TIME VALUE OF MONEY AND BOND CHARACTARIZATION AND VALUATION

I – TIME VALUE OF MONEY

1. The concept

The notion of time value of money can be described in two different, but somewhat similar perspectives.

On the one hand it can be viewed as the purchasing power of money across time. Today with 10,000 euros, for instance I can buy some goods, but with same amount, five years from now, I will not be able to purchase the same goods. If, then, I will need 11,000 euros to buy the same goods, we can say that 10,000 euros today are equivalent of 11,000 in five years‘ time. This, notion, as expected, is basically related with the concept of inflation that leads to an increase of prices over time.

Another way of seeing the notion of time value of money is by seeing it as a way of increasing the total amount invested through the accumulation of interests (investing money today to earn interest), in order to get a higher value in the future (or, alternatively, borrowing today an amount that will allow me to buy something that I want and which I will repay in the future, needing then a higher amount captured by the interest of the loan). Assuming, for simplicity a similar rate for investing or borrowing, that rate will represent the driver of value of money across time.

2. Real interest rates

Putting together the two previous perspectives of seeing the time value of money, we can address the question of how value of money really changes. Let´s assume that a given good costs today 250 euros and that the inflation rate in the next year will be 10%. The good will cost 275 euros one year from now. Let´s now assume that I decide to invest those 250 euros for one year and earning an interest rate of 12%. One year from the investment I will receive the 250 plus 30 of interest, that is, 280. In this case I will be able to buy the same good one year from now (for 275) and keep a surplus (approximately 5). I can say that my wealth increased (I can buy more one year from now than today). This wealth increase was due to investing with a positive real interest rate, that is, an interest rate which is above the inflation rate.

In a simple formula:

$$r_r = \frac{1+r_n}{1+i} - 1$$

Being:

r_r – real interest rate

r_n – nominal interest rate (the rate that we get in the investment)

i – inflation rate

In our illustration

$$r_r = \frac{1+0.12}{1.10} - 1 = 0.0812\ (1.82\%)$$

This rate reflects the increase in our wealth. Often, for simplicity, the real rate is presented as subtraction between the nominal interest rate and the inflation rate, in our example 12-10 = 2. If the two rates are not very different, the simplification is acceptable.

Sometimes, we have in the market negative real rates, which incentivizes borrowing and consumption, instead of investing our money (as the investment will lead to a real wealth decrease).

3. Future values and the concept of compounding interest

Let´s assume that I am investing today 10,000 euros in a one year banking deposit, with an interest rate of 4%. One year from now I will receive the principal (10,000) plus the interest (400 or 10,000 x 0.04).

Let´s now assume that I reinvest the money (10,400, principal plus interest) for one year more. One year after, I will receive the new principal, 10,400 plus the interest (416 or 10,400 x 0.04). Therefore, after two years, I will have 10,816.

This amount could be easily computed as:

$V = 10{,}000 \times (1+0.04)^2 = 10{,}816$

The term $(1+0.04)^2$ represents the compounding factor that incorporates the increase in the invested amount, assuming the interest earned in each period will be added to the principal and reinvested in the next period.

More generally, the future value (FV) of any amount (A), taking into account a given interest rate (r) and a given time frame (n periods) can be computed as:

$FV = A \times (1+r)^n$

This formula allows us to transport a given amount of money from a previous moment to any future moment.

4. Present values

Let´s now suppose that we have an opposite problem. We are going to receive 10,000 euros in three years' time and we want to know what its value is today. We will use the same framework but now to identify the value today:

$10{,}000 = V \times (1+0.04)^3$

$V = 10{,}000/[(1+0.04)^3]= 8{,}890$

Therefore, 10,000 in three years' time represent 8,890 today. The basic notion is that if we invest today 8,890 for 3 years at an annual interest rate of 4%, we will have 10,000 euros three years from now (always assuming the compounding effect, that is, the interest earned in each period is reinvested).

