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PROBABILISTIC MODELING AND ANALYSIS OF MOTORCARS RECYCLING AND DISMANTLING PROCESS IN A SCARCE OIL ENVIRONMENT

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ABSTRACT

With the basis on the $M|G|\infty$ queue system it is modelled and analysed the idle motorcars dismantling and recycling process in a scarce conventional energy ambience. The model allows concluding that the dismantling or recycling time hazard rate function plays a key role in the process control. Moreover, through a cost-benefit analysis based on this model, it is possible to obtain criteria to evaluate the interest of the recycling and dismantling process or to compare the dismantling process against the recycling process.

Keywords: Motorcars, recycling, dismantling, infinite servers queues, hazard rate function.

1. INTRODUCTION

Conventional sources of energy are exploited at a very fast rate and they will be exhausted in a near future, maybe within just a few decades, see [2]. If this scenario happens, the owners of motorcars which function on the basis of oil, in a huge number, will look for an alternative use with new sources of energy; or simply they give them for dismantling. So this justifies that the model to be presented, for this situation, is based on the use of infinite servers queues, more concretely: the $M|G|\infty$ queue.

The most important conclusion from this model is that the rythm at which the recycling and dismanting actions are performed is determinant, being

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important in this analysis the dismantling or recycling time hazard rate function, see [7], that works as reference rate.

The hazard rate function is very much used in reliability theory, see again [7]. For instance, if the variable under study is the lifetime of a device, the hazard rate function is the rate at which occur the devices failures making them stop working. In the situation under study in this paper, the dismantling or recycling time hazard rate function is the rate at which the recycling or dismantling services end. So it incorporates technologigal information on the dismantling and recycling processes, respectively.

In the next section the model is outlined. In the following it is presented a cost-benefit analysis based on the model. Then some conclusions are outlined. The paper ends with a short list of references.

2. CONSTRUCTING THE MODEL

According to the Kendall´s notation the M in $M|G|\infty$ queue, see [3; 4], means that the customers arrivals process is a Poisson one at rate λ. The G means that it is considered any (general) service time distribution function $G(.)$, being its mean value designated α. The ∞ symbol signs that there are infinite servers in the queue, that is: when a customer arrives it always finds an available server (or that there is no distinction between the customer and the server: the customer is its own server). The traffic intensity, called $\rho = \lambda\alpha$, is an important parameter, not dimensional, that measures the ability (speed) of the servers to deal with the customers. The M | G | ∞ queue has neither losses nor waiting.

In the model, intended to be constructed, the costumers are the motor cars that become idle and the arrivals rate is the rate at which they so become. The service time for each one is the time that goes from the moment they become idle till the moment that end either their recycling or dismantling service.

The service time hazard rate function, *h (t)*, is, see [7] :

$$h(t) = \frac{g(t)}{1 - G(t)} \qquad (1)$$

being $g(.)$ is the probability density function associated to $G(.)$. From[4]:

If $G(t) < 1, t > 0$ **continuous and differentiable and**

$$h(t) \geq \lambda,\ t > 0 \qquad (2)$$

the probability that the queue is empty at time t is non- decreasing.

So

-If the rate at which the services end is greater or equal than the customers' arrivals rate the probability that the queue is empty at time t is non- decreasing.

-Considering exponential service times, the service time hazard rate function is constant and given by $h(t) = {}^{1}/_{\alpha}$, (2) is equivalent to

$$\rho \leq 1 \qquad (3)$$

It is evidenced by equation (2) that if either the recycling or the dismantling rate is greater or equal than the rate at which the motorcars become idle, the probability that that there are no idle motorcars at instant t, does not decrease with t. So the system has a propensity to become balanced as far as time goes on. Again from[4]:

If $G(t)<1, t>0$ continuous and differentiable and

$$h(t) \leq \lambda,\ t>0 \qquad (4)$$

the mean number of the customers in the queue is non- decreasing.

Then

-If the rate at which the services end is lesser or equal than the customer's arrivals rate, the mean number of the customers in the queue is non- decreasing.

- Considering exponential service times, (4) is equivalent to

$$\rho \geq 1 \qquad (5)$$

It results from equation (4) that if either the recycling or the dismantling rate is lesser or equal than the rate at which the motorcars become idle, the mean number of idle motor cars in time t does not decrease with t. This means that the system has a tendency to become unbalanced as far as time goes on.

Note that for exponential service times, in both criteria considered, the condition that guarantees the system balance possibility, $\rho \leq 1$, does not depend on t.

3. PERFORMING COST-BENEFIT ANALYSIS

The model built over the $M|G|\infty$ queue, to study the motorcars recycling and dismantling process, in the former section emphasized the key role of $h(t)$ to monitor the way the system is managed. It is also important to determine λ, a characteristic of the system, so exactly as possible. To perform an economic type analysis, based on the model presented behind, both *h(t)* and λ are determinant. Also it must be considered an additional parameter *p*: the probability, or percentage, of the motorcars arrivals intended to the recycling being consequently *1-p* the same to the intended to the dismantling. Call $h_i(t), c_i(t)$ and $b_i(t), i = 1,2$ the hazard rate function, the mean cost and the mean benefit, respectively for recycling when $i = 1$ and for dismantling when $i = 2$.

