11.0%	9.7%	6.6%
ParamsGT	18.1%	17.7%	3.1%	9.8%	13.7%	22.9%	37.0%	14.9%	14.4%	3.3%
ParamsGTNoR	19.6%	18.6%	6.3%	8.3%	12.8%	29.2%	64.1%	17.6%	15.8%	7.9%
ParamsGTEWM	35.1%	32.6%	8.0%	26.5%	28.3%	51.5%	89.6%	34.2%	31.6%	8.8%
ParamsGTEWMANoR	36.0%	33.0%	9.8%	26.3%	28.4%	54.7%	143.9%	35.7%	32.3%	11.0%
NormalEqW	17.1%	15.9%	7.6%	6.8%	11.4%	25.5%	146.5%	15.7%	12.4%	12.4%
RiskMetrics	34.8%	31.6%	12.1%	24.8%	27.1%	53.8%	195.7%	36.4%	32.2%	16.3%
Historical	8.6%	8.2%	3.0%	2.3%	4.4%	14.2%	22.5%	7.2%	6.8%	2.4%
HistVolAdj	43.2%	39.2%	18.9%	19.3%	26.8%	74.1%	303.7%	41.6%	37.7%	18.2%

Table A.6: This table shows the summary NRMSE statistics of VaR estimates at the 2.5% and 5% significance levels for the 1000 simulations of $SGT(0.02\%, 1.25\%, 2, 3, 0.3)$ returns. All models estimate daily VaR using a sample size of 1000 observations, including to estimate all parameters, except for the volatility estimate with equally weighted method that uses a sample size of 500. The summary statistics for the 1% significance level were presented on Table 4.6.

SGT(0.02%, 1.25%, 2, 3, 0.3) returns, n = 1000														
$\alpha = 2.5\%$														
Model	Mean	Median	StDev	Min	5%	95%	Max	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	9.6%	8.8%	4.1%	3.3%	5.3%	16.4%	47.6%	8.2%	7.6%	3.6%	2.6%	4.7%	13.7%	55.8%
GARCHvolknownNoR	8.5%	7.2%	7.2%	0.0%	1.3%	19.1%	105.1%	7.4%	6.4%	5.7%	0.0%	1.4%	16.8%	47.3%
GARCHQR	12.3%	11.3%	5.7%	4.6%	6.8%	21.0%	78.2%	11.1%	9.8%	6.2%	3.3%	5.9%	18.9%	109.3%
GARCHQRNoR	9.3%	7.9%	8.6%	0.3%	1.6%	20.5%	150.6%	8.0%	6.7%	7.8%	0.1%	1.6%	17.5%	183.9%
voltEqW	15.2%	12.9%	10.8%	4.3%	7.0%	29.3%	180.6%	14.6%	12.2%	10.3%	3.8%	6.7%	28.9%	162.9%
voltEqWNoR	17.7%	14.1%	15.1%	3.3%	7.0%	36.9%	302.5%	17.5%	14.1%	15.0%	3.4%	6.8%	38.3%	300.1%
voltEWMA	40.5%	36.1%	18.0%	21.4%	26.1%	66.9%	339.7%	39.6%	35.2%	17.8%	21.7%	25.6%	66.8%	335.9%
voltEWMANoR	40.5%	35.6%	20.0%	21.2%	24.9%	71.2%	393.9%	39.7%	34.9%	19.3%	20.6%	24.9%	70.7%	374.1%
c+voltEqW	11.0%	10.7%	3.7%	4.1%	6.8%	16.6%	52.6%	8.4%	8.0%	2.7%	2.9%	5.1%	12.7%	37.7%
c+voltEqWNoR	13.9%	10.0%	15.4%	0.2%	2.3%	35.9%	201.2%	11.0%	7.8%	13.6%	0.1%	1.8%	28.5%	232.0%
c+voltEWMA	9.9%	9.3%	3.8%	4.0%	5.6%	15.7%	54.2%	7.6%	7.2%	2.8%	2.3%	4.4%	11.7%	34.4%
c+voltEWMANoR	9.4%	8.0%	6.4%	0.3%	2.1%	21.2%	66.5%	7.2%	6.2%	5.1%	0.2%	1.8%	15.2%	76.3%
ParamSGT	33.6%	32.4%	7.9%	14.6%	23.9%	47.2%	82.1%	24.5%	23.8%	5.5%	10.7%	17.3%	34.5%	60.9%
ParamSGTNoR	36.1%	32.6%	20.1%	5.5%	19.7%	61.3%	420.8%	28.4%	24.8%	19.3%	5.5%	12.0%	54.8%	383.3%
ParamSGTEWMA	50.5%	47.1%	16.5%	28.3%	35.2%	77.6%	300.7%	44.1%	40.8%	14.5%	25.7%	31.1%	67.4%	261.6%
ParamSGTEWMANoR	51.7%	46.9%	23.1%	21.4%	32.3%	85.5%	448.9%	45.9%	41.2%	21.8%	20.9%	27.8%	77.9%	411.6%
NormalEqW	41.7%	38.3%	19.4%	18.3%	26.4%	65.9%	460.3%	50.0%	46.4%	20.4%	25.5%	33.9%	75.0%	492.0%
RiskMetrics	56.3%	51.4%	23.7%	30.6%	37.3%	91.5%	470.7%	63.5%	58.3%	24.8%	35.8%	43.5%	99.9%	501.4%
Historical	6.0%	5.8%	2.1%	1.6%	3.1%	9.8%	15.7%	4.6%	4.4%	1.6%	1.6%	2.5%	7.5%	14.5%
HistVolAdj	44.6%	40.2%	20.2%	22.7%	28.3%	73.6%	387.3%	43.5%	38.8%	19.9%	22.9%	27.7%	73.3%	372.0%

A.1.2 NRMSD results with 500 sample size

Table A.7: This table shows the summary NRMSD statistics of VaR estimates at the 1% significance level for the 1000 simulations of GARCH returns, generated with $(\omega, \gamma, \beta) = (0.1, 0.15, 0.8)$ as Zheng et al. (2018). All models estimate daily VaR using a sample size of 500 observations.

GARCH(1,1) returns, Zheng et al. (2018)'s parametrization,, $n = 500$, $\alpha = 1\%$							
Model	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	11.4%	11.1%	2.2%	5.9%	8.3%	15.3%	22.6%
GARCHvolknownNoR	10.4%	9.8%	5.4%	1.5%	2.9%	20.5%	33.7%
GARCH2G	13.5%	13.0%	2.6%	7.5%	9.9%	18.2%	28.0%
GARCH2GNoR	11.7%	10.9%	5.4%	2.1%	4.5%	21.9%	44.5%
voltEqW	36.3%	34.5%	8.7%	22.8%	26.8%	53.5%	114.6%
voltEqWNoR	40.4%	36.3%	17.7%	23.0%	27.5%	65.8%	426.2%
voltEWMA	19.5%	18.8%	4.0%	12.9%	15.1%	26.9%	69.2%
voltEWMANoR	18.3%	17.3%	4.2%	12.3%	14.0%	26.6%	52.1%
c+voltEqW	42.6%	37.1%	21.5%	23.5%	27.4%	75.3%	376.5%
c+voltEqWNoR	44.5%	36.8%	31.3%	23.0%	27.5%	76.4%	425.5%
c+voltEWMA	19.6%	18.9%	4.0%	12.9%	15.1%	26.9%	69.2%
c+voltEWMANoR	18.3%	17.3%	4.3%	12.3%	14.0%	26.8%	52.1%
ParamSGT	34.7%	32.7%	7.9%	21.0%	26.0%	50.2%	94.8%
ParamSGTNoR	33.9%	32.1%	7.3%	21.3%	26.1%	48.1%	97.6%
ParamSGTEWMA	18.8%	18.0%	3.7%	12.2%	14.0%	26.4%	34.8%
ParamSGTEWMANoR	17.8%	17.2%	3.3%	11.7%	13.8%	23.9%	39.1%
NormalEqW	32.0%	30.8%	6.1%	21.5%	25.1%	42.2%	85.2%
RiskMetrics	15.8%	15.4%	2.3%	11.8%	13.3%	20.0%	40.7%
Historical	37.6%	34.8%	11.6%	23.1%	26.9%	55.5%	196.5%
HistVolAdj	19.5%	18.9%	3.8%	12.9%	15.1%	26.3%	63.1%

Table A.8: This table shows the summary NRMSD statistics of VaR estimates at the 2.5% significance level for the 1000 simulations of GARCH returns, generated with $(\omega, \gamma, \beta) = (0.1, 0.15, 0.8)$ as Zheng et al. (2018). All models estimate daily VaR using a sample size of 500 observations.

GARCH(1,1) returns, Zheng et al. (2018)'s parametrization, $n = 500$, $\alpha = 2.5\%$							
Model	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	9.7%	9.5%	1.9%	5.7%	6.9%	13.2%	22.3%
GARCHvolknownNoR	9.1%	8.2%	4.8%	1.5%	2.8%	17.7%	30.7%
GARCH2G	12.0%	11.7%	2.2%	6.8%	8.8%	15.8%	26.9%
GARCH2GNoR	10.4%	9.6%	4.8%	1.1%	4.3%	19.4%	38.0%
voltEqW	33.2%	31.8%	6.4%	21.9%	26.1%	44.9%	86.9%
voltEqWNoR	37.9%	35.1%	14.9%	22.5%	27.3%	56.2%	403.3%
voltEWMA	18.6%	17.9%	3.7%	13.0%	14.6%	24.3%	72.2%
voltEWMANoR	17.5%	16.7%	3.3%	12.0%	14.0%	23.5%	45.8%
c+voltEqW	41.4%	36.7%	20.0%	22.4%	27.5%	69.3%	405.9%
c+voltEqWNoR	43.5%	36.1%	38.8%	23.7%	27.3%	73.4%	947.2%
c+voltEWMA	18.6%	18.0%	3.7%	13.1%	14.7%	24.3%	72.2%
c+voltEWMANoR	17.5%	16.7%	3.3%	12.0%	13.9%	23.5%	45.8%
ParamSGT	32.3%	30.9%	6.0%	21.1%	25.2%	43.5%	73.4%
ParamSGTNoR	32.3%	30.8%	6.7%	21.3%	25.3%	44.7%	94.0%
ParamSGTEWMA	16.1%	15.8%	1.9%	12.0%	13.5%	19.6%	25.7%
ParamSGTEWMANoR	16.2%	15.7%	2.6%	12.0%	13.4%	20.6%	50.5%
NormalEqW	32.0%	30.8%	6.1%	21.5%	25.1%	42.2%	85.2%
RiskMetrics	15.8%	15.4%	2.3%	11.8%	13.3%	20.0%	40.7%
Historical	33.8%	32.0%	8.9%	22.6%	26.0%	46.5%	175.7%
HistVolAdj	18.5%	18.0%	3.5%	12.7%	14.5%	24.0%	64.1%

Table A.9: This table shows the summary NRMSD statistics of VaR estimates at the 5% significance level for the 1000 simulations of GARCH returns, generated with $(\omega, \gamma, \beta) = (0.1, 0.15, 0.8)$ as Zheng et al. (2018). All models estimate daily VaR using a sample size of 500 observations.