More generally, the present value (PV) of any future amount (A), placed n periods in the future and considering a given interest rate (r) is:

$PV = A/[(1+r)^n]$

5. Annuities and perpetuities

Let´s now assume that we want to compute the PV of a stream of values with two common characteristics: same amount and same periodicity (for instance, receiving 100 euros every year for ten years).

Using the general formula presented before we can compute the PV:

$$PV = \frac{A}{(1+r)^1} + \frac{A}{(1+r)^2} + \frac{A}{(1+r)^3} + \cdots + \frac{A}{(1+r)^n}$$

Being A the constant amount and n the number of periods.

The computation of the PV of this stream of constant values can be simplified by applying the following formula:

$$PV = A \times \left(\frac{1}{r} - \frac{1}{r \times (1+r)^n}\right)$$

In the special case of having a perpetual (infinite) stream of A values:

$$PV = \frac{A}{r}$$

The PV of a long stream of values points out to a key feature that should be taken into account in many valuation exercises. Let´s assume a constant value of 100 every year and an interest rate of 10%. The PV of a twenty year annuity and of, instead, a perpetuity will be:

$$PV_{(20\ years)} = 100 \times \left(\frac{1}{0.1} - \frac{1}{0.1 \times (1.1)^{10}}\right) = 851$$

$$PV_{(\infty)} = \frac{100}{0.1} = 1{,}000$$

The first 20 years represents more than 85% of the value of the perpetual stream. If we increase (decrease) the level of interest rates, this effect of the first years will be less (more) pronounced (if we increase interest rates, the first years will contribute less to the perpetual value).

6. Nominal vs effective rates

Typically, interest rates of any given financial operation are presented on a nominal annual basis. However, the effective rate of the operation will depend on the periodicity of the payment of interest (usually up to one year).

Let´s assume a one year loan with a nominal annual rate of 6% and with interest being paid every quarter. The effective rate will be:

$$r_e = (1 + \frac{0.06}{4})^4 - 1 = 0.0614$$

The meaning of the effective rate (in this case of 6.14%) is the following:

- The proposal of a 6% annual rate **with quarterly interest payments** results, in the end, in an annual effective rate of 6.14%, due to compounding effect (interests on interests).

More generally:

$$r_e = \left(1 + \frac{r_n}{p}\right)^4 - 1$$

Being:

r_n - nominal annual rate of the operation

p – number of interest payments per year

II. BOND CHARACTERIZATION AND VALUATION

1. The instrument

The first and most important concept regarding a bond is that it is a debt instrument. The issuer of a bond (typically a Government or a corporation) is borrowing funds from the buyers of the bond, for a pre-established period (maturity of the bond) and a given interest rate.

2. Key elements of a plain vanilla bond

<u>Face value</u>

It is the value that the issuer has to repay to the bond holder at the maturity of the bond.

Issue value

The bond can be issued at its face value (issue at par) or above or below the face value (at premium or at discount, respectively).

The issue value represents the amount that the buyer will pay to acquire the bond. If the bond is issued at premium or discount is because it has some characteristic that justifies the payment of a value above or below the face value (the value that will be received at maturity) namely because it offers a higher or lower interest rate than similar bonds are currently offering.

Interest

Usually designated as the coupon[1] of the bond, interest can be paid with different periodicities (annually, quarterly, etc.). The interest rate can be a fixed rate (more common) or a floating rate (less usual), in this case being linked to a certain index (for instance LIBOR + 1%). The interests are computed against the face value of the bond (and not against the issue value).

Trading a bond

Usually, after being issued, the bonds are listed in an exchange, meaning that the initial buyer can sell them to other investors. The same bond can be held by several investors until its maturity. This characteristic offers an appealing level of liquidity to prospective investors.

