Assessing the efficiency of maintenance operators: a case study of turning railway wheelsets on an under-floor wheel lathe

Andrade, A. R.[1,2] and Stow, J. M.[3]

Abstract

The present paper assesses the technical efficiency of different operators turning railway wheelsets on a under-floor wheel lathe. This type of lathe is a Computer Numerical Control (CNC) machine used to turn wheelsets in-situ on the train. As railway wheels are turned, a certain amount of the wheel diameter is lost to restore the tread profile and full flange thickness of the wheel. The technical efficiencies of the different wheel lathe operators are assessed using a Stochastic Frontier Analysis (SFA), whilst controlling for other explaining variables such as the flange thickness and the occurrence of rolling contact fatigue (RCF) defects, wheel flats and cavities. Different model specifications for the SFA are compared with Linear Mixed Model (LMM) specifications, showing that the SFA model exhibits a better Akaike Information Criterion (AIC).

Keywords: Technical Efficiency; Railway maintenance; Stochastic Frontier Analysis; Linear Mixed Models; Performance modelling.

1- Introduction

An important factor in the life-cycle of a railway wheelset is the turning maintenance operations. Turning is conducted using an under-floor wheel lathe while the wheelset remains in-situ on the vehicle. Wheels are typically turned to restore the shape of the tread profile (which changes due to wear) and to remove tread damage such as rolling contact fatigue, wheel flats and cavities. Turning may be undertaken at fixed mileage intervals or using a condition-based strategy. However, as the wheel reaches a minimum diameter – the scrap diameter, turning is no longer possible and the wheel has to be renewed. Therefore, in order to maximise wheelset life, wheel lathe operators should try to remove the minimum amount of diameter possible, whilst removing all tread defects and/or restoring the original wheel profile.

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[1] Assistant Professor (Invited), Instituto Universitário de Lisboa (ISCTE-IUL), Business Research Unit (BRU-IUL), Lisboa, Portugal Corresponding author e-mail: araes@iscte.pt

[2] Institute of Railway Research, University of Huddersfield, UK. Queensgate, Huddersfield HD1 3DH, UK.

[3] Assistant Director – Principal Enterprise Fellow, Institute of Railway Research, University of Huddersfield, Queensgate, Huddersfield HD1 3DH, UK.

In generic terms, a railway wheel lathe can be regarded as a maintenance system in which humans and machines interact, i.e. the operators interact with the wheel lathe. This 'maintenance system' receives as input the wheel condition pre-turning, including the wear and damage defects suffered during operation and the pre-turning diameter (D^{pre}), as well as the technician/operator and their attitude and experience; and it provides as output: the wheel condition post-turning, namely its final/post-turning diameter (D^{post}). The diameter loss due to turning (ΔD_T) is then the difference between the pre-turning diameter and the post-turning diameter, i.e. $\Delta D_T = D^{pre} - D^{post}$, and it is a measure that can be used to assess how efficient a wheel operator is in the turning operation controlling for any other influencing factor. The wheel lathe operator decides how much material to remove whilst the lathe will advise how much is required to restore the profile. When removing damaged material, the operator has to decide how much to remove to get underneath the damaged material.

Two research questions can be formulated: i) which factors may contribute to explain the variability in the diameter loss due to turning? and ii) controlling for those factors, do different operators exhibit significant differences in their performance using the wheel lathe?

To answer these research questions, we made use of a Stochastic Frontier Analysis (SFA) model, which is a common statistical model in economics, management and business sciences for benchmarking. This was then compared with a Linear Mixed Model (LMM) to understand the effect of variability in the decisions taken by different technicians on the statistical modelling of diameter loss due to turning. The main advantage of using SFA, comparing with other benchmarking techniques, is that it allows a separation between noise and inefficiency[1].

The main novelty of the present paper is the application of SFA in the risk and reliability area in a mechanical system, by showing that SFA provides a better fit than LMM, which are complex models currently being used in statistically modelling wear and damage of railway wheelsets[2]. Therefore, the paper provides an example of why risk and reliability researchers should start paying attention to SFA as an alternative technique to statistically model the degradation of mechanical components in a system.

The outline of this paper is as follows: this first section introduces the need to assess the technical efficiency of different wheel lathe operators in statistical modelling of the diameter loss due to turning, whereas the second section provides some background on the SFA topic. The third section discusses the statistical methods used in this paper, namely SFA and LMM, and the fourth section provides details on a sample dataset from a wheel lathe. Then, section fifth applies SFA and LMM models to a, comparing several model specifications for the SFA and LMM approaches. Finally, the last section highlights the main conclusions and some directions for future research.