GARCH(1,1) returns, Zheng et al. (2018)'s parametrization, $n = 500$, $\alpha = 5\%$							
Model	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	9.3%	9.1%	1.9%	4.6%	6.7%	12.5%	26.2%
GARCHvolknownNoR	8.7%	8.1%	4.3%	1.5%	2.5%	16.7%	24.5%
GARCH2G	11.6%	11.4%	2.2%	6.5%	8.5%	15.5%	29.7%
GARCH2GNoR	10.0%	9.2%	4.5%	1.1%	4.0%	18.2%	32.6%
voltEqW	32.2%	30.8%	5.8%	21.0%	25.6%	43.3%	71.7%
voltEqWNoR	36.4%	34.2%	11.3%	23.2%	27.1%	52.0%	296.2%
voltEWMA	18.0%	17.5%	3.3%	12.7%	14.7%	22.9%	62.9%
voltEWMANoR	17.0%	16.4%	2.9%	11.9%	13.7%	22.3%	49.3%
c+voltEqW	39.5%	35.5%	16.8%	21.6%	27.4%	63.7%	333.8%
c+voltEqWNoR	42.3%	35.3%	28.9%	22.8%	27.4%	70.3%	548.7%
c+voltEWMA	18.1%	17.5%	3.4%	12.9%	14.7%	22.9%	62.9%
c+voltEWMANoR	17.1%	16.4%	3.0%	11.9%	13.7%	22.4%	49.3%
ParamSGT	31.4%	30.2%	5.4%	21.2%	25.1%	41.6%	71.7%
ParamSGTNoR	31.9%	30.4%	6.5%	21.5%	25.0%	43.9%	91.4%
ParamSGTEWMA	15.7%	15.4%	1.9%	11.9%	13.6%	18.8%	33.4%
ParamSGTEWMANoR	16.2%	15.7%	2.7%	12.3%	13.6%	20.4%	65.5%
NormalEqW	32.0%	30.8%	6.1%	21.5%	25.1%	42.2%	85.2%
RiskMetrics	15.8%	15.4%	2.3%	11.8%	13.3%	20.0%	40.7%
Historical	32.2%	31.0%	6.8%	21.9%	25.6%	42.4%	150.4%
HistVolAdj	17.9%	17.4%	3.2%	12.4%	14.5%	22.8%	61.5%

Table A.10: This table shows the summary NRMSD statistics of VaR estimates at the 1% significance level for the 1000 simulations of GARCH returns, generated with $(\omega, \gamma, \beta) = (3.125 \times 10^{-7}, 0.05, 0.9)$. All models estimate daily VaR using a sample size of 500 observations.

GARCH(1,1) returns, realistic parametrization,, $n = 500$, $\alpha = 1\%$							
Model	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	11.6%	11.4%	1.7%	7.5%	9.2%	14.6%	22.4%
GARCHvolknownNoR	10.9%	10.0%	5.0%	0.4%	3.9%	20.6%	40.5%
GARCH2G	12.8%	12.4%	2.9%	8.6%	10.0%	16.3%	59.7%
GARCH2GNoR	11.4%	10.4%	5.1%	1.9%	4.9%	20.5%	43.2%
voltEqW	13.3%	13.1%	1.8%	9.8%	10.9%	16.5%	24.3%
voltEqWNoR	15.4%	14.3%	4.1%	9.4%	11.0%	23.4%	43.4%
voltEWMA	11.3%	11.0%	1.8%	7.2%	8.9%	14.6%	19.6%
voltEWMANoR	10.6%	9.6%	3.4%	6.5%	7.4%	17.4%	28.3%
c+voltEqW	19.9%	19.2%	4.0%	11.6%	14.7%	27.4%	41.1%
c+voltEqWNoR	25.1%	19.7%	16.1%	9.4%	11.4%	56.2%	182.7%
c+voltEWMA	11.6%	11.4%	2.1%	6.2%	8.5%	15.4%	20.1%
c+voltEWMANoR	10.5%	8.9%	6.3%	2.6%	3.4%	23.0%	58.4%
ParamSGT	12.2%	12.0%	1.5%	9.3%	10.1%	14.8%	19.3%
ParamSGTNoR	12.1%	11.8%	1.6%	8.9%	9.9%	15.0%	19.2%
ParamSGTEWMA	9.2%	9.0%	1.1%	6.2%	7.5%	11.3%	14.1%
ParamSGTEWMANoR	9.1%	8.8%	1.4%	6.0%	7.3%	11.8%	16.0%
NormalEqW	11.4%	11.3%	1.2%	8.7%	9.6%	13.8%	17.1%
RiskMetrics	8.3%	8.3%	0.8%	5.6%	7.0%	9.7%	12.7%
Historical	13.3%	12.9%	2.1%	8.9%	10.6%	17.3%	24.7%
HistVolAdj	11.9%	11.6%	2.1%	7.3%	9.0%	15.9%	21.7%

Table A.11: This table shows the summary NRMSD statistics of VaR estimates at the 2.5% significance level for the 1000 simulations of GARCH returns, generated with $(\omega, \gamma, \beta) = (3.125 \times 10^{-7}, 0.05, 0.9)$. All models estimate daily VaR using a sample size of 500 observations.

GARCH(1,1) returns, realistic parametrization, $\alpha = 2.5\%$							
Model	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	9.2%	9.0%	1.6%	5.2%	6.9%	11.9%	15.4%
GARCHvolknownNoR	8.5%	7.8%	4.7%	0.5%	2.2%	17.0%	42.4%
GARCH2G	10.8%	10.4%	3.0%	5.9%	8.1%	13.8%	57.8%
GARCH2GNoR	9.1%	8.4%	4.7%	0.4%	2.8%	17.6%	42.2%
voltEqW	12.6%	12.4%	1.5%	9.2%	10.4%	15.4%	20.7%
voltEqWNoR	14.7%	14.0%	3.2%	9.3%	11.0%	21.5%	33.1%
voltEWMA	10.6%	10.4%	1.5%	7.1%	8.6%	13.3%	17.8%
voltEWMANoR	9.9%	9.3%	2.3%	6.4%	7.4%	14.5%	22.5%
c+voltEqW	18.3%	17.8%	3.4%	10.9%	13.9%	25.0%	40.9%
c+voltEqWNoR	23.7%	18.0%	17.3%	9.4%	11.0%	54.7%	193.1%
c+voltEWMA	10.1%	9.9%	1.7%	5.6%	7.5%	12.9%	17.0%
c+voltEWMANoR	9.2%	8.0%	5.1%	2.5%	3.4%	18.6%	39.5%
ParamSGT	11.7%	11.6%	1.2%	9.2%	10.0%	14.0%	17.3%
ParamSGTNoR	11.7%	11.6%	1.3%	8.9%	9.9%	14.1%	17.4%
ParamSGTEWMA	8.7%	8.6%	0.9%	6.0%	7.3%	10.3%	12.5%
ParamSGTEWMANoR	8.7%	8.5%	1.1%	6.0%	7.2%	10.6%	13.5%
NormalEqW	11.4%	11.3%	1.2%	8.7%	9.6%	13.8%	17.1%
RiskMetrics	8.3%	8.3%	0.8%	5.6%	7.0%	9.7%	12.7%
Historical	12.6%	12.4%	1.6%	8.3%	10.3%	15.4%	20.7%
HistVolAdj	11.1%	10.9%	1.7%	7.0%	8.7%	14.4%	17.6%

Table A.12: This table shows the summary NRMSD statistics of VaR estimates at the 5% significance level for the 1000 simulations of GARCH returns, generated with $(\omega, \gamma, \beta) = (3.125 \times 10^{-7}, 0.05, 0.9)$. All models estimate daily VaR using a sample size of 500 observations.

GARCH(1,1) returns, realistic parametrization, $n = 500$, $\alpha = 5\%$							
Model	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	9.9%	9.7%	1.7%	5.9%	7.5%	12.8%	18.5%
GARCHvolknownNoR	9.3%	8.6%	4.9%	1.0%	3.1%	17.7%	52.7%
GARCH2G	11.6%	11.2%	3.4%	6.8%	8.8%	15.0%	77.8%
GARCH2GNoR	9.8%	9.2%	4.9%	1.3%	3.4%	18.1%	52.4%
voltEqW	12.3%	12.2%	1.3%	9.0%	10.4%	14.6%	17.1%
voltEqWNoR	14.4%	13.8%	2.8%	9.3%	11.0%	20.1%	27.2%
voltEWMA	10.3%	10.2%	1.3%	7.2%	8.4%	12.6%	18.2%
voltEWMANoR	9.6%	9.2%	2.0%	6.4%	7.4%	13.6%	20.1%
c+voltEqW	17.9%	17.5%	3.0%	11.4%	13.7%	23.7%	31.7%
c+voltEqWNoR	22.7%	17.5%	15.8%	9.2%	10.9%	54.2%	146.7%
c+voltEWMA	9.7%	9.5%	1.6%	5.6%	7.3%	12.6%	16.6%
c+voltEWMANoR	9.0%	8.1%	4.7%	2.4%	3.4%	17.7%	34.4%
ParamSGT	11.5%	11.4%	1.1%	9.0%	9.9%	13.7%	16.2%
ParamSGTNoR	11.6%	11.4%	1.2%	9.1%	9.8%	13.7%	16.4%
ParamSGTEWMA	8.5%	8.4%	0.8%	6.0%	7.2%	9.9%	11.3%
ParamSGTEWMANoR	8.5%	8.4%	0.9%	6.0%	7.1%	10.1%	11.7%
NormalEqW	11.4%	11.3%	1.2%	8.7%	9.6%	13.8%	17.1%
RiskMetrics	8.3%	8.3%	0.8%	5.6%	7.0%	9.7%	12.7%
Historical	12.3%	12.1%	1.5%	8.7%	10.3%	14.9%	20.4%
HistVolAdj	10.7%	10.6%	1.5%	7.1%	8.6%	13.4%	19.7%

Table A.13: This table shows the summary NRMSD statistics of VaR estimates at the 1% significance level for the 1000 simulations of $SGT(0.02\%, 1.25\%, 2, \infty, 0)$ returns. All models estimate daily VaR using a sample size of 500 observations.