Price of a bond

Initially, the price corresponds to the issue value (the face value, for instance, if the issue is at par). Until the maturity of the bond, its price can change due to several factors, such as the economic and financial profile of the issuer or the general

[1] It's called a "coupon" because traditionally the paper bonds included physical coupons that we could tear off and redeem for interest. Nowadays, bonds are dematerialized, being simply a record that is kept electronically.

evolution of the interest rates (if the bond has a fixed rate). For instance, if there is a deterioration of the economic condition of the issuer, it means that the risk of the bond increased and, therefore any prospective buyer will demand a higher return, obtained by buying the bond at a lower price, as the interest rate (coupon) is pre-established and so is the principal to be received at the maturity of the bond.

3. Pricing a bond

3.1. A simple approach to annual coupon plain vanilla bonds

Pricing (valuing) a bond is an exercise that can be carried out at any moment during its life. Looking at the exercise from an investment perspective, we can define the purchase of a bond as the purchase of a future stream of cash flows: the interest that will be periodically received from the issuer and the face value at the maturity of the bond. Therefore, the fair price of a bond should be the PV of all future cash flows that that the bond will produce. In order to compute the PV we need a discount rate.

Let´s assume that we are analyzing a bond with a face value of 100 an annual coupon of 5% and a maturity of 3 years. In addition let´s also assume that currently in the market there are similar bonds (in terms of risk and maturity) that offer a return of 6%. Rationally, any investor will only be willing to purchase our bond if the investment will provide a similar return of 6% (otherwise he/she would prefer to invest the funds in similar bonds offering that return). Having an alternative offering a 6% annual return this should be the appropriate discount rate.

$$Price = \frac{5}{(1+0.06)^1} + \frac{5}{(1+0.06)^2} + \frac{5}{(1+0.06)^3} + \frac{100}{(1+0.06)^3} = 97.33$$

Therefore the fair price of the bond should be 97.33. Although the investor receives a coupon of only 5% (compared to the required return of 6%), he will be compensated by buying the bond at 97.33 and receiving 100 of principal (the face value) at maturity. This difference of 2.67 will offset the loss of a lower 1% coupon (if we compute the PV of 1% coupon in years 1,2 and 3, we will get exactly 2.67).

More generally:

$$P = \frac{coupon}{(1+r)^1} + \frac{coupon}{(1+r)^2} + \cdots + \frac{coupon}{(1+r)^n} + \frac{face\ value}{(1+r)^n}$$

Being r the return offered by similar bonds and n the maturity of the bond.

As we can see in the general formula there is an inverse relationship between the price of the bond and the discount rate (return offered by similar bonds). If the discount rate increases the price of the bond will decrease and vice-versa. This relationship is intuitive. If the discount rate increases for instance, it means that we can get a higher return, bearing the same risk, elsewhere. Therefore, prospective buyers will only buy this bond if it offers a similar increase in its return. As the coupon and the face value are fixed, this extra return will only be generated through a lower price (vs. the face value that will be received at maturity). This price sensitivity to changes in the discount rate is related to the length of the life of the bond. Other things equal, the longest the maturity of the bond, the more sensitive is its price to changes in the discount rate.[2] A practical consequence of this relationship between the price and the discount rate is that important changes in the market price of the bond can happen throughout its life (due to a general variation of interest rates in the economy or to a change in the risk profile of the company placing it in a different risk category). This variation of price is less important if the bondholder intends to carry his/her investment until the maturity of the bond (because, then, the face value will be received[3]) assuming that those price changes do not reflect a serious deterioration of the company's ability to pay interests and, ultimately, principal. But in terms of an active management of the

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[2] A good proxy of this sensitivity is given by the duration of the bond. In annex this concept is presented.

[3] Holding the bond until maturity and ignoring the reinvestment risk of the coupons will generate a return similar to the yield to maturity (ytm) computed when the bond was acquired.

investment (meaning that the bonds will be eventually sold before maturity), it can represent a relevant risk.