2- A brief background

The assessment of technical efficiency has its roots in the economic literature under the topics of benchmarking and quantitative performance evaluation. Many studies have been published in areas like economics, operation research, management and business, and though this is a mature topic in economic literature, it is not common in mechanical engineering and especially in modelling physical phenomena in general, or in the context of human-machine interaction in a maintenance system.

In the economic literature, the classical reference on this topic is Farrell[3] who proposed a method to measure productive efficiency. The introduction of the SFA as a robust statistical method was put forward twenty years later in 1977. According to Kumbhakar and Lovell[4], SFA was first proposed by Meeusen and Broeck[5], Aigner et al.[6] and Battese and Corra[7]. These SFA models specified two error components: i) a first component associated with statistical/measurement noise and ii) a second non-negative component associated with technical inefficiency. These three different SFA models were distinct in the sense that they specified different distributions for the second error component: an exponential[5], a half-normal[7] and both distributions[6].

In transportation systems, some references on measuring technical efficiency using SFA can be found in various contexts. For the airway system, Michaelides et al.[8] explored SFA for international air carriers, analysing a dataset of the world's largest network airlines and comparing estimates of technical efficiency using SFA and Data Envelopment Analysis (DEA). Scotti et al.[9] analysed the role of airport competition on the technical efficiency of 38 Italian airports by applying an SFA approach. For the road transport system, Welde and Odeck[10] compared the technical efficiency of road toll companies operating in Norway, using both SFA and DEA techniques. Filippini et al.[11] used SFA to assess differences in levels of cost efficiency of bus lines operated under competitively tendered contracts versus performance-based negotiated contracts in Swiss public transport. For the railway system, Smith[12] applied the SFA technique to estimate the efficiency gap between Network Rail and other European rail infrastructure managers to provide a quantitative basis for fair regulation. Farsi et al.[13] applied several statistical models, including the SFA technique to measure cost efficiency in Swiss railways for a panel of 50 railway companies operating over a 13-year period. Other applications of SFA can also be found in a literature review on the economic performance of waste management [14].

To the best of our knowledge, the SFA method to statistically compare the performance of different machine operators in a maintenance system has not been applied before, and it provides an opportunity to compare it with other statistical models such as LMM. Moreover, free access to R packages[15] called *Benchmarking* and *lme4* has equipped researchers and practitioners with routines to conduct SFA[1] and to estimate LMM[16] in a straightforward way. The next section provides details on these two statistical techniques.

3- Statistical methods

This section discusses the statistical methods used to model the diameter loss due to turning (ΔD_T), namely a) SFA and b) LMM.

a. SFA

SFA is a method typically used in benchmarking, especially in economic literature to assess the technical efficiency of different firms/agents. In simple terms, given a set of data (typically an output and some input), the basic research question is to find a frontier, above which it is technically impossible to increase the output for that level of input. This is called a 'production frontier'. SFA is a method used to assess technical efficiency of different agents in producing some outputs provided a certain amount of inputs. An agent or a firm, as it is usually referred to in microeconomics literature, will be more efficient if it produces maximum output with the least inputs needed. Therefore, the central idea of SFA is to try to define a frontier of efficiency, where each agent would be 100% efficient and cannot be more efficient than that level, i.e. the outputs are maxima for the same level of inputs, or the inputs are minima for the same level of output.

SFA includes two stochastic terms: i) a term v associated with some measurement errors and the stochastic nature of a production function, and ii) a term u associated with possible inefficiency of a given agent or firm. The SFA model will then assume the following expression:

$$y_i = f(X_i|\beta) + v_i - u_i \qquad (1)$$

In which: y_i is the dependent variable (output) that we are interested in modelling for observation i; X_i are the explaining/independent variables; β are parameters describing the parametric functional form f; v_i is the random measurement error for observation i and u_i is an error for observation i associated with inefficiency.

Some assumptions on the error terms v and u must be made. They are assumed to be independent and the inefficiency term u assumes only nonnegative values, i.e. u follows a one-sided distribution. The most typical assumptions are that v_i is normally distributed with mean zero and a certain variance, i.e. $v_i \sim N(0, \sigma_v^2)$ and u_i is half-normally distributed, i.e. $u_i \sim N_+(0, \sigma_u^2)$. In case $u_i = 0$ then the firm or agent is 100% efficient, whereas if $u_i > 0$, there is some inefficiency.