$SGT(0.02\%, 1.25\%, 2, \infty, 0)$ returns, $n = 500$, $\alpha = 1\%$							
Model	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	9.1%	8.9%	1.8%	4.2%	6.5%	12.2%	18.2%
GARCHvolknownNoR	7.9%	7.3%	4.9%	0.0%	1.0%	16.6%	34.4%
GARCH2G	7.3%	7.2%	1.8%	3.2%	4.6%	10.6%	16.5%
GARCH2GNoR	8.3%	7.7%	4.9%	0.2%	1.4%	17.2%	34.4%
voltEqW	6.7%	6.6%	1.5%	2.6%	4.5%	9.6%	12.7%
voltEqWNoR	6.6%	5.8%	3.7%	1.3%	2.4%	14.1%	27.2%
voltEWMA	14.9%	14.6%	1.7%	11.3%	12.7%	18.0%	22.0%
voltEWMANoR	14.6%	13.5%	3.2%	10.5%	11.6%	21.2%	31.0%
c+voltEqW	13.4%	13.0%	2.8%	6.2%	9.2%	18.5%	24.5%
c+voltEqWNoR	16.9%	12.7%	14.6%	0.2%	2.8%	45.0%	109.1%
c+voltEWMA	10.5%	10.4%	1.9%	5.3%	7.5%	13.7%	18.7%
c+voltEWMANoR	9.7%	8.9%	5.4%	0.3%	2.3%	20.1%	32.8%
ParamSGT	4.6%	4.5%	1.1%	1.9%	3.1%	6.7%	9.4%
ParamSGTNoR	4.4%	3.9%	1.9%	1.3%	2.2%	8.2%	14.5%
ParamSGTEWMA	12.8%	12.8%	0.8%	10.5%	11.5%	14.3%	16.1%
ParamSGTEWMANoR	12.8%	12.7%	1.0%	10.5%	11.4%	14.6%	17.9%
NormalEqW	3.2%	3.1%	0.8%	1.5%	2.0%	4.5%	6.6%
RiskMetrics	12.3%	12.3%	0.7%	9.9%	11.2%	13.6%	14.9%
Historical	6.8%	6.6%	1.7%	2.8%	4.3%	9.8%	14.7%
HistVolAdj	15.7%	15.4%	2.0%	11.8%	13.0%	19.6%	24.9%

Table A.14: This table shows the summary NRMSD statistics of VaR estimates at the 2.5% significance level for the 1000 simulations of $SGT(0.02\%, 1.25\%, 2, \infty, 0)$ returns. All models estimate daily VaR using a sample size of 500 observations.

$SGT(0.02\%, 1.25\%, 2, \infty, 0)$ returns, $n = 500$, $\alpha = 2.5\%$							
Model	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	6.1%	6.1%	1.5%	2.9%	3.9%	8.7%	11.9%
GARCHvolknownNoR	5.1%	4.3%	4.0%	0.0%	0.4%	13.1%	21.1%
GARCH2G	6.1%	6.0%	1.5%	2.7%	3.9%	8.7%	11.6%
GARCH2GNoR	6.0%	5.2%	4.1%	0.1%	0.7%	13.4%	23.6%
voltEqW	5.7%	5.5%	1.4%	2.5%	3.7%	8.1%	11.9%
voltEqWNoR	5.9%	4.8%	3.4%	1.4%	2.4%	12.9%	22.7%
voltEWMA	13.9%	13.9%	1.2%	11.0%	12.2%	16.2%	18.0%
voltEWMANoR	13.7%	13.2%	2.0%	10.6%	11.5%	17.6%	24.7%
c+voltEqW	11.9%	11.7%	2.4%	6.1%	8.1%	15.9%	20.9%
c+voltEqWNoR	14.8%	11.0%	14.0%	0.3%	2.5%	39.6%	161.0%
c+voltEWMA	8.9%	8.8%	1.6%	4.7%	6.4%	11.8%	15.6%
c+voltEWMANoR	8.2%	7.6%	4.6%	0.1%	1.9%	16.6%	28.1%
ParamSGT	4.0%	3.8%	0.9%	2.0%	2.6%	5.7%	7.8%
ParamSGTNoR	3.8%	3.6%	1.4%	1.4%	2.1%	6.8%	10.8%
ParamSGTEWMA	12.6%	12.5%	0.8%	10.3%	11.3%	13.8%	15.1%
ParamSGTEWMANoR	12.6%	12.5%	0.8%	10.5%	11.3%	14.0%	16.5%
NormalEqW	3.2%	3.1%	0.8%	1.5%	2.0%	4.6%	6.7%
RiskMetrics	12.4%	12.3%	0.7%	9.9%	11.2%	13.6%	14.9%
Historical	5.9%	5.8%	1.5%	2.7%	3.7%	8.5%	11.4%
HistVolAdj	14.6%	14.5%	1.4%	11.4%	12.6%	17.1%	19.2%

Table A.15: This table shows the summary NRMSD statistics of VaR estimates at the 5% significance level for the 1000 simulations of $SGT(0.02\%, 1.25\%, 2, \infty, 0)$ returns. All models estimate daily VaR using a sample size of 500 observations.

$SGT(0.02\%, 1.25\%, 2, \infty, 0)$ returns, $n = 500$, $\alpha = 5\%$							
Model	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	7.7%	7.6%	1.4%	4.5%	5.5%	10.2%	13.5%
GARCHvolknownNoR	7.0%	6.4%	4.0%	0.1%	1.3%	14.3%	20.3%
GARCH2G	6.3%	6.2%	1.3%	3.1%	4.4%	8.5%	12.3%
GARCH2GNoR	7.3%	6.9%	4.2%	0.1%	1.5%	14.2%	47.4%
voltEqW	5.3%	5.3%	1.2%	2.4%	3.5%	7.3%	10.7%
voltEqWNoR	5.8%	5.1%	3.0%	1.3%	2.4%	12.2%	17.5%
voltEWMA	13.6%	13.5%	1.0%	11.0%	12.1%	15.5%	18.8%
voltEWMANoR	13.4%	13.0%	1.7%	10.4%	11.4%	16.7%	23.9%
c+voltEqW	11.6%	11.4%	2.2%	5.6%	8.2%	15.5%	19.0%
c+voltEqWNoR	14.3%	11.2%	12.6%	0.3%	2.5%	38.8%	134.0%
c+voltEWMA	8.5%	8.4%	1.5%	4.5%	6.3%	11.1%	13.9%
c+voltEWMANoR	7.7%	7.1%	4.4%	0.6%	1.9%	16.1%	28.0%
ParamSGT	3.6%	3.5%	0.9%	1.7%	2.4%	5.1%	7.4%
ParamSGTNoR	3.5%	3.3%	1.1%	1.5%	2.1%	5.8%	9.3%
ParamSGTEWMA	12.4%	12.4%	0.7%	10.4%	11.3%	13.7%	14.9%
ParamSGTEWMANoR	12.4%	12.4%	0.8%	10.5%	11.3%	13.8%	15.7%
NormalEqW	3.3%	3.2%	0.8%	1.5%	2.1%	4.7%	6.8%
RiskMetrics	12.4%	12.3%	0.7%	9.9%	11.2%	13.7%	14.9%
Historical	5.6%	5.5%	1.3%	2.1%	3.5%	7.8%	11.2%
HistVolAdj	14.1%	14.0%	1.2%	11.3%	12.4%	16.4%	20.0%

Table A.16: This table shows the summary NRMSD statistics of VaR estimates at the 1% significance level for the 1000 simulations of $SGT(0.02\%, 1.25\%, 2, 3, 0)$ returns. All models estimate daily VaR using a sample size of 500 observations.

$SGT(0.02\%, 1.25\%, 2, 3, 0)$ returns, $n = 500$, $\alpha = 1\%$							
Model	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	22.5%	20.7%	8.5%	9.5%	13.7%	38.7%	99.3%
GARCHvolknownNoR	18.8%	15.6%	14.0%	0.5%	3.6%	46.3%	118.8%
GARCH2G	18.6%	17.3%	6.6%	4.7%	10.8%	31.5%	53.0%
GARCH2GNoR	20.1%	15.6%	18.8%	0.3%	3.9%	48.7%	247.7%
voltEqW	17.4%	16.1%	6.1%	7.7%	10.6%	29.0%	58.2%
voltEqWNoR	19.5%	16.2%	12.7%	4.3%	6.9%	44.1%	96.5%
voltEWMA	40.1%	36.7%	13.5%	21.2%	26.6%	65.2%	128.7%
voltEWMANoR	39.1%	34.5%	15.6%	19.9%	24.4%	67.4%	160.6%
c+voltEqW	54.9%	42.6%	44.5%	14.4%	22.4%	132.2%	600.8%
c+voltEqWNoR	56.6%	30.8%	77.5%	0.4%	6.6%	193.2%	755.1%
c+voltEWMA	26.2%	24.1%	9.1%	14.1%	16.8%	41.3%	95.3%
c+voltEWMANoR	23.8%	20.0%	17.3%	0.5%	5.8%	52.6%	214.8%
ParamSGT	10.0%	9.3%	3.7%	3.4%	5.7%	16.6%	33.6%
ParamSGTNoR	12.7%	10.2%	9.5%	3.5%	6.1%	25.8%	139.5%
ParamSGTEWMA	31.6%	29.0%	9.2%	20.9%	23.2%	47.8%	123.7%
ParamSGTEWMANoR	32.6%	29.3%	11.5%	20.9%	23.4%	52.1%	166.0%
NormalEqW	18.4%	17.0%	8.7%	7.4%	12.2%	27.1%	103.7%
RiskMetrics	33.8%	30.9%	10.5%	25.6%	27.2%	48.5%	148.6%
Historical	16.4%	15.2%	5.5%	4.4%	9.8%	27.1%	44.5%
HistVolAdj	45.9%	42.4%	16.4%	22.5%	29.1%	75.6%	139.9%

Table A.17: This table shows the summary NRMSD statistics of VaR estimates at the 2.5% significance level for the 1000 simulations of $SGT(0.02\%, 1.25\%, 2, 3, 0)$ returns. All models estimate daily VaR using a sample size of 500 observations.

$SGT(0.02\%, 1.25\%, 2, 3, 0)$ returns, $n = 500$, $\alpha = 2.5\%$							
Model	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	12.0%	11.7%	3.5%	4.8%	7.4%	18.2%	43.0%
GARCHvolknownNoR	9.9%	8.4%	7.8%	0.0%	0.8%	23.3%	62.7%
GARCH2G	11.7%	11.3%	3.0%	5.1%	7.4%	17.2%	27.7%
GARCH2GNoR	12.0%	9.7%	11.0%	0.0%	1.3%	27.6%	147.6%
voltEqW	13.6%	12.7%	5.1%	5.5%	8.0%	22.3%	50.0%
voltEqWNoR	17.1%	14.4%	10.6%	3.9%	6.8%	36.6%	93.9%
voltEWMA	37.2%	33.9%	12.5%	21.9%	25.2%	60.6%	135.2%
voltEWMANoR	36.8%	33.0%	14.0%	19.8%	24.2%	62.2%	158.6%
c+voltEqW	32.8%	25.3%	32.8%	12.0%	16.0%	74.8%	502.3%
c+voltEqWNoR	39.0%	21.5%	58.5%	1.0%	5.0%	127.8%	1040.5%
c+voltEWMA	18.3%	17.3%	5.7%	8.5%	12.0%	27.9%	66.5%
c+voltEWMANoR	16.9%	14.3%	11.9%	0.3%	3.7%	38.9%	106.2%
ParamSGT	7.1%	6.8%	2.1%	3.2%	4.5%	10.6%	22.6%
ParamSGTNoR	13.6%	11.0%	9.8%	4.0%	5.9%	29.4%	143.2%
ParamSGTEWMA	31.1%	28.8%	8.7%	20.5%	22.9%	46.2%	117.0%
ParamSGTEWMANoR	33.1%	30.0%	11.5%	20.1%	23.7%	52.9%	169.6%
NormalEqW	14.9%	10.8%	14.2%	3.4%	5.6%	37.4%	133.0%
RiskMetrics	34.6%	30.7%	14.8%	18.9%	23.0%	57.8%	181.2%
Historical	11.3%	10.9%	3.1%	4.4%	6.8%	17.0%	27.8%
HistVolAdj	41.3%	37.9%	14.4%	23.5%	27.1%	66.1%	149.5%

Table A.18: This table shows the summary NRMSD statistics of VaR estimates at the 5% significance level for the 1000 simulations of $SGT(0.02\%, 1.25\%, 2, 3, 0)$ returns. All models estimate daily VaR using a sample size of 500 observations.