3.2. Yield-to-maturity (YTM) of a bond

Returning to the previous example, we saw that by buying the bond for a price of 97.33 we will get a return of 6%. We can get this same return be equalizing the price to the future stream of cash flows and computing the discount rate that equals the price and the bond's cash flows. This rate is called the yield to maturity (YTM) of the bond:

$$97.33 = \frac{5}{(1+ytm)^1} + \frac{5}{(1+ytm)^2} + \frac{5}{(1+ytm)^3} + \frac{100}{(1+ytm)^3}$$

ytm = 6%

This measure allows us, from observing the market price of any bond and identifying its future cash flows until maturity, to compute the average annual return[4] that the bond will offer (if we buy it at its current market price), assuming that we will hold the bond until maturity.

3.3. Bonds with other type of periodic coupons

As mentioned earlier, there are different possibilities regarding the periodicity of the coupon, from annual to monthly coupons. The valuation principles employed still hold but we need to adjust them to the specific periodicity of the bond.

Let´s assume a bond with a face value of 100, an interest rate of 4% with interest being paid quarterly, and a maturity of 2 years. Let´s also consider that similar bonds offer a 3.25% annual return.

First of all is important to notice that the interest rate of any given bond is always presented on a nominal and annual basis and **the computation of interest**

[4] It is an average return as it has two components: the periodic coupon and the difference between the price and the face value.

always follows a simple proportional rule. In this case, the quarterly interest will be:

Interest = 100 x 0.04/4 = 1

Computing the price of the bond:

$$P = \frac{1}{(1+0.0325)^{1/4}} + \frac{1}{(1+0.0325)^{2/4}} + \cdots + \frac{1}{(1+0.0325)^{\frac{8}{4}}} + \frac{100}{(1+0.0325)^{2}} = 101.5$$

If instead, we had a given market price and we wanted to compute the ytm of the bond, we would use as discount factor $(1+\text{ytm})^{n/8}$, being n the quarterly interest payments.

3.4. Zero coupon bonds

A zero coupon bond, as the designation implies, doesn´t offer any explicit interest. The return is obtained through a discount price over the face value. Consider, for instance a zero coupon bond with a face value of 100 and a maturity of 4 years. If similar bonds offer an annual return of 5%, its fair price should be:

$$P = \frac{100}{(1+0.05)^{4}} = 82.27$$

Conversely, having a giving market price, for instance, 85, its ytm will be:

$$85 = \frac{100}{(1+ytm)^{4}} =$$

ytm = 4.15%

One of the advantages of the zero coupon bonds, in contrast with the plain vanilla bonds, is that they haven´t any reinvestment risk. If the investor wants to fully use the time horizon until the maturity of the bond, he/she will not be worried with the reinvestment, during this period, of any coupon, while if it was a regular bond, he/she would have to reinvest every coupon. In fact, when we compute the ytm of a plain vanilla bond and we say that the ytm represents the average annual return of the investment, we are implicitly assuming that every coupon will be reinvested at the ytm until the maturity of the bond.

4. Other types of bonds

Besides the plain vanilla and zero coupon bonds already described in this text, there are many other types of bonds with some distinctive features. Below, is presented a brief description of some more well-known cases.

Callable bonds

These bonds have an embedded option conceded to the holder or the issuer (or both) to demand an early termination of the bond. From an issuer's perspective it makes sense to trigger such a clause if there are available cheaper sources of funds or if there are enough cash that doesn´t justify the holding of that ongoing debt. From the investor's perspective, the early termination will be appropriate if he/she finds a similar investment offering a higher return or if he/she anticipates that the risk profile of the bond will increase. Typically, the early redemption has an associated penalty is defined (timing, penalties, etc.) when the bonds are issued.