In the case that the output of the system (y_i) is not in the form 'the more, the better' as in a typical production function, but instead is in the form 'the less, the better', a simple transformation $y_i' = -y_i$ can be applied to the original dependent variable y_i so that the new variable y_i' is in the form 'the more, the better'. For the case under analysis, we will see that the output (i.e. the diameter loss due to turning) is in the form 'the less, the better', so the simple transformation will be applied. The results are presented in the original form for the output diameter loss due to turning (ΔD_T), i.e. in the form 'the less, the better'.

b. LMM

LMM are flexible linear models that can tackle the fixed effects of different controlling variables ($\boldsymbol{X_i\beta}$) in the expected mean of the dependent variable ($\boldsymbol{y_i}$), as well as the random effects associated with some factor or group ($\boldsymbol{Z_i b_i}$). In mathematical terms, if one considers a single grouping level, LMMs can be formulated as[16]:

$$\boldsymbol{y_i} = \boldsymbol{X_i\beta} + \boldsymbol{Z_i b_i} + \boldsymbol{\varepsilon_i} \quad (2)$$

In which: $\boldsymbol{y_i}$ is the dependent variable for group i, $\boldsymbol{X_i}$ is the design matrix for that group i, $\boldsymbol{\beta}$ is the slope parameter and $\boldsymbol{\varepsilon_i}$ is the residual error for group i. $\boldsymbol{Z_i}$ is the matrix of covariates corresponding to random effects and $\boldsymbol{b_i}$ are the corresponding random effects for each group i.

Some assumptions then have to be made on the random components:

$$\boldsymbol{b_i} \sim N(\mathbf{0}, \boldsymbol{\mathcal{D}}),\ \boldsymbol{\varepsilon_i} \sim N(\mathbf{0}, \boldsymbol{\mathcal{R}_i}), \text{ with } \boldsymbol{b_i} \perp \boldsymbol{\varepsilon_i} \quad (3)$$

The random effects associated with a given group ($\boldsymbol{b_i}$) and the residual error for each group ($\boldsymbol{\varepsilon_i}$) are normally distributed with zero mean and co-variance matrices equal to $\boldsymbol{\mathcal{D}}$ and $\boldsymbol{\mathcal{R}_i}$ respectively. Both error terms are assumed to be independent between each other (for the same group i and between different groups). Additionally, the co-variance matrices are specified with an unknown scaling parameter σ^2:

$$\boldsymbol{\mathcal{D}} = \sigma^2 \boldsymbol{D} \text{ and } \boldsymbol{\mathcal{R}_i} = \sigma^2 \boldsymbol{R_i} \quad (4)$$

Some additional constraints on the matrices $\boldsymbol{D}$ and $\boldsymbol{R_i}$ have to be made to guarantee identifiability[16], which are usually simplifications leading to choices of the matrices $\boldsymbol{D}$ and $\boldsymbol{R_i}$ that are multiples of the identity matrix.

The main difference between these two statistical methods is that the LMM approach provides the 'average' production function, whereas the SFA approach estimates the frontier that is only achievable if there are no inefficiencies.

4- Sample description

The sample refers to a set of railway wheels that were maintained at a single depot on an under-floor wheel lathe. The dataset was collected in a railway maintenance depot from a fleet of modern multiple units, in the time period between December 2006 and July 2012 (i.e. a 7-year period), representing a total of 6,246 observations of railway turned wheelsets. All modern multiple unit have exactly three vehicles, and each vehicle has eight wheels (i.e. four wheelsets). For further details, the reader is referred to our previous work on wear and damage of railway wheelsets[2].

Table 1 provides the variables, their description and some statistics of the dataset collected. The dependent variable is the diameter loss due to turning ($\Delta \mathrm{D_T}$) and the remaining variables are used as independent/explaining variables or factors, namely: flange thickness pre-turning (F_t), occurrences of Rolling Contact Fatigue (RCF), of cavities (CAV) and of wheel flats (FLAT), mileage since last turning, wheelset type (motored, internal or leading trailer), unit number (in a total of 51 units), vehicle type (in 3 types: Driving Motor Composite (DMC), Motor Second (MS), Driving Motor Second (DMS)) and the month of measurement (in a total of 68 months).

5- Applying SFA and LMM

This section starts with a brief description of the sample of turning records at the wheel lathe and then applies the SFA and LMM statistical methods described above to the sample in order to assess the technical efficiencies of the wheel lathe operators.