$SGT(0.02\%, 1.25\%, 2, 3, 0)$ returns, $n = 500$, $\alpha = 5\%$							
Model	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	13.4%	12.8%	3.9%	5.8%	8.9%	20.3%	52.3%
GARCHvolknownNoR	11.8%	10.8%	8.1%	0.0%	1.5%	25.3%	77.1%
GARCH2G	9.5%	9.2%	2.4%	4.9%	6.3%	13.8%	19.4%
GARCH2GNoR	12.2%	10.8%	8.4%	0.0%	2.0%	25.9%	71.7%
voltEqW	12.2%	11.2%	4.7%	4.9%	7.0%	20.3%	48.5%
voltEqWNoR	15.9%	13.2%	10.0%	3.8%	6.6%	34.0%	83.1%
voltEWMA	35.9%	32.8%	12.0%	20.7%	25.0%	56.9%	148.3%
voltEWMANoR	36.0%	32.3%	13.7%	19.8%	23.8%	60.0%	163.5%
c+voltEqW	21.7%	18.3%	22.1%	9.2%	12.4%	34.7%	395.6%
c+voltEqWNoR	30.4%	17.2%	43.4%	0.2%	3.7%	96.0%	576.0%
c+voltEWMA	14.8%	14.0%	4.2%	8.1%	10.1%	21.7%	53.9%
c+voltEWMANoR	13.9%	11.9%	9.2%	0.2%	3.2%	30.9%	69.0%
ParamSGT	6.7%	6.5%	1.8%	3.0%	4.4%	9.6%	20.7%
ParamSGTNoR	15.0%	12.3%	10.2%	4.2%	6.0%	33.7%	145.0%
ParamSGTEWMA	31.1%	28.9%	8.5%	20.5%	23.1%	45.7%	112.2%
ParamSGTEWMANoR	33.7%	30.9%	11.4%	19.8%	23.9%	53.4%	171.4%
NormalEqW	25.8%	21.8%	16.3%	6.6%	12.8%	51.6%	158.3%
RiskMetrics	41.7%	37.6%	17.3%	19.5%	26.9%	68.4%	208.8%
Historical	9.1%	8.8%	2.4%	3.9%	5.6%	13.3%	18.7%
HistVolAdj	39.3%	35.8%	13.8%	21.9%	26.6%	62.8%	169.7%

Table A.19: This table shows the summary NRMSD statistics of VaR estimates at the 1% significance level for the 1000 simulations of $SGT(0.02\%, 1.25\%, 2, 3, -0.3)$ returns. All models estimate daily VaR using a sample size of 500 observations.

$SGT(0.02\%, 1.25\%, 2, 3, -0.3)$ returns, $n = 500$, $\alpha = 1\%$							
Model	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	25.0%	22.3%	27.2%	7.4%	14.6%	39.9%	843.3%
GARCHvolknownNoR	20.7%	17.6%	15.0%	0.5%	3.8%	50.8%	122.2%
GARCH2G	20.2%	18.7%	7.1%	6.2%	11.2%	33.7%	54.5%
GARCH2GNoR	22.3%	17.9%	20.5%	0.4%	4.4%	56.4%	330.5%
voltEqW	18.0%	16.7%	6.6%	6.5%	10.7%	29.3%	84.9%
voltEqWNoR	20.9%	17.9%	13.0%	3.8%	7.7%	45.6%	147.5%
voltEWMA	44.1%	40.1%	17.6%	20.7%	27.1%	73.0%	232.5%
voltEWMANoR	41.3%	36.2%	18.5%	19.7%	25.4%	74.3%	232.9%
c+voltEqW	71.3%	57.6%	50.4%	19.5%	28.6%	163.3%	562.4%
c+voltEqWNoR	71.8%	37.4%	113.1%	2.6%	9.9%	222.5%	1617.9%
c+voltEWMA	29.7%	26.3%	25.2%	12.1%	17.6%	46.2%	646.5%
c+voltEWMANoR	26.7%	22.6%	19.0%	1.2%	6.7%	58.9%	212.8%
ParamSGT	21.6%	21.7%	3.3%	11.9%	16.2%	27.1%	38.9%
ParamSGTNoR	23.0%	22.1%	7.1%	11.5%	16.9%	30.8%	129.1%
ParamSGTEWMA	36.6%	34.5%	8.0%	28.0%	30.7%	48.6%	144.3%
ParamSGTEWMANoR	37.2%	34.7%	9.8%	28.3%	30.7%	52.7%	160.7%
NormalEqW	30.6%	30.7%	4.4%	17.5%	25.1%	35.3%	109.5%
RiskMetrics	41.4%	39.9%	7.4%	33.1%	37.1%	50.3%	156.5%
Historical	17.8%	16.7%	5.6%	5.8%	10.6%	28.8%	47.5%
HistVolAdj	52.1%	47.3%	21.9%	22.0%	29.8%	88.3%	257.5%

Table A.20: This table shows the summary NRMSD statistics of VaR estimates at the 2.5% significance level for the 1000 simulations of $SGT(0.02\%, 1.25\%, 2, 3, -0.3)$ returns. All models estimate daily VaR using a sample size of 500 observations.

$SGT(0.02\%, 1.25\%, 2, 3, -0.3)$ returns, $n = 500$, $\alpha = 2.5\%$							
Model	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	13.5%	13.0%	4.2%	3.9%	8.4%	20.8%	55.7%
GARCHvolknownNoR	10.9%	9.2%	8.3%	0.1%	0.9%	26.4%	53.3%
GARCH2G	13.0%	12.6%	3.5%	4.1%	8.2%	19.4%	29.2%
GARCH2GNoR	15.3%	11.8%	17.9%	0.1%	1.3%	37.5%	269.7%
voltEqW	14.8%	13.5%	5.9%	5.4%	8.6%	25.3%	77.3%
voltEqWNoR	19.1%	15.9%	12.3%	3.7%	7.3%	41.3%	159.5%
voltEWMA	40.6%	36.7%	16.1%	22.0%	26.5%	67.0%	246.0%
voltEWMANoR	39.8%	35.2%	17.6%	19.8%	25.2%	69.7%	247.2%
c+voltEqW	49.0%	36.2%	41.7%	14.2%	21.3%	129.0%	477.3%
c+voltEqWNoR	53.9%	27.7%	121.0%	0.8%	6.4%	160.2%	2452.2%
c+voltEWMA	20.8%	19.6%	5.8%	8.8%	14.1%	32.0%	54.6%
c+voltEWMANoR	19.5%	16.6%	14.6%	0.2%	3.8%	44.3%	172.0%
ParamSGT	17.2%	17.2%	2.6%	9.6%	13.1%	21.4%	34.9%
ParamSGTNoR	20.3%	18.7%	8.8%	7.9%	13.2%	32.9%	141.3%
ParamSGTEWMA	34.8%	32.6%	8.6%	25.6%	28.1%	48.8%	147.8%
ParamSGTEWMANoR	36.3%	33.4%	11.3%	25.0%	28.3%	54.9%	180.6%
NormalEqW	17.1%	15.9%	7.6%	6.8%	11.4%	25.5%	146.5%
RiskMetrics	34.8%	31.6%	12.1%	24.8%	27.1%	53.8%	195.7%
Historical	12.9%	12.4%	3.6%	3.9%	8.0%	19.4%	29.1%
HistVolAdj	46.1%	41.9%	19.1%	23.6%	28.6%	76.6%	295.6%

Table A.21: This table shows the summary NRMSD statistics of VaR estimates at the 5% significance level for the 1000 simulations of $SGT(0.02\%, 1.25\%, 2, 3, -0.3)$ returns. All models estimate daily VaR using a sample size of 500 observations.

$SGT(0.02\%, 1.25\%, 2, 3, -0.3)$ returns, $n = 500$, $\alpha = 5\%$							
Model	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	15.5%	14.7%	4.7%	7.7%	10.2%	23.9%	52.6%
GARCHvolknownNoR	13.5%	12.0%	9.1%	0.1%	2.1%	29.3%	71.2%
GARCH2G	11.0%	10.8%	2.6%	5.3%	7.3%	15.5%	20.5%
GARCH2GNoR	15.8%	12.8%	18.3%	0.4%	2.5%	33.4%	246.7%
voltEqW	13.4%	12.2%	5.6%	4.8%	7.6%	24.1%	69.8%
voltEqWNoR	17.9%	14.9%	11.3%	3.7%	7.3%	39.2%	126.7%
voltEWMA	38.9%	35.0%	14.6%	20.8%	26.1%	63.5%	195.2%
voltEWMANoR	38.9%	34.6%	17.2%	19.9%	24.9%	67.5%	245.2%
c+voltEqW	32.3%	25.7%	28.1%	10.3%	16.3%	77.3%	365.6%
c+voltEqWNoR	43.1%	23.3%	93.4%	0.7%	5.1%	127.2%	2404.5%
c+voltEWMA	17.4%	16.4%	5.4%	8.5%	11.8%	26.5%	74.3%
c+voltEWMANoR	16.8%	14.0%	13.5%	0.5%	3.5%	38.4%	226.5%
ParamSGT	13.4%	13.2%	2.3%	7.9%	10.1%	16.7%	34.0%
ParamSGTNoR	18.3%	15.9%	10.8%	5.1%	9.4%	35.6%	153.5%
ParamSGTEWMA	33.8%	31.5%	9.5%	22.6%	25.8%	48.9%	156.8%
ParamSGTEWMANoR	36.1%	32.7%	12.8%	20.8%	26.0%	57.1%	201.2%
NormalEqW	15.7%	12.4%	12.4%	3.7%	6.0%	36.0%	183.5%
RiskMetrics	36.4%	32.2%	16.3%	18.7%	23.5%	64.2%	234.5%
Historical	10.5%	10.3%	2.6%	4.2%	6.6%	15.2%	20.4%
HistVolAdj	43.4%	39.2%	17.9%	22.8%	27.6%	73.6%	265.7%

Table A.22: This table shows the summary NRMSD statistics of VaR estimates at the 1% significance level for the 1000 simulations of $SGT(0.02\%, 1.25\%, 2, 3, 0.3)$ returns. All models estimate daily VaR using a sample size of 500 observations.