Convertible bonds

These bonds have as main feature the possibility of being redeemed not with cash but with shares of the company, according to a pre-established conversion ratio (for instance a 2/1 conversion ratio in a 10 face value bond, means that two bonds can be exchanged by one share, that is, the holder of the bond will buy shares at a price of 20 each by exercising the conversion option). The conversion can be exercised at the maturity of the bond (more common) or in specific pre-established dates before maturity (eventually having different conversion ratios for different conversion dates). The conversion is always an option of the bondholder. If he/she decides not to convert he/she will receive the face value of the bond at maturity. Typically, convertible bonds offer a lower return than plain vanilla bonds of the same risk class. The difference of returns can be viewed as premium paid by the investor to have the option, later, of converting the bonds into shares and getting a gain in that conversion (in the previous example if the shares of the company were traded at 25, the investor would gain 5 in the conversion). From the perspective of the issuer, it will have a lower cost of funds and an eventual equity increase (in the case of conversion), although that equity increase may be carried out at an unfair (lower) price for the other shareholders (dilution effect).

Perpetual bonds

As the name implies these bonds have an infinite maturity. Typically, they are issued by Governments (as the life of a corporation tends always to be limited).

5. Bond risk: rating

5.1. Concept

Rating represents an assessment regarding the capacity of the debtor to timely repay the interest and principal of the outstanding debt (typically bonds) that is being assessed. It is important to notice that this assessment, in the case of corporations, doesn't attempt to evaluate profitability or performance. Naturally, profits and returns are key variables in the analysis of the company conducted by the rating agencies. However, it is perfectly plausible that a given company has a better rating than another company although it may have a lower profitability but, instead, it has a more stable pattern of margins and profits or a more solid capital structure (less debt). Once again, rating only measures the capacity of the company to repay its debt (and associated interests).

In spite of using a complex and sophisticated framework to assess companies, rating became a popular tool mainly because of the simplicity of its final result (a grade/score), that everyone can easily read and understand. Rating, in practice, replaced, in an efficient and costless way, the research that each investor should undertake before buying a bond (in order to identify the risk of the bond issuer).

5.2. Rating agencies

In 1909 John Moody, owner of a small company dedicated to the analysis of railway companies, Moody's, decided to assign grades to the bonds issued by those railway companies, leading the way to the development of a new activity within the financial information sector. During several decades this rating activity was basically confined to the US, which had a large bond market. Following the globalization of the world economy in the 90’s, rating became a tool used worldwide not only applied to companies but to countries as well, since these were becoming active players in the global bond market.

Currently, the global rating industry is dominated by three companies: Moody’s and Standard and Poor’s (S&P), fighting for the number one position, and Fitch. There are other rating agencies, but smaller and mainly working locally.

Generally, the assignment of a rating grade is a service paid by the issuers (companies or States) who want to have a grade. This grade is publicly announced and therefore everyone has freely access to the grade. However, the access to a more detailed report, supporting the grade, is a paid service.

The rating agencies have the obligation to closely monitor the situation of each issuer whose bonds were graded, and they can, in any moment, modify (upgrade or downgrade) the rating if they conclude that the financial risk profile of the issuer has changed. These sudden modifications have a huge impact in the price of the bonds. For instance, if the rating decreases, investors will demand a higher rate of return to buy those bonds, reducing their prices (the price of an outstanding bond is the present value of the interest and principal payments that will occur until the maturity of the bond).

5.3. Rating grades

There are different rating grade scales for long term debt and for short term debt (with a maturity up to one year). The next table provides a summary of the scale used by Fitch, S&P and Moody’s for long term debt.

Table: Rating Agencies scales

S&P e FITCH	MOODY'S
Tier Investment Grade (Good probability of meeting debt obligations)	
AAA	Aaa
AA	Aa
A	A
BBB	Baa
Tier Non – Investment Grade (Junk)	
BB	Ba
B	B
CCC	Caa
CC	Ca
C (1)	C
D (1)	

(1) S&P has, below C a grade of CI (interest payment failure) and within D, has the grades of DDD, DD and D.