Several model specifications were run to provide a basis for comparison between SFA and LMM models: 4 model specifications for SFA (M0.SFA-ΔD_T up to M3.SFA-ΔD_T) and 7 model specifications for LMM (M0.LMM-ΔD_T up to M6.LMM-ΔD_T). For the SFA, each model specification sequentially adds more explaining variables, i.e. first model only considers the flange thickness (F_t), the second model adds the occurrence of damage defects (Y_{RCF}, Y_{FLAT}, Y_{CAV}), the third model adds the wheelset type (W_t) and the fourth model adds some interaction terms with mileage since turning and damage defects ($M \times Y_{RCF}$, $M \times Y_{FLAT}$, $M \times Y_{CAV}$). Similarly, for the LMMs each specification sequentially adds fixed effects and random effects, i.e. the first model also only considers the flange thickness (F_t) as a fixed effect, the second model adds the occurrence of wheel tread damage and the wheelset type (Y_{RCF}, Y_{FLAT}, Y_{CAV}, W_t) as fixed effects, the third model adds the technician (T) as a random effect, the fourth model adds the month of measurement (M_n) as a random effect, the fifth model adds the unit (U) as a random effect, the sixth model adds the vehicle (V) as a random effect. The final seventh LMM model specification also adds the interaction terms with mileage since turning and damage defects ($M \times Y_{RCF}$, $M \times Y_{FLAT}$, $M \times Y_{CAV}$) for a fair comparison with the fourth SFA model specification, i.e. so that models M3.SFA-ΔD_T and M6.LMM-ΔD_T have exactly the same explaining variables. Table 2 provides details on each of the estimated model and 'goodness-of-fit' statistics for easier comparison.

Several 'goodness-of-fit' measures are computed for both models. The Log-likelihood value and the –2 Restricted Log-likelihood value are computed for the SFA and LMM models, respectively, and the Akaike Information Criterion (AIC) value is computed for all model specifications. The AIC is used for comparing between different SFA and LMM model specifications. It combines a goodness-of-fit measure with a measure of model complexity, i.e. the -2 Log-likelihood plus 2 times the number of parameters. AIC provides a criterion to compare different models, in which the preferred model is the one with the lowest AIC value.

Tables 3 and 4 show the estimates for the parameters of all SFA and LMM model specifications, respectively. Regarding Table 3, all variables are statistically significant at the 5% significance level for all model specifications, except for the parameter associated with motored wheelsets (β_{Motor}). The flange thickness (F_t) has a negative effect, i.e. the lower the flange thickness, the more diameter a wheel will lose due to turning; whereas the damage defects (Y_{RCF}, Y_{FLAT}, Y_{CAV}) have a positive effect, i.e. the occurrence of tread damage increases the diameter lost due to turning. Note that the damage defects have all positive interaction terms with mileage since turning, i.e. the diameter loss required to remove tread damage increases as the mileage since turning increases. Furthermore, the scale parameters show that the term associated with inefficiency provides a higher value of variance than the term associated with random noise, i.e. $\sigma_u > \sigma_v$ resulting into a value for $\lambda = \frac{\sigma_u}{\sigma_v}$ higher than 1. This shows that the error component associated with inefficiency (u_i) dominates the variability around the mean of the diameter loss due to turning, controlling for the explaining variables.

Figure 1 provides a contrast between the SFA specified in model M3.SFA-ΔD_T, an Ordinary Least Square (OLS) estimation without considering inefficiency terms and a Corrected Ordinary Least Square (COLS) approach. The OLS and COLS have the same slopes, though the COLS line is shifted to the minimum diameter loss observed. A box-and-whisker plot is presented in Figure 2, for the total residual above the SFA line for different technicians based on the residuals estimated from model M3.SFA-ΔD_T. One interesting finding from the statistical modelling regards the variability between wheel lathe operators. The model showed that, whilst three of the operators removed very similar amounts of material above the minimum possible (around 2.0 mm diameter on average), one operator (number '3') removed significantly more (more than an average of 10.0 mm). The model is constructed to carefully control for other factors which would influence the minimum possible value to remove. For example, if operator '3' was considered the most experienced and therefore given the most damaged wheels to turn. As the wheel damage types, depths, times of turning etc. were found to be similar for all operators, the analysis therefore suggests that there is an underlying difference in the turning approach adopted by operator '3'. This has the potential to significantly affect the overall wheelset life. Indeed the lathe operator was found to be one of the most statistically significant factors in amount of material removed at turning. Although it is beyond the scope of this paper, it would be interesting to investigate whether wheels turned by Operator '3' subsequently had shorter or longer intervals to next turning. It may be that removing more material is more effective in ensuring that RCF damage is fully removed preventing early recurrence. Alternatively it may be that operator '3' removed more material than necessary shortening the wheels life.