$SGT(0.02\%, 1.25\%, 2, 3, 0.3)$ returns, $n = 500$, $\alpha = 1\%$							
Model	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	20.1%	18.6%	7.5%	7.8%	11.9%	32.5%	90.8%
GARCHvolknownNoR	16.9%	14.4%	12.2%	0.3%	2.5%	39.3%	95.7%
GARCH2G	15.5%	14.5%	5.3%	5.9%	8.9%	25.6%	46.9%
GARCH2GNoR	18.8%	14.7%	19.2%	0.5%	3.2%	44.9%	267.4%
voltEqW	17.4%	16.0%	7.1%	6.5%	10.1%	29.7%	73.2%
voltEqWNoR	20.1%	16.5%	12.9%	3.6%	7.4%	46.2%	116.3%
voltEWMA	44.1%	40.8%	15.3%	22.5%	28.3%	72.1%	159.9%
voltEWMANoR	43.6%	38.1%	18.4%	20.4%	25.3%	79.2%	186.1%
c+voltEqW	32.9%	25.8%	26.8%	12.1%	16.3%	69.9%	400.4%
c+voltEqWNoR	45.9%	25.6%	78.8%	0.9%	5.7%	135.3%	1693.6%
c+voltEWMA	23.1%	21.4%	9.0%	10.3%	14.1%	36.7%	115.1%
c+voltEWMANoR	20.9%	16.9%	17.5%	0.8%	4.3%	52.7%	194.6%
ParamSGT	41.9%	41.1%	9.0%	18.9%	28.9%	58.7%	80.4%
ParamSGTNoR	43.6%	40.9%	14.4%	13.8%	28.1%	68.5%	152.4%
ParamSGTEWMA	57.0%	54.0%	15.0%	29.4%	39.0%	84.1%	174.5%
ParamSGTEWMANoR	57.8%	54.0%	18.2%	24.7%	37.9%	90.8%	195.8%
NormalEqW	27.1%	23.9%	17.7%	6.4%	13.3%	49.3%	402.8%
RiskMetrics	44.9%	40.1%	21.3%	22.4%	27.9%	76.8%	415.2%
Historical	13.6%	12.7%	4.4%	5.7%	8.1%	21.6%	42.4%
HistVolAdj	49.0%	45.3%	17.4%	24.3%	30.4%	80.5%	178.6%

Table A.23: This table shows the summary NRMSD statistics of VaR estimates at the 2.5% significance level for the 1000 simulations of $SGT(0.02\%, 1.25\%, 2, 3, 0.3)$ returns. All models estimate daily VaR using a sample size of 500 observations.

$SGT(0.02\%, 1.25\%, 2, 3, 0.3)$ returns, $n = 500$, $\alpha = 2.5\%$							
Model	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	10.1%	9.6%	3.0%	4.0%	6.1%	16.0%	28.8%
GARCHvolknownNoR	8.3%	7.0%	6.8%	0.0%	0.7%	21.0%	87.3%
GARCH2G	9.5%	9.0%	2.3%	4.5%	6.5%	14.2%	22.9%
GARCH2GNoR	10.8%	8.6%	13.6%	0.0%	1.1%	26.5%	258.7%
voltEqW	14.1%	12.9%	6.6%	5.4%	7.7%	24.7%	68.1%
voltEqWNoR	18.1%	15.2%	11.6%	3.7%	6.9%	38.6%	128.7%
voltEWMA	41.4%	38.1%	13.9%	22.1%	27.3%	66.9%	129.5%
voltEWMANoR	41.4%	37.0%	16.1%	21.2%	25.7%	72.7%	173.2%
c+voltEqW	18.8%	16.6%	15.9%	7.7%	11.3%	29.3%	326.5%
c+voltEqWNoR	29.9%	18.2%	45.6%	0.1%	3.8%	89.5%	927.1%
c+voltEWMA	15.0%	14.2%	4.5%	7.4%	9.9%	22.6%	67.1%
c+voltEWMANoR	14.0%	12.0%	9.9%	0.2%	3.4%	32.4%	106.6%
ParamSGT	31.6%	31.2%	5.3%	16.6%	23.3%	41.1%	52.6%
ParamSGTNoR	35.7%	33.5%	14.1%	10.7%	19.1%	60.0%	131.4%
ParamSGTEWMA	49.8%	47.3%	12.8%	28.0%	35.4%	72.8%	157.4%
ParamSGTEWMANoR	51.6%	48.1%	17.6%	24.0%	32.0%	85.0%	187.2%
NormalEqW	41.7%	38.3%	19.4%	18.3%	26.4%	65.9%	460.3%
RiskMetrics	56.3%	51.4%	23.7%	30.6%	37.3%	91.5%	470.7%
Historical	9.0%	8.7%	2.5%	3.9%	5.6%	13.7%	23.9%
HistVolAdj	45.7%	42.2%	15.6%	23.9%	29.0%	75.7%	143.3%

Table A.24: This table shows the summary NRMSD statistics of VaR estimates at the 5% significance level for the 1000 simulations of $SGT(0.02\%, 1.25\%, 2, 3, 0.3)$ returns. All models estimate daily VaR using a sample size of 500 observations.

$SGT(0.02\%, 1.25\%, 2, 3, 0.3)$ returns, $n = 500$, $\alpha = 5\%$							
Model	Mean	Median	StDev	Min	5%	95%	Max
GARCHvolknown	13.4%	12.8%	3.9%	5.8%	8.9%	20.3%	52.3%
GARCHvolknownNoR	11.8%	10.8%	8.1%	0.0%	1.5%	25.3%	77.1%
GARCH2G	9.5%	9.2%	2.4%	4.9%	6.3%	13.8%	19.4%
GARCH2GNoR	12.2%	10.8%	8.4%	0.0%	2.0%	25.9%	71.7%
voltEqW	12.2%	11.2%	4.7%	4.9%	7.0%	20.3%	48.5%
voltEqWNoR	15.9%	13.2%	10.0%	3.8%	6.6%	34.0%	83.1%
voltEWMA	35.9%	32.8%	12.0%	20.7%	25.0%	56.9%	148.3%
voltEWMANoR	36.0%	32.3%	13.7%	19.8%	23.8%	60.0%	163.5%
c+voltEqW	21.7%	18.3%	22.1%	9.2%	12.4%	34.7%	395.6%
c+voltEqWNoR	30.4%	17.2%	43.4%	0.2%	3.7%	96.0%	576.0%
c+voltEWMA	14.8%	14.0%	4.2%	8.1%	10.1%	21.7%	53.9%
c+voltEWMANoR	13.9%	11.9%	9.2%	0.2%	3.2%	30.9%	69.0%
ParamSGT	6.7%	6.5%	1.8%	3.0%	4.4%	9.6%	20.7%
ParamSGTNoR	15.0%	12.3%	10.2%	4.2%	6.0%	33.7%	145.0%
ParamSGTEWMA	31.1%	28.9%	8.5%	20.5%	23.1%	45.7%	112.2%
ParamSGTEWMANoR	33.7%	30.9%	11.4%	19.8%	23.9%	53.4%	171.4%
NormalEqW	25.8%	21.8%	16.3%	6.6%	12.8%	51.6%	158.3%
RiskMetrics	41.7%	37.6%	17.3%	19.5%	26.9%	68.4%	208.8%
Historical	9.1%	8.8%	2.4%	3.9%	5.6%	13.3%	18.7%
HistVolAdj	39.3%	35.8%	13.8%	21.9%	26.6%	62.8%	169.7%

A.2 Results Summary, 10000 simulations

A.2.1 NRMSD results with 1000 sample size

Table A.25: This table shows the summary NRMSE statistics of VaR estimates at the 1%, 2.5% and 5% significance levels for the 10000 simulations of GARCH returns, generated with $(\omega, \gamma, \beta) = (0.1, 0.15, 0.8)$ as Zheng et al. (2018). All models estimate daily VaR using a sample size of 1000 observations, including to estimate all parameters, except for the volatility estimate with equally weighted method that uses a sample size of 500.

GARCH(1,1) returns, Zheng et al. (2018)'s parametrization, $n = 1000$															
Model	$\alpha = 1\%$					$\alpha = 2.5\%$					$\alpha = 5\%$				
	Mean	Median	StDv	5%	95%	Mean	Median	StDv	5%	95%	Mean	Median	StDv	5%	95%
GARCHvolknownNoR	7.4%	6.8%	3.7%	2.4%	14.3%	6.4%	5.9%	3.2%	2.1%	12.4%	6.0%	5.6%	2.9%	2.1%	11.4%
GARCHQRNoR	8.9%	8.3%	3.9%	3.6%	15.9%	8.0%	7.5%	3.4%	3.3%	14.2%	7.7%	7.2%	3.3%	3.2%	13.8%
volEqWNoR	37.0%	34.5%	11.0%	26.3%	55.4%	34.1%	32.3%	8.7%	25.8%	47.6%	33.1%	31.6%	7.6%	25.7%	44.9%
volEWMANoR	17.5%	16.8%	3.4%	13.8%	23.8%	16.9%	16.3%	2.8%	13.6%	21.9%	16.5%	16.1%	2.6%	13.6%	20.9%
c+volEqWNoR	39.3%	35.3%	17.3%	26.5%	64.5%	36.7%	33.5%	13.8%	26.1%	56.9%	35.4%	32.8%	12.2%	26.0%	52.7%
c+volEWMANoR	17.5%	16.8%	3.4%	13.8%	23.9%	16.9%	16.3%	2.9%	13.6%	21.9%	16.6%	16.1%	2.6%	13.6%	21.0%
ParamSGTNoR	34.4%	32.9%	8.0%	26.2%	47.1%	32.6%	31.2%	7.4%	25.2%	44.3%	32.1%	30.7%	7.1%	25.2%	43.2%
ParamSGTEWMANoR	18.3%	17.7%	4.5%	14.0%	24.1%	16.3%	15.7%	3.9%	13.4%	20.4%	16.3%	15.8%	3.8%	13.7%	20.1%
NormalEqW	32.3%	30.9%	7.4%	25.1%	43.7%	32.3%	30.9%	7.4%	25.1%	43.7%	32.3%	30.9%	7.4%	25.1%	43.7%
RiskMetrics	15.9%	15.5%	2.3%	13.4%	19.8%	15.9%	15.5%	2.3%	13.4%	19.8%	15.9%	15.5%	2.3%	13.4%	19.8%
Historical	36.3%	34.2%	10.7%	26.4%	52.5%	32.9%	31.6%	7.0%	25.6%	44.1%	31.8%	30.7%	5.9%	25.3%	41.9%
HistVolAdj	18.4%	17.8%	3.5%	14.2%	24.6%	17.6%	17.0%	3.1%	14.0%	22.9%	17.1%	16.6%	2.9%	13.8%	22.1%

Table A.26: This table shows the summary NRMSE statistics of VaR estimates at the 1%, 2.5% and 5% significance levels for the 10000 simulations of GARCH returns, generated with $(\omega, \gamma, \beta) = (3.125 \times 10^{-7}, 0.05, 0.9)$. All models estimate daily VaR using a sample size of 1000 observations, including to estimate all parameters, except for the volatility estimate with equally weighted method that uses a sample size of 500.