These grades can have two additional features:

<u>Sub grades</u>

Each grade (except the first, AAA/Aaa, and the scales below B) is divided in three sub grades:

S&P and Fitch: each grade can have a +, nothing, or - (for instance, AA^+, AA and AA^-)

Moodys: each grade can have 1, 2 or 3 (for instance Aa1, Aa2 and Aa3)

The variation from one sub grade to the next sub grade (for instance from A to A^-), that is, the minimum variation, is usually called one notch.

<u>Perspectives</u>

Associated with the grade there is also an indication of how the rating agency views the future evolution of the grade, usually using the mention *"positive outlook"*, “*negative outlook*” or “*stable outlook*”, indicating that on a 6 to 12 months period there is a strong possibility of reviewing the grade upwards, downwards or maintaining it unchanged. It is important to notice that any rating upgrade or downgrade can be done at any time. It can also be changed in several notches (from A^+ to BBB^-, for instance), and not just by one notch.

There is also another feature, a negative one, called "credit watch", indicating the sudden appearance of a fact or an eventual upcoming event, that may cause the downgrade of the rating in the short term (up to three months).

Regarding the grades for short term debt, agencies use a smaller scale. S&P has the grades of A-1, A-2 and A-3 and Moody’s P-1, P-2 and P-3 in the investment grade tier. S&P has B, C and D in the junk tier while Moody’s has only the grade NP for all the remaining cases.

5.4. Assessment criteria

Rating agencies make available, at no cost, the methodologies, criteria and metrics employed to produce the rating grades. Essentially, companies are analyzed regarding two main areas: business risk and financial risk. Regarding business risk, the key areas of analysis are the economic status of the home country, the industry risk profile, the competitive position in the market, operational efficiency, profitability (trend and comparison with main competitors). Subjects like the corporate control and the quality of corporate governance are also relevant. In terms of financial risk, topics such as cash flow adequacy versus the company’s financial needs, the capital structure equilibrium, the liquidity of assets or the availability of unused credit lines are taken into serious consideration. Specific criteria taking into account the nature and characteristics of the company´s industry will also be used.

One interesting feature is the definition of benchmarks for several (financial) ratios, associated with each grade. By other words, the company will know what values in those ratios should be met in order to maintain (or to improve) a given grade.

5.5. Company´s rating vs. Home country´s rating

In the majority of the cases the rating of a company is conditioned by the rating of the home country. This relationship is based on fact that the public debt of a given country is, typically, the safest investment in that country and the debt of the local companies will tend to offer a higher risk. However, there are cases in which this rule is not followed.

The most common case of companies having a better rating than the home country is when the local activities represent a small fraction of the global activity, that is, companies with an extensive international exposure. If these international activities are well diversified (or in spite of being in just a couple of countries, those countries have a better rating than the home country), it is plausible that the company will have a better rating than the home country. Usually, though, rating agencies establish a cap allowing a maximum of 3 notches above the country´s rating.

ANNEX I

DURATION OF A BOND AND MEASUREMENT OF PRICE SENSITIVITY

The duration of a bond is a weighted average of all discounted cash flows produced during its life against the bond price.

Let´s use the following example:

Face value: 100

Coupon: 6%, annually paid

Maturity: 4 years

Let´s now assume that similar risk bonds offered currently a return of 8%. First of all we need to compute the price of the bond:

$$P = \frac{6}{(1+0.08)^1} + \frac{6}{(1+0.08)^2} + \frac{6}{(1+0.08)^3} + \frac{6}{(1+0.08)^4} + \frac{100}{(1+0.08)^4} = 93.37$$

To compute the duration of the bond we need to weight each discounted cash flow against the price:

Year 1 $\frac{6/(1.08)^1}{93.37}$ = 0.0595

Year 2 $\frac{6/(1.08)^2}{93.37}$ = 0.0551

Year 3 $\frac{6/(1.08)^3}{93.37}$ = 0.0510

Year 4 $\frac{106/(1.08)^4}{93.37}$ = 0.8344

The sum will be 1.