In microeconomics literature, it is also common to represent the same box-and-whisker plot for different agents/firms but measuring technical efficiency (i.e. e^{-u_i}). Figure 3 represents the technical efficiency of the different wheel lathe operators.

All the LMM models shown in Table 4 exhibited statistically significant estimates at the 5% significance level, except for the parameter associated with motored wheelsets (β_{Motor}) for the model M6.LMM-ΔD_T. The factor associated with different technicians (T) was the random effect that showed the greatest variability, followed by the random effects associated with month of measurement (M_n), unit (U) and vehicle (V).

Finally, comparing model M6.LMM-ΔD_T with model M3.SFA-ΔD_T, the best SFA model has a lower AIC value (27083.64), than the best LMM models with an AIC of 28157.69. This indicates that the SFA model performed better than the LMM model. The finding suggests that using an error component structure associated with inefficiencies, by mathematically adding a one-sided distribution, may considerably enhance the statistical models, even when comparing with a complex statistical model like the LMM.

6- Conclusions and further research

This paper provided evidence of the importance of modelling the performance of different wheel lathe operators in the maintenance of railway wheelsets. By applying an SFA model, we were able to identify the technical efficiency of each wheel lathe operator, when compared to a 'best practice' frontier, and thus, isolate the bias due to inefficiencies of each operator, while controlling for other factors that contribute to explain the variability of the diameter loss due to turning. It also highlights the need to provide lathe operators with clear guidance and training so that they understand the effect of their decisions on wheel life. Therefore, current maintenance managers should apply this technique to identify maintenance operators, which might be able to improve their performance, and recommend them specific training.

The comparison between the SFA models and LMM models showed that the error component structure that tackles technical inefficiencies provides a significant enhancement in the AIC value. This suggests that the application of statistical techniques, such as SFA, previously applied in economic analysis can also prove useful in modelling physical phenomena.

For further research, it would be interesting to add more variables describing the attitudes and experience of the different technicians, trying to answer for instance whether or not more experienced technicians perform better than others. Moreover, it would also be useful to conduct further analysis on whether turning more material off might be beneficial in preventing the re-occurrence of RCF damage and cavities. In that sense, we recommend an extension of the assessment of technicians' performance, which would necessarily imply collecting other sources of data (not available in our sample), more usual in research areas like human-computer interaction, human factors and usability engineering. Finally, we believe that the combination of mixed factors with SFA models through a hierarchical Bayesian model might provide even better results, but this is a step for the future, in which a good starting point is Griffin and Steel[17], which could be combined with a previous work[18].

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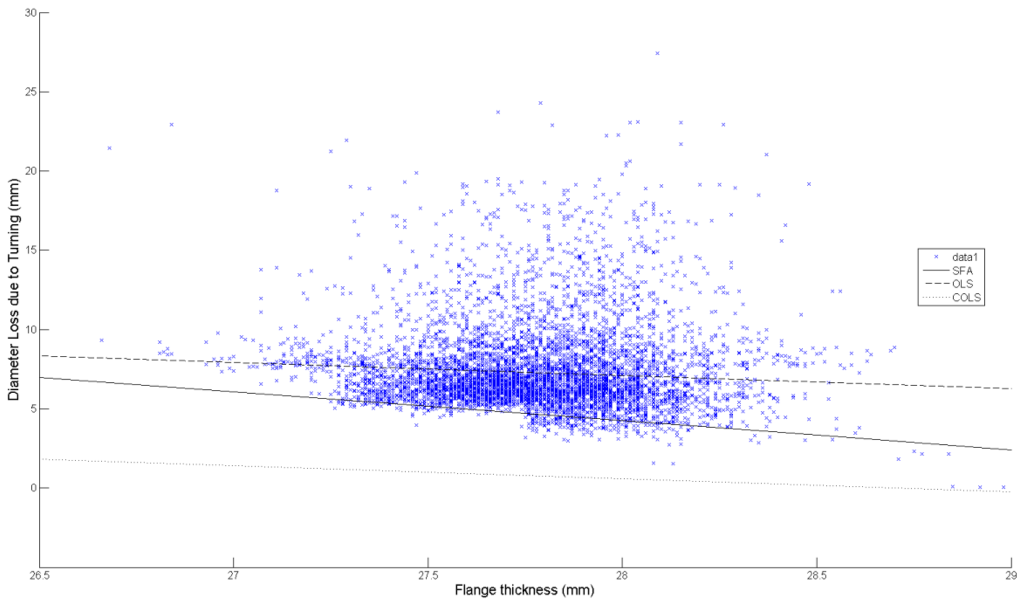

Figure 1 - Comparison between OLS, COLS and SFA approaches.