GARCH(1,1) returns, realistic parametrization, $n = 1000$															
$\alpha = 1\%$															
$\alpha = 2.5\%$															
$\alpha = 5\%$															
Model	Mean	Median	StdV	5%	95%	Mean	Median	StdV	5%	95%	Mean	Median	StdV	5%	95%
GARCHvolknownNoR	8.6%	8.4%	3.6%	3.1%	14.8%	6.1%	6.0%	3.1%	1.6%	11.8%	6.6%	6.2%	3.1%	2.6%	12.3%
GARCHQRNoR	9.1%	8.8%	3.6%	3.6%	15.5%	6.9%	6.6%	3.3%	2.0%	12.6%	7.4%	6.9%	3.2%	2.9%	13.1%
volEqWNoR	12.9%	12.3%	2.4%	10.1%	17.6%	12.4%	12.1%	1.9%	10.0%	16.0%	12.3%	12.0%	1.7%	10.0%	15.6%
volEWMANoR	9.8%	9.2%	2.1%	7.4%	14.1%	9.3%	9.0%	1.6%	7.4%	12.4%	9.1%	8.9%	1.4%	7.3%	11.8%
c+volEqWNoR	16.9%	14.3%	7.9%	10.4%	32.5%	15.7%	13.5%	6.8%	10.2%	28.6%	15.3%	13.3%	6.4%	10.2%	27.2%
c+volEWMANoR	7.5%	6.7%	3.6%	3.2%	14.4%	6.6%	5.9%	3.0%	3.1%	12.4%	6.3%	5.6%	2.8%	3.0%	11.7%
ParamSGTNoR	11.8%	11.6%	1.4%	9.8%	14.4%	11.6%	11.4%	1.3%	9.8%	13.9%	11.5%	11.3%	1.2%	9.8%	13.7%
ParamSGTEWMANoR	8.8%	8.6%	1.1%	7.2%	10.7%	8.5%	8.4%	0.9%	7.1%	10.0%	8.4%	8.4%	0.8%	7.1%	9.8%
NormalEqW	11.4%	11.3%	1.2%	9.7%	13.7%	11.4%	11.3%	1.2%	9.7%	13.7%	11.4%	11.3%	1.2%	9.7%	13.7%
RiskMetrics	8.3%	8.3%	0.8%	7.1%	9.7%	8.3%	8.3%	0.8%	7.1%	9.7%	8.3%	8.3%	0.8%	7.1%	9.7%
Historical	12.4%	12.1%	1.7%	10.1%	15.6%	11.9%	11.7%	1.4%	9.9%	14.4%	11.7%	11.6%	1.2%	9.9%	13.9%
HistVolAdj	10.7%	10.4%	1.9%	8.2%	14.2%	10.1%	9.9%	1.5%	8.0%	12.9%	9.8%	9.7%	1.3%	7.9%	12.3%

Table A.27: This table shows the summary NRMSE statistics of VaR estimates at the 1%, 2.5% and 5% significance levels for the 10000 simulations of $SGT(0.02\%, 1.25\%, 2, \infty, 0)$ returns. All models estimate daily VaR using a sample size of 1000 observations, including to estimate all parameters, except for the volatility estimate with equally weighted method that uses a sample size of 500.

SGT(0.02%, 1.25%, 2, ∞, 0) returns, n = 1000															
Model	α = 1%					α = 2.5%					α = 5%				
	Mean	Median	Stdv	5%	95%	Mean	Median	Stdv	5%	95%	Mean	Median	Stdv	5%	95%
GARCHvolknownNoR	5.5%	5.0%	3.6%	0.6%	12.0%	4.7%	4.3%	3.0%	0.6%	10.2%	4.8%	4.6%	2.7%	1.1%	9.7%
GARCHQRNoR	5.7%	5.3%	3.5%	0.9%	12.0%	4.9%	4.4%	3.0%	0.8%	10.4%	5.1%	4.8%	2.8%	1.2%	10.1%
voltEqWNoR	5.2%	4.5%	2.6%	2.3%	10.2%	4.7%	4.1%	2.1%	2.2%	8.9%	4.5%	4.0%	2.0%	2.2%	8.4%
voltEWMANoR	13.7%	13.2%	2.0%	11.5%	17.8%	13.1%	12.8%	1.4%	11.4%	15.9%	12.9%	12.7%	1.2%	11.3%	15.0%
c+voltEqWNoR	10.0%	7.8%	8.3%	1.9%	26.0%	8.8%	6.8%	7.5%	1.5%	23.2%	8.3%	6.4%	7.1%	1.5%	21.8%
c+voltEWMANoR	6.6%	6.1%	3.6%	1.7%	13.1%	5.6%	5.2%	3.1%	1.4%	11.4%	5.3%	4.9%	2.9%	1.3%	10.6%
ParamSGTNoR	3.8%	3.6%	1.4%	2.1%	6.6%	3.5%	3.3%	1.1%	2.1%	5.7%	3.4%	3.2%	1.0%	2.0%	5.2%
ParamSGTEWMANoR	12.6%	12.5%	0.9%	11.3%	14.1%	12.4%	12.4%	0.8%	11.2%	13.8%	12.4%	12.4%	0.8%	11.2%	13.7%
NormalEqW	3.2%	3.1%	0.8%	2.0%	4.6%	3.2%	3.1%	0.8%	2.0%	4.6%	3.2%	3.1%	0.8%	2.0%	4.7%
RiskMetrics	12.3%	12.3%	0.7%	11.2%	13.6%	12.3%	12.3%	0.7%	11.2%	13.6%	12.4%	12.4%	0.7%	11.2%	13.6%
Historical	4.8%	4.6%	1.6%	2.5%	7.7%	4.1%	3.9%	1.3%	2.2%	6.5%	3.9%	3.7%	1.3%	2.1%	6.2%
HistVolAdj	14.6%	14.4%	1.8%	12.2%	17.9%	13.8%	13.7%	1.3%	12.0%	16.2%	13.5%	13.4%	1.1%	11.8%	15.5%

Table A.28: This table shows the summary NRMSE statistics of VaR estimates at the 1%, 2.5% and 5% significance levels for the 10000 simulations of $SGT(0.02\%, 1.25\%, 2, 3, 0)$ returns. All models estimate daily VaR using a sample size of 1000 observations, including to estimate all parameters, except for the volatility estimate with equally weighted method that uses a sample size of 500.

Model	$\alpha = 1\%$					$\alpha = 2.5\%$					$\alpha = 5\%$				
	Mean	Median	Stdv	5%	95%	Mean	Median	Stdv	5%	95%	Mean	Median	Stdv	5%	95%
GARCHvolknownNoR	13.5%	11.2%	11.1%	2.3%	31.2%	9.6%	8.3%	7.4%	1.1%	22.2%	8.1%	7.1%	6.1%	1.1%	18.3%
GARCHQRNoR	14.3%	11.8%	12.2%	2.4%	33.9%	10.3%	8.7%	8.9%	1.5%	23.3%	9.1%	7.7%	7.5%	1.6%	20.2%
volEqWNoR	17.6%	14.2%	15.5%	6.9%	37.4%	16.5%	13.1%	15.0%	6.6%	35.1%	16.1%	12.7%	14.8%	6.4%	34.7%
volEWMANoR	38.2%	33.6%	18.9%	23.8%	66.6%	37.0%	32.7%	18.6%	23.6%	63.8%	36.2%	32.1%	17.6%	23.5%	61.7%
c+volEqWNoR	29.2%	18.4%	41.8%	3.9%	86.7%	20.3%	13.0%	34.8%	2.8%	58.8%	15.7%	10.2%	22.0%	2.3%	44.9%
c+volEWMANoR	16.2%	14.0%	11.1%	3.6%	36.3%	11.5%	10.0%	8.0%	2.5%	25.0%	9.3%	8.1%	6.6%	2.1%	20.0%
ParamSGTNoR	13.4%	10.2%	13.6%	6.0%	30.3%	13.9%	10.8%	13.4%	6.1%	31.1%	14.9%	11.7%	13.4%	6.2%	33.0%
ParamSGTEWMANoR	33.6%	29.7%	15.7%	23.1%	56.3%	33.7%	30.0%	15.4%	23.3%	56.2%	34.1%	30.6%	15.1%	23.6%	56.0%
NormalEqW	18.2%	17.0%	8.9%	12.2%	24.5%	14.3%	10.7%	13.9%	5.5%	33.3%	25.0%	21.6%	16.1%	12.3%	46.6%
RiskMetrics	33.8%	30.8%	12.3%	27.1%	49.7%	34.6%	30.6%	16.8%	22.9%	58.9%	41.8%	37.5%	19.5%	26.5%	70.4%
Historical	11.2%	10.5%	4.2%	5.8%	19.1%	7.7%	7.3%	2.7%	4.0%	12.7%	6.2%	5.9%	2.1%	3.3%	10.1%
HistVolAdj	43.6%	39.5%	19.9%	26.7%	73.1%	41.1%	37.0%	18.9%	26.1%	69.1%	39.7%	35.7%	18.6%	25.6%	66.5%

Table A.29: This table shows the summary NRMSE statistics of VaR estimates at the 1%, 2.5% and 5% significance levels for the 10000 simulations of $SGT(0.02\%, 1.25\%, 2, 3, -0.3)$ returns. All models estimate daily VaR using a sample size of 1000 observations, including to estimate all parameters, except for the volatility estimate with equally weighted method that uses a sample size of 500.