The total cost as referred to the unit of time for motor cars recycling and dismantling is:

$$C(t) = pc_1(t)\,\lambda + (1-p)c_2(t)\,\lambda \quad (6)$$

And the benefit as referred to the unit of time resulting from recycling and dismantling

$$B(t) = b_1(t)h_1(t) + b_2(t)h_2(t) \quad (7)$$

Making an option for an economic point of view quite extreme, see [6], it may be stated that

$$B(t) > C(t), \text{for every } t \quad (8)$$

So it results that it is interesting recycling if

$$b_1(t) > \frac{pc_1(t)\,\lambda + (1-p)c_2(t)\,\lambda - b_2(t)h_2(t)}{h_1(t)} \quad (9)$$

When $G_1(t)$ and $G_2(t)$ are both exponential, being α_1 and α_2 the respective means, (9) becomes

$$b_1(t) > \left(pc_1(t) + (1-p)c_2(t)\right)\rho_1 - \frac{\alpha_1 b_2(t)}{\alpha_2} \;; \rho_1 = \lambda\,\alpha_1 \quad (10)$$

It is economically interesting dismantling if

$$b_2(t) > \frac{pc_1(t)\,\lambda + (1-p)c_2(t)\,\lambda - b_1(t)h_1(t)}{h_2(t)} \quad (11)$$

Supposing that $G_1(t)$ and $G_2(t)$ are both exponential, being α_1 and α_2 the respective means, (11) becomes

$$b_2(t) > \left(pc_1(t) + (1-p)c_2(t)\right)\rho_2 - \frac{\alpha_2 b_1(t)}{\alpha_1}; \rho_2 = \lambda\,\alpha_2 \quad (12)$$

In a more realistic approach, not so extreme, consider the economic equilibrium over a period of time with length T, instead of the instantaneous one. That is, be

$$\int_0^T B(t)dt > \int_0^T C(t)dt \quad (13)$$

When $b_1(t)$ and $b_2(t)$ are both constant in $[0,T]$ with values b_1, b_2 respectively, it may be concluded that

- Recycling is economically interesting if

$$b_1 > \frac{\lambda\,[pC_1^T + (1-p)C_2^T] - b_2 \ln\dfrac{1-G_2(0)}{1-G_2(T)}}{\ln\dfrac{1-G_1(0)}{1-G_1(T)}} \qquad (14)$$

- Dismantling is economically interesting if

$$b_2 > \frac{\lambda\,[pC_1^T + (1-p)C_2^T] - b_1 \ln\dfrac{1-G_1(0)}{1-G_1(T)}}{\ln\dfrac{1-G_2(0)}{1-G_2(T)}} \qquad (15)$$

where

$$C_i^T = \int_0^T c_i(t)dt\,, i = 1,2 \quad (16)$$

If $G_1(t)$ and $G_2(t)$ are both exponential, being α_1 and α_2 the respective means, (14) becomes

$$b_1 > \frac{\rho_1[pC_1^T + (1-p)C_2^T] - b_2\dfrac{\alpha_1 T}{\alpha_2}}{T}\ ;\ \rho_1 = \lambda\,\alpha_1 \qquad (17)$$

and (15)

$$b_2 > \frac{\rho_2[pC_1^T + (1-p)C_2^T] - b_1\dfrac{\alpha_2 T}{\alpha_1}}{T}\ ;\ \rho_2 = \lambda\,\alpha_2 \qquad (18)$$

4. CONCLUSIONS

It is an evident conclusion that the most important parameters in the model presented are λ and $h(t)$. The first is a characteristic of the system. The second may be influenced (controlled) by engineers.

To apply this model, and get useful conclusions, it is important to check if customers' arrivals occur according to a Poisson process, see for instance [1]. In this kind of problem this is pacific, in general, due to the huge quantity of motorcars whose owners certainly will look for these services.

It results from this discussion that the greatest possibility for managing the system is to act on the service time hazard rate function, for instance, choosing, or even creating and improving, the dismantling and recycling processes to be used.

From the Cost-Benefit analysis it results that there are levels under which it is not economically interesting recycling or dismantling. In a broader perspective it is more efficient the activity that corresponds to a lower level for the minimum interesting benefit.

In [5] it is presented another example of infinite servers queues use, now to model repair systems.

REFERENCES

[1] F. P. Kelly. Reversibility and Stochastic Networks. John Wiley & Sons, New York (1979).

[2] J. H. Kunstler. O Fim do Petróleo- O Grande Desafio do Século XXI. Editorial Bizâncio, Lisboa (2006).

[3] L. Takács. An Introduction to Queueing Theory. Oxford University Press, New York (1962).

[4] M. A. M. Ferreira. Comportamento Transeunte e Período de Ocupação de Sistemas de Filas de Espera sem Espera. PhD Thesis discussed in ISCTE, Lisboa (1995).

[5] M. A. M. Ferreira. Modelling and differential costs evaluation of a two echelons repair system through infinite servers nodes queuing networks. Applied Mathematical Sciences 7(2013), 112, 5567-5576. DOI:10.12988/ams.2013.38478.

[6] M. A. M. Ferreira, J. A. Filipe, M. Coelho. A queue model-recycling and dismantling motor vehicles. International Journal of Latest Trends in Finance and Economic Sciences 1(2011), 3, 137-141.

[7] S. Ross. Stochastic Processes. John Wiley & Sons, New York (1983).