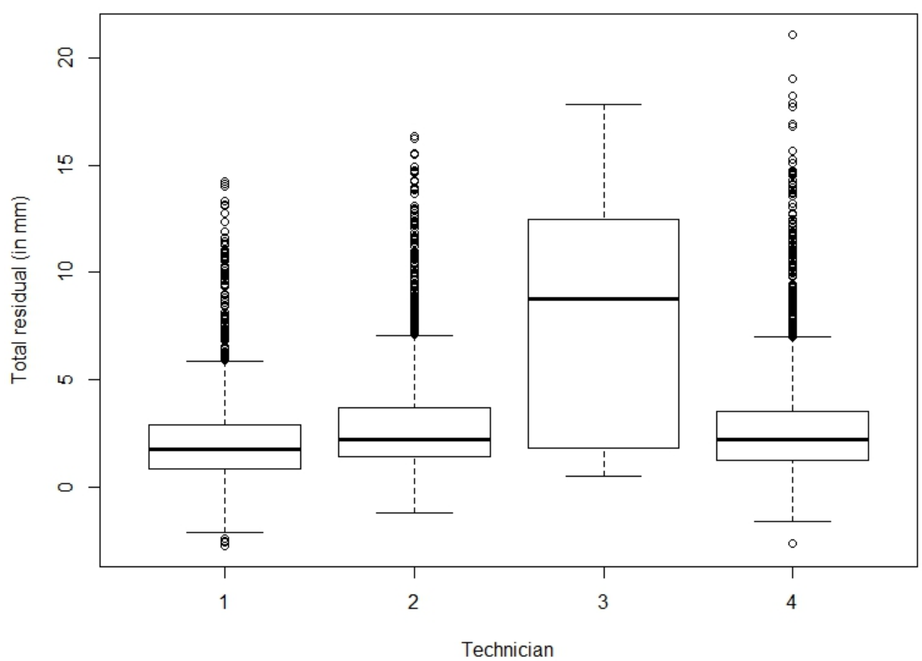

Figure 2 – Box-and-whisker[4] plot for the total residual (i.e. $v_i - u_i$) (in mm) for different wheel lathe operators/technicians.

[4] The box-and-whisker is a typical plot in statistics that helps to show the variability of a given sample. The box refers to the first (Q1) and third (Q3) quartiles, and the line in the box marks the median value (i.e. the second quartile –

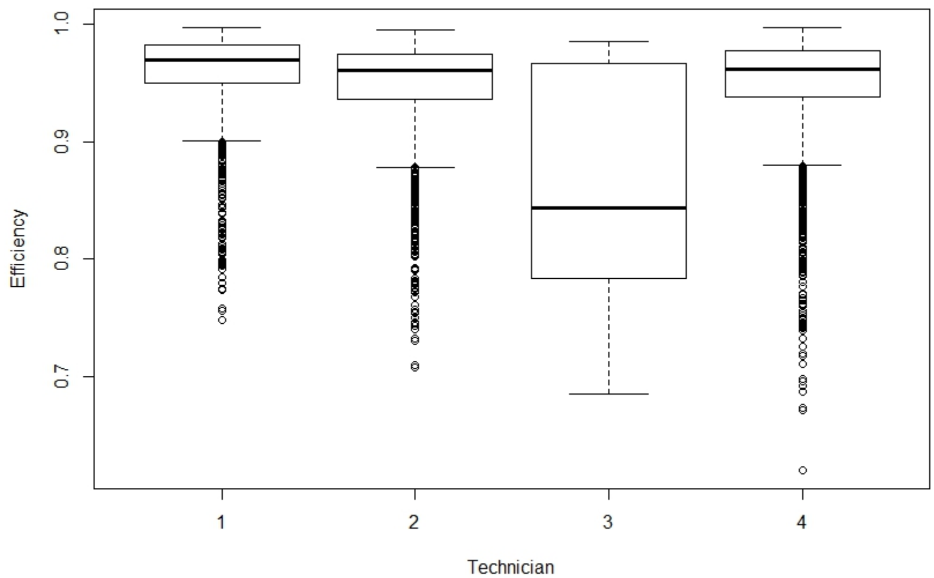

Figure 3 - Technical Efficiency (i.e. e^{-u_i}) for different wheel lathe operators/technicians.

Q2). The whiskers go from the lower limit (Q1-1.5×IQR) to the upper limit (Q3+1.5×IQR), in which IQR is the interquartile range, i.e. the difference between Q3 and Q1 (IQR=Q3-Q1). The observations that go outside the whiskers range are considered outliers and are identified as simple points.