SGT(0.02%, 1.25%, 2, 3, -0.3) returns, n = 1000															
Model	α = 1%					α = 2.5%					α = 5%				
	Mean	Median	StDv	5%	95%	Mean	Median	StDv	5%	95%	Mean	Median	StDv	5%	95%
GARCHvolknownNoR	14.4%	12.2%	11.3%	2.3%	33.6%	10.6%	9.1%	7.8%	2.0%	23.3%	9.3%	8.2%	6.6%	1.4%	20.3%
GARCHQRNoR	15.2%	12.5%	12.7%	2.5%	35.6%	11.4%	9.6%	9.5%	2.1%	25.9%	10.3%	8.8%	8.4%	1.8%	22.9%
voltEqWNoR	19.0%	15.4%	15.9%	7.3%	41.2%	18.0%	14.4%	15.7%	7.1%	39.6%	17.7%	14.1%	15.4%	7.0%	38.9%
voltEWMANoR	40.5%	35.4%	20.4%	24.8%	71.0%	39.2%	34.5%	19.8%	24.6%	67.3%	38.4%	33.9%	19.2%	24.5%	66.0%
c+voltEqWNoR	35.6%	21.6%	51.6%	4.5%	108.5%	25.1%	15.1%	39.3%	3.1%	76.4%	19.6%	12.4%	28.9%	2.8%	58.1%
c+voltEWMANoR	17.7%	15.3%	13.4%	4.2%	38.8%	12.9%	11.2%	9.2%	2.9%	28.1%	10.8%	9.3%	7.7%	2.5%	23.5%
ParamSGTNoR	22.6%	21.6%	9.5%	15.8%	29.9%	20.2%	18.4%	10.7%	12.6%	31.7%	18.3%	15.8%	12.3%	9.4%	34.2%
ParamSGTEWMANoR	37.2%	34.3%	12.3%	30.5%	52.6%	36.4%	33.2%	13.4%	28.3%	54.3%	36.1%	32.6%	14.6%	26.4%	56.6%
NormalEqW	31.3%	30.9%	10.0%	25.2%	35.9%	18.0%	16.2%	14.7%	11.4%	27.3%	16.4%	12.1%	19.9%	6.0%	38.0%
RiskMetrics	41.5%	39.9%	9.3%	37.0%	50.9%	35.5%	31.8%	14.3%	27.3%	54.8%	37.5%	33.1%	18.8%	23.9%	65.0%
Historical	12.2%	11.5%	4.6%	6.2%	20.7%	8.7%	8.3%	3.0%	4.6%	14.2%	7.2%	6.9%	2.4%	3.8%	11.7%
HistVolAdj	47.6%	42.6%	23.2%	27.6%	82.3%	44.3%	39.7%	21.6%	27.2%	75.7%	42.7%	38.1%	20.6%	26.7%	72.5%

Table A.30: This table shows the summary NRMSE statistics of VaR estimates at the 1%, 2.5% and 5% significance levels for the 10000 simulations of $SGT(0.02\%, 1.25\%, 2, 3, 0.3)$ returns. All models estimate daily VaR using a sample size of 1000 observations, including to estimate all parameters, except for the volatility estimate with equally weighted method that uses a sample size of 500.

$SGT(0.02\%, 1.25\%, 2, 3, 0.3)$ returns, $n = 1000$													
Model	$\alpha = 1\%$					$\alpha = 2.5\%$					$\alpha = 5\%$		
	Mean	Median	Stdv	5%	95%	Mean	Median	Stdv	5%	95%	Mean	Median	Stdv
GARCHvolknownNoR	11.8%	10.0%	9.9%	1.7%	27.7%	8.4%	7.3%	6.5%	1.2%	18.5%	7.1%	6.3%	5.2%
GARCHQRNoR	12.7%	10.3%	10.9%	2.1%	30.2%	9.3%	7.8%	7.8%	1.5%	21.4%	7.8%	6.6%	6.2%
volEqWNoR	18.9%	15.1%	15.7%	7.3%	41.2%	17.8%	14.2%	15.2%	7.0%	38.7%	17.5%	13.9%	15.1%
volEWMANoR	42.3%	37.2%	21.4%	25.4%	73.7%	41.1%	36.1%	20.5%	25.1%	70.5%	40.2%	35.4%	20.1%
c+volEqWNoR	23.1%	15.0%	30.9%	3.3%	67.1%	15.1%	10.0%	19.7%	2.2%	43.3%	11.4%	7.6%	15.5%
c+volEWMANoR	13.9%	11.7%	10.6%	3.1%	31.9%	9.2%	7.9%	6.7%	2.0%	20.5%	7.1%	6.1%	5.3%
ParamSGTNoR	44.7%	40.9%	20.5%	28.0%	71.8%	36.6%	33.1%	19.4%	19.5%	62.6%	28.8%	25.6%	18.6%
ParamSGTEWMANoR	58.8%	53.7%	24.8%	38.0%	93.0%	52.2%	47.5%	23.6%	31.9%	85.0%	46.4%	41.7%	22.2%
NormalEqW	28.2%	23.9%	23.2%	13.5%	54.0%	43.0%	38.4%	25.6%	26.7%	70.4%	51.3%	46.6%	26.9%
RiskMetrics	45.4%	40.5%	22.1%	27.9%	76.7%	56.9%	51.7%	24.6%	37.0%	91.2%	64.1%	58.7%	25.8%
Historical	9.3%	8.8%	3.4%	4.8%	15.7%	6.1%	5.8%	2.1%	3.2%	10.0%	4.7%	4.4%	1.6%
HistVolAdj	47.1%	42.3%	22.1%	28.9%	77.6%	45.3%	40.6%	21.6%	28.4%	75.4%	44.1%	39.3%	21.5%

A.2.2 NRMSD results with 500 sample size

Table A.31: This table shows the summary NRMSE statistics of VaR estimates at the 1%, 2.5% and 5% significance levels for the 10000 simulations of GARCH returns, generated with $(\omega, \gamma, \beta) = (0.1, 0.15, 0.8)$ as Zheng et al. (2018). All models estimate daily VaR using a sample size of 500 observations.

Model	$\alpha = 1\%$					$\alpha = 2.5\%$					$\alpha = 5\%$				
	Mean	Median	Stdv	5%	95%	Mean	Median	Stdv	5%	95%	Mean	Median	Stdv	5%	95%
GARCHvolknownNoR	7.4%	6.8%	3.7%	2.4%	14.3%	6.4%	5.9%	3.2%	2.1%	12.4%	6.0%	5.6%	2.9%	2.1%	11.4%
GARCHQNoR	8.9%	8.3%	3.9%	3.6%	15.9%	8.0%	7.5%	3.4%	3.3%	14.2%	7.7%	7.2%	3.3%	3.2%	13.8%
volEqWNoR	36.9%	34.5%	10.8%	26.3%	55.2%	34.0%	32.3%	8.4%	25.8%	47.4%	33.0%	31.6%	7.3%	25.7%	44.6%
volEWMANoR	17.5%	16.8%	3.4%	13.8%	23.8%	16.9%	16.3%	2.8%	13.6%	21.9%	16.5%	16.1%	2.6%	13.6%	20.9%
c+volEqWNoR	39.3%	35.3%	17.2%	26.5%	64.2%	36.6%	33.5%	13.6%	26.1%	56.6%	35.4%	32.8%	12.0%	26.0%	52.3%
c+volEWMANoR	17.5%	16.8%	3.4%	13.8%	23.9%	16.9%	16.3%	2.9%	13.6%	21.9%	16.6%	16.1%	2.6%	13.6%	21.0%
ParamSGTNoR	34.3%	32.9%	7.6%	26.2%	47.0%	32.5%	31.2%	7.0%	25.2%	44.1%	32.0%	30.7%	6.7%	25.2%	43.1%
ParamSGTEWMANoR	18.2%	17.7%	3.4%	14.0%	24.0%	16.2%	15.7%	2.5%	13.4%	20.4%	16.2%	15.8%	2.4%	13.7%	20.0%
NormalEqW	32.3%	30.9%	7.4%	25.1%	43.7%	32.3%	30.9%	7.4%	25.1%	43.7%	32.3%	30.9%	7.4%	25.1%	43.7%
RiskMetrics	15.9%	15.5%	2.3%	13.4%	19.8%	15.9%	15.5%	2.3%	13.4%	19.8%	15.9%	15.5%	2.3%	13.4%	19.8%
Historical	36.3%	34.2%	10.7%	26.4%	52.5%	32.9%	31.6%	7.0%	25.6%	44.1%	31.8%	30.7%	5.9%	25.3%	41.9%
HistVolAdj	18.4%	17.8%	3.5%	14.2%	24.6%	17.6%	17.0%	3.1%	14.0%	22.9%	17.1%	16.6%	2.9%	13.8%	22.1%

Table A.32: This table shows the summary NRMSE statistics of VaR estimates at the 1%, 2.5% and 5% significance levels for the 10000 simulations of GARCH returns, generated with $(\omega, \gamma, \beta) = (3.125 \times 10^{-7}, 0.05, 0.9)$. All models estimate daily VaR using a sample size of 500 observations.

GARCH(1,1) returns, realistic parametrization, $n = 500$													
$\alpha = 1\%$							$\alpha = 2.5\%$						
Model	Mean	Median	StDv	5%	95%	Mean	Median	StDv	5%	95%	Mean	Median	StDv
GARCHvolknownNoR	11.2%	10.3%	5.0%	4.4%	20.3%	8.6%	7.7%	4.7%	2.4%	17.3%	9.2%	8.4%	4.9%
GARCHQRNoR	11.6%	10.7%	5.1%	5.1%	20.9%	9.1%	8.4%	4.6%	2.9%	17.5%	9.7%	9.0%	4.9%
voltEqWNoR	15.2%	14.2%	4.0%	11.0%	23.0%	14.6%	13.9%	3.3%	10.9%	20.9%	14.4%	13.8%	3.0%
voltEWMANoR	10.7%	9.6%	3.2%	7.5%	17.1%	9.9%	9.3%	2.3%	7.5%	14.7%	9.7%	9.2%	2.0%
c+voltEqWNoR	25.2%	19.6%	17.5%	11.2%	58.5%	22.9%	18.0%	15.0%	11.0%	51.1%	22.2%	17.6%	14.6%
c+voltEWMANoR	10.6%	9.3%	5.9%	3.7%	22.0%	9.2%	8.1%	5.0%	3.4%	18.9%	8.8%	7.7%	4.8%
ParamSGTNoR	12.0%	11.8%	1.6%	9.9%	15.0%	11.7%	11.5%	1.4%	9.8%	14.1%	11.6%	11.4%	1.2%
ParamSGTEWMANoR	9.1%	8.8%	1.4%	7.3%	11.8%	8.7%	8.6%	1.0%	7.2%	10.5%	8.5%	8.5%	0.9%
NormalEqW	11.4%	11.3%	1.2%	9.7%	13.7%	11.4%	11.3%	1.2%	9.7%	13.7%	11.4%	11.3%	1.2%
RiskMetrics	8.3%	8.3%	0.8%	7.1%	9.7%	8.3%	8.3%	0.8%	7.1%	9.7%	8.3%	8.3%	0.8%
Historical	13.4%	13.1%	2.1%	10.7%	17.2%	12.7%	12.5%	1.6%	10.5%	15.7%	12.4%	12.2%	1.4%
HistVolAdj	12.0%	11.7%	2.2%	9.0%	16.0%	11.1%	11.0%	1.7%	8.7%	14.1%	10.7%	10.6%	1.5%

Table A.33: This table shows the summary NRMSE statistics of VaR estimates at the 1%, 2.5% and 5% significance levels for the 10000 simulations of $SGT(0.02\%, 1.25\%, 2, \infty, 0)$ returns. All models estimate daily VaR using a sample size of 500 observations.