Variables	Description	Type	Mean	St. Dev.	Min	Max
ΔD_T	Diameter loss due to turning (in mm)	Continuous	7.5253	2.7696	0.037	27.443
F_t	Flange thickness pre-turning (in mm)	Continuous	27.782	0.2596	26.66	28.98
Y_{RCF}	1 if a Rolling Contact Fatigue (RCF) defect occurred, 0 otherwise.	Binary	0.1002	0.3003	0	1
Y_{CAV}	1 if a cavity defect occurred, 0 otherwise.	Binary	0.0195	0.1384	0	1
Y_{FLAT}	1 if a wheel flat defect occurred, 0 otherwise.	Binary	0.1313	0.3377	0	1
M	Mileage since turning (in 1000 miles)	Continuous	111.54	50.67	0.02	235.98
T	Technician (4 different operators/technicians)	Nominal	-	-	-	-
W	Wheelset type (3 types: motored, internal or leading trailer)	Nominal	-	-	-	-
U	Unit number (51 units)	Nominal	-	-	-	-
V	Vehicle type (3 types: DMC, MS and DMS)	Nominal	-	-	-	-
M_n	Month of measurement (68 months)	Nominal	-	-	-	-

Table 1 – Variables, their description, type and some statistics for a total of 6,246 observations.

Model	Explaining Variables	Log Likelihood	-2 Restricted Log Likelihood	Number of parameters (df)	AIC
M0.SFA	F_t	-13900.79	-	4	27809.58
M1.SFA	F_t, Y_{RCF}, Y_{FLAT}, Y_{CAV}	-13606.70	-	7	27227.40
M2.SFA	F_t, Y_{RCF}, Y_{FLAT}, Y_{CAV}, W_t	-13566.25	-	9	27150.50
M3.SFA	F_t, Y_{RCF}, Y_{FLAT}, Y_{CAV}, W_t, $M\times Y_{RCF}$, $M\times Y_{FLAT}$, $M\times Y_{CAV}$	-13529.82	-	12	27083.64
M0.LMM	FE: F_t RE: -	-	-	3	30455.72
M1.LMM	FE: F_t, Y_{RCF}, Y_{FLAT}, Y_{CAV}, W_t RE: -	-	-	8	29233.51
M2.LMM	FE: F_t, Y_{RCF}, Y_{FLAT}, Y_{CAV}, W_t RE: (T)	-	29041.38	9	29059.38
M3.LMM	FE: F_t, Y_{RCF}, Y_{FLAT}, Y_{CAV}, W_t RE: (T, M_n)	-	28441.62	10	28461.62
M4.LMM	FE: F_t, Y_{RCF}, Y_{FLAT}, Y_{CAV}, W_t RE: (T, M_n, U)	-	28289.20	11	28311.20
M5.LMM	FE: F_t, Y_{RCF}, Y_{FLAT}, Y_{CAV}, W_t RE: (T, M_n, U, V)	-	28168.70	12	28192.70
M6.LMM	FE: F_t, Y_{RCF}, Y_{FLAT}, Y_{CAV}, W_t, $M\times Y_{RCF}$, $M\times Y_{FLAT}$, $M\times Y_{CAV}$, RE: (T, M_n, U, V)	-	28127.69	15	28157.69

Table 2 – Explaining variables and comparison of the fit statistics from different models estimated for the dependent variable diameter loss due to turning (ΔD_T). Note 1: All models included an intercept constant value (β_0). Note 2: For the LMM models, the Fixed Effects (FE) are presented first and the Random Effects (RE) are included in parenthesis.

Model Label	Parameter	M0.SFA-ΔD_T	M1.SFA-ΔD_T	M2.SFA-ΔD_T	M3.SFA-ΔD_T
1	β_0	54.425	57.797	55.441	52.290
		(2.3554)	(2.4344)	(2.5211)	(2.0379)
F_t	β_{F_t}	-1.794	-1.920	-1.829	-1.714
		(0.0850)	(0.0879)	(0.0910)	(0.0738)
Y_{RCF}	β_{RCF}	-	1.613	1.604	0.530
			(0.0740)	(0.0755)	(0.2726)
Y_{FLAT}	β_{flat}	-	1.146	1.100	0.698
			(0.0718)	(0.0741)	(0.1251)
Y_{CAV}	β_{cav}	-	1.452	1.495	0.684
			(0.1720)	(0.165)	(0.3228)
W_t	β_{Motor}	-	-	-0.036	-0.059
				(0.0631)	(0.0736)
	$\beta_{Trailer}$	-	-	-0.447	-0.462
				(0.0693)	(0.0812)
	$\beta_{Leading}$	-	-	0^b	0^b
$M \times Y_{RCF}$	$\beta_{M\times RCF}$	-	-	-	0.009
					(0.0021)
$M \times Y_{FLAT}$	$\beta_{M\times FLAT}$	-	-	-	0.006
					(0.0015)
$M \times Y_{CAV}$	$\beta_{M\times CAV}$	-	-	-	0.013
					(0.0037)
Scale	σ_v	0.5921	0.6881	0.6923	0.6884
	σ_u	4.0172	3.7159	3.6828	3.6610
	λ	6.784	5.400	5.319	5.318
		(0.2619)	(0.2030)	(0.2084)	(0.1697)
Log Likelihood		-13900.79	-13606.70	-13566.25	-13529.82
AIC		27809.58	27227.40	27150.50	27083.64
Number of parameters (df)		4	7	9	12

Table 3– Estimates for the parameters of different models M0.SFA-M3.SFA for the dependent variable diameter loss due to turning (ΔD_T).

Model Label	Parameter	M0.LMM-ΔD_T	M1.LMM-ΔD_T	M2.LMM-ΔD_T	M3.LMM-ΔD_T	M4.LMM-ΔD_T	M5.LMM-ΔD_T	M6.LMM-ΔD_T
Fixed Effects								
1	β_0	16.8587 (3.7467)	34.48553 (3.50410)	39.58786 (3.64597)	45.9905 (3.99044)	44.50749 (3.97864)	46.51300 (3.97040)	38.36797 (4.00547)
F_t	β_{F_t}	-0.3360 (0.1349)	-0.98135 (0.12646)	-1.12562 (0.12487)	-1.35069 (0.13599)	-1.29974 (0.13629)	-1.36727 (0.13515)	-1.12076 (0.13796)
Y_{RCF}	β_{RCF}	-	3.53981 (0.10808)	3.61747 (0.10705)	3.45374 (0.10805)	3.38064 (0.10723)	3.26511 (0.10677)	2.13286 (0.28118)
Y_{FLAT}	β_{FLAT}	-	1.50080 (0.09762)	1.43816 (0.09617)	1.45948 (0.10985)	1.47444 (0.10990)	1.51732 (0.10888)	1.09268 (0.15382)
Y_{CAV}	β_{CAV}	-	2.68745 (0.23063)	2.90104 (0.22731)	2.91918 (0.22543)	2.87964 (0.22390)	2.83966 (0.22174)	1.30192 (0.38160)
W	β_{Motor}	-	-0.44570 (0.09046)	-0.42377 (0.08894)	-0.47616 (0.08403)	-0.49889 (0.08260)	-0.59099 (0.08472)	-0.05067 (0.08282)
	$\beta_{Trailer}$	-	-0.24949 (0.09476)	-0.21310 (0.09322)	-0.21884 (0.08808)	-0.22938 (0.08652)	-0.37037 (0.09170)	-0.54323 (0.09191)
	$\beta_{Leading}$	-	0[b]	0[b]	0[b]	0[b]	0[b]	0[b]
$M \times Y_{RCF}$	$\beta_{M \times RCF}$	-	-	-	-	-	-	0.00958 (0.00213)
$M \times Y_{FLAT}$	$\beta_{M \times FLAT}$	-	-	-	-	-	-	0.00544 (0.00155)
$M \times Y_{CAV}$	$\beta_{M \times CAV}$	-	-	-	-	-	-	0.02091 (0.00443)
Random Effects								
T	$\sqrt{d_T}$	-	-	2.264	2.558	2.422	2.514	2.278
M_n	$\sqrt{d_{Mn}}$	-	-	-	1.023	1.050	1.062	1.049
U	$\sqrt{d_U}$	-	-	-	-	0.489	0.500	0.501
V	$\sqrt{d_V}$	-	-	-	-	-	0.423	0.330
Scale	σ	2.769	2.510	2.467	2.318	2.273	2.248	2.237
-2 Restricted Log Likelihood		-	-	29041.38	28441.62	28289.20	28168.70	28127.69
AIC value		30455.72	29233.51	29059.38	28461.62	28311.20	28192.70	28157.69
Number of parameters (df)		3	8	9	10	11	12	15

Table 4 – Restricted Maximum Likelihood (REML) estimates for the parameters of models M0.LMM-M6.LMM for the dependent variable Diameter loss due to turning (ΔD_T).

[a] Approximate Standard Errors for Fixed Effects are included in parentheses. [b] This parameter is redundant.