Model	$\alpha = 1\%$					$\alpha = 2.5\%$					$\alpha = 5\%$				
	Mean	Median	StdV	5%	95%	Mean	Median	StdV	5%	95%	Mean	Median	StdV	5%	95%
GARCHvolknownNoR	7.9%	7.4%	5.1%	0.9%	16.9%	5.1%	4.3%	3.8%	0.4%	12.4%	6.9%	6.4%	3.8%	1.5%	13.8%
GARCHQRNoR	8.1%	7.5%	5.3%	1.2%	16.9%	5.9%	5.2%	4.1%	0.7%	13.6%	7.2%	6.7%	4.0%	1.7%	14.0%
volEqWNoR	6.6%	5.6%	3.8%	2.4%	14.2%	5.9%	5.0%	3.2%	2.3%	12.3%	5.7%	4.9%	3.0%	2.3%	11.6%
volEWMANoR	14.4%	13.5%	3.0%	11.6%	20.5%	13.6%	13.1%	2.0%	11.5%	17.7%	13.4%	13.0%	1.7%	11.5%	16.7%
c+volEqWNoR	17.2%	12.8%	15.0%	2.9%	46.0%	15.1%	11.3%	13.3%	2.6%	39.9%	14.8%	11.1%	12.8%	2.6%	39.7%
c+volEWMANoR	9.5%	8.7%	5.4%	2.3%	19.2%	8.1%	7.4%	4.6%	2.0%	16.7%	7.8%	7.1%	4.4%	2.0%	15.8%
ParamSGTNoR	4.4%	3.9%	1.9%	2.2%	8.2%	3.9%	3.6%	1.4%	2.1%	6.6%	3.5%	3.3%	1.1%	2.1%	5.7%
ParamSGTEWMANoR	12.8%	12.6%	1.1%	11.3%	14.8%	12.6%	12.5%	0.9%	11.3%	14.1%	12.4%	12.4%	0.8%	11.2%	13.8%
NormalEqW	3.2%	3.1%	0.8%	2.0%	4.6%	3.2%	3.1%	0.8%	2.0%	4.6%	3.2%	3.1%	0.8%	2.0%	4.7%
RiskMetrics	12.3%	12.3%	0.7%	11.2%	13.6%	12.3%	12.3%	0.7%	11.2%	13.6%	12.4%	12.4%	0.7%	11.2%	13.6%
Historical	6.8%	6.7%	1.7%	4.2%	9.9%	5.9%	5.8%	1.5%	3.8%	8.5%	5.6%	5.5%	1.4%	3.6%	8.1%
HistVolAdj	15.7%	15.4%	2.0%	12.9%	19.3%	14.7%	14.5%	1.5%	12.6%	17.3%	14.2%	14.1%	1.2%	12.4%	16.4%

Table A.34: This table shows the summary NRMSE statistics of VaR estimates at the 1%, 2.5% and 5% significance levels for the 10000 simulations of $SGT(0.02\%, 1.25\%, 2, 3, 0)$ returns. All models estimate daily VaR using a sample size of 500 observations.

SGT(0.02%, 1.25%, 2, 3, 0) returns, n = 500															
Model	α = 1%					α = 2.5%					α = 5%				
	Mean	Median	StDv	5%	95%	Mean	Median	StDv	5%	95%	Mean	Median	StDv	5%	95%
GARCHvolknownNoR	19.2%	16.1%	15.0%	3.5%	45.9%	9.8%	8.1%	8.0%	0.8%	24.1%	12.0%	10.7%	10.0%	1.7%	26.2%
GARCHQRNoR	20.1%	16.3%	17.6%	3.8%	48.2%	12.4%	9.9%	12.3%	1.2%	30.2%	13.0%	11.1%	10.9%	2.4%	27.6%
voltEqWNoR	19.8%	16.2%	15.0%	7.0%	43.1%	17.4%	14.3%	13.8%	6.5%	37.8%	16.2%	13.2%	13.0%	6.3%	34.4%
voltEWMANoR	39.7%	34.5%	19.5%	24.1%	71.4%	37.7%	33.3%	17.9%	23.8%	64.4%	36.9%	32.6%	17.3%	23.6%	62.4%
c+voltEqWNoR	59.5%	32.4%	97.8%	7.2%	193.8%	40.1%	22.6%	85.9%	5.0%	122.0%	29.9%	17.8%	54.3%	4.0%	91.3%
c+voltEWMANoR	24.6%	20.6%	19.1%	5.6%	56.8%	17.3%	14.7%	12.9%	3.7%	39.3%	14.2%	12.1%	10.6%	3.1%	31.9%
ParamSGTNoR	13.3%	10.2%	12.6%	6.0%	29.7%	14.0%	10.9%	12.9%	6.0%	31.4%	15.3%	12.0%	13.3%	6.1%	34.2%
ParamSGTEWMANoR	33.4%	29.6%	15.1%	23.1%	55.1%	33.8%	30.1%	15.2%	23.2%	55.7%	34.5%	30.9%	15.2%	23.5%	56.0%
NormalEqW	18.2%	17.0%	8.9%	12.2%	24.5%	14.3%	10.7%	13.9%	5.5%	33.3%	25.0%	21.6%	16.1%	12.3%	46.6%
RiskMetrics	33.8%	30.8%	12.3%	27.1%	49.7%	34.6%	30.6%	16.8%	22.9%	58.9%	41.8%	37.5%	19.5%	26.5%	70.4%
Historical	16.4%	15.5%	5.3%	9.7%	26.2%	11.3%	11.0%	3.0%	7.1%	16.8%	9.0%	8.7%	2.3%	5.6%	13.1%
HistVolAdj	47.3%	43.0%	20.9%	28.5%	78.5%	42.4%	38.5%	18.5%	27.3%	69.7%	40.4%	36.4%	17.8%	26.4%	66.8%

Table A.35: This table shows the summary NRMSE statistics of VaR estimates at the 1%, 2.5% and 5% significance levels for the 10000 simulations of $SGT(0.02\%, 1.25\%, 2, 3, -0.3)$ returns. All models estimate daily VaR using a sample size of 500 observations.

Model	$\alpha = 1\%$					$\alpha = 2.5\%$					$\alpha = 5\%$				
	Mean	Median	Stdv	5%	95%	Mean	Median	Stdv	5%	95%	Mean	Median	Stdv	5%	95%
GARCHvolknownNoR	20.6%	17.2%	15.6%	3.9%	49.1%	11.0%	9.2%	8.7%	1.0%	27.1%	13.4%	11.9%	9.6%	2.0%	29.0%
GARCHQRNoR	21.7%	17.5%	18.9%	4.1%	52.6%	13.9%	11.2%	13.1%	1.4%	34.5%	14.6%	12.7%	12.6%	2.7%	31.3%
volEqWNoR	21.5%	17.8%	15.5%	7.7%	47.3%	19.5%	16.0%	14.5%	7.4%	42.2%	18.3%	15.0%	14.0%	7.2%	38.9%
volEWMANoR	42.0%	36.5%	20.1%	25.4%	74.5%	40.1%	35.4%	18.7%	24.9%	68.9%	39.3%	34.8%	18.0%	24.9%	67.4%
c+volEqWNoR	74.1%	39.5%	112.2%	8.1%	246.8%	51.8%	27.8%	87.7%	6.2%	159.7%	40.1%	22.0%	73.3%	4.7%	127.5%
c+volEWMANoR	26.1%	22.1%	18.5%	5.8%	59.3%	19.5%	16.5%	13.8%	4.3%	44.1%	16.5%	14.0%	12.0%	3.6%	37.1%
ParamSGTNoR	23.0%	22.0%	8.6%	16.4%	30.5%	20.4%	18.5%	10.1%	13.0%	33.1%	18.5%	15.8%	11.9%	9.3%	36.2%
ParamSGTEWMANoR	37.5%	34.7%	10.9%	30.9%	52.2%	36.7%	33.6%	12.2%	28.3%	54.0%	36.5%	33.1%	13.6%	26.1%	56.7%
NormalEqW	31.3%	30.9%	10.0%	25.2%	35.9%	18.0%	16.2%	14.7%	11.4%	27.3%	16.4%	12.1%	19.9%	6.0%	38.0%
RiskMetrics	41.5%	39.9%	9.3%	37.0%	50.9%	35.5%	31.8%	14.3%	27.3%	54.8%	37.5%	33.1%	18.8%	23.9%	65.0%
Historical	17.9%	16.9%	5.7%	10.5%	28.4%	12.7%	12.3%	3.4%	7.9%	18.9%	10.4%	10.2%	2.7%	6.6%	15.2%
HistVolAdj	52.8%	47.7%	23.2%	30.1%	90.6%	46.7%	42.3%	20.2%	28.8%	77.9%	44.0%	39.6%	19.2%	27.9%	73.3%

Table A.36: This table shows the summary NRMSE statistics of VaR estimates at the 1%, 2.5% and 5% significance levels for the 10000 simulations of $SGT(0.02\%, 1.25\%, 2, 3, 0.3)$ returns. All models estimate daily VaR using a sample size of 500 observations.

SGT(0.02%, 1.25%, 2, 3, 0.3) returns, n = 500															
Model	$\alpha = 1\%$					$\alpha = 2.5\%$					$\alpha = 5\%$				
	Mean	Median	StDv	5%	95%	Mean	Median	StDv	5%	95%	Mean	Median	StDv	5%	95%
GARCHvolknownNoR	17.1%	14.6%	13.3%	2.7%	40.5%	8.4%	7.0%	7.1%	0.7%	20.7%	10.6%	9.3%	7.4%	1.9%	22.7%
GARCHQRNoR	18.3%	14.9%	17.5%	3.2%	43.3%	11.4%	8.9%	13.3%	1.1%	27.5%	11.6%	9.7%	10.7%	2.1%	25.9%
voltEqWNoR	20.3%	16.4%	16.2%	7.5%	44.7%	18.2%	14.6%	15.1%	6.9%	39.2%	17.2%	13.8%	14.8%	6.6%	36.7%
voltEWMANoR	43.7%	38.0%	22.3%	25.5%	78.0%	41.8%	36.9%	20.6%	25.2%	72.4%	40.9%	36.1%	20.1%	25.2%	69.8%
c+voltEqWNoR	46.3%	25.6%	75.9%	5.9%	149.7%	29.9%	17.4%	63.8%	3.8%	90.1%	22.6%	13.3%	39.6%	3.0%	68.1%
c+voltEWMANoR	21.6%	17.7%	17.4%	4.6%	51.9%	14.5%	12.0%	11.4%	3.0%	34.6%	11.3%	9.4%	8.7%	2.4%	26.4%
ParamSGTNoR	42.9%	39.5%	20.8%	23.8%	68.5%	35.5%	32.2%	20.1%	17.8%	60.8%	28.6%	25.3%	19.6%	11.3%	53.8%
ParamSGTEWMANoR	57.2%	52.4%	24.3%	34.8%	91.0%	51.4%	46.8%	23.3%	30.7%	83.9%	46.3%	41.7%	22.1%	27.4%	77.5%
NormalEqW	28.2%	23.9%	23.2%	13.5%	54.0%	43.0%	38.4%	25.6%	26.7%	70.4%	51.3%	46.6%	26.9%	34.3%	80.0%
RiskMetrics	45.4%	40.5%	22.1%	27.9%	76.7%	56.9%	51.7%	24.6%	37.0%	91.2%	64.1%	58.7%	25.8%	43.1%	99.9%
Historical	13.9%	13.0%	4.6%	8.1%	22.5%	9.2%	8.8%	2.6%	5.6%	13.9%	7.0%	6.7%	2.0%	4.3%	10.5%
HistVolAdj	49.5%	44.9%	22.1%	30.4%	81.3%	46.1%	41.5%	21.1%	29.2%	75.9%	44.4%	39.8%	20.6%	28.3%	73.9%