Now, the duration (D) is nothing more than the average life of the bond using those weights:

Duration = 1 x 0.0595 + 2 x 0.0551 + 3 x 0.051 + 4 x 0.8344 =3.66 years.

Another measurement is what is called the modified duration (MD):

MD = D/1+r

In our illustration:

MD = 3.66/1.08 = 3.39

The MD can be used as a sensitivity measure of the impact of a change in the discount rate in the price of the bond. It is not 100% accurate, but it is a good and simple proxy.

Change in Price (ΔP) = ΔRate x MD

To end our illustration, let´s assume that the discount rate increases from the current 8% to 8.5%.

Computing the new price:

$$P = \frac{6}{(1+0.085)^1} + \frac{6}{(1+0.085)^2} + \frac{6}{(1+0.085)^3} + \frac{6}{(1+0.085)^4} + \frac{100}{(1+0.085)^4} = 91.81$$

The change in the price was:

ΔP = (91.81 – 93.37)/93.37 = -0.0167 (1.67%)

Using instead the concept of MD:

ΔP = 0.5% x 3.39 = 1.69%

The results are very similar but not identical and the divergence tends to increase if we have a larger change in the discount rate. In any case, the MD is a useful tool to quickly analyze the price sensitivity of a bond against the change in the discount rate.

The duration of a bond, as the previous example showed, is quite different from its maturity and the divergence tends to increase with the maturity of the bond, as the last and important cash flow, the repayment of the face value, will have a lesser weight in the price (induced by the geometrical growth of the discount factor). The single exception is a zero coupon bond, whose duration equals the maturity, as this bond has only one future cash flow, at its maturity.

ANNEX II

BOND VALUATION, ZERO COUPON RATES AND YIELD CURVES

When we analyzed the valuation of a bond, we used as the discount rate the return offered by a similar risk bond. This return is the ytm of that similar bond. We stated in the text that the use of ytm has an associated assumption: the coupons produced by the bond will be reinvested at the ytm. This assumption has two holes. One is that through the life of a bond, most likely the interest rates will change. The other is that the return offered for different maturities will also likely be different (consider a bond with a maturity of 5 years; the reinvestment of the coupons for the maturity of the bond will represent investments with a maturity of 4, 3, 2 and 1 years).

It would be a different matter if we were using zero coupon bonds in which the reinvestment issue doesn’t exist. However, using a ytm of a zero coupon bond as the discount rate of a plain vanilla bond creates another problem. Let´s assume that we were pricing a plain vanilla bond with a face value of 100, an interest rate of 6% and a maturity of 3 years. It wouldn´t be appropriate to use a 3 year zero coupon bond ytm as the discount rate, because the plain vanilla bond will generate cash flows in year 1, 2 and 3 (coupons) and all would be discounted by a 3 year interest rate. A way of overcoming this problem is to see the plain vanilla bond in a different way. Using the example we can see the bond as an investment portfolio that generates a cash flow of 6 in year 1, 6 in year 2 and 106 in year 3. If we have zero coupon bonds of the same risk class with maturities of 1, 2 and 3 years, we will use a specific rate to discount each cash flow of the plain vanilla bond. Let´s assume that zero coupon bonds offer a return of 4%, 4.5% and 5% for the maturities of 1 to 3 years. The price should be:

$$P = \frac{6}{(1+0.04)^1} + \frac{6}{(1+0.045)^2} + \frac{6}{(1+0.05)^3} + \frac{100}{(1+0.05)^3} = 102.83$$

More generally:

$$P = \frac{coupon}{(1+r_{(0,1)})^1} + \frac{coupon}{(1+r_{(0,2)})^2} + \cdots + \frac{coupon}{(1+r_{(0,n)})^n} + \frac{face\ value}{(1+r_{(0,n)})^n}$$

Being:

$r_{(0,n)}$ - Zero coupon rate for a maturity of n

Sometimes, we don´t have available in the market zero coupon bonds for the all the maturities that we need. But, provided that there is available a zero coupon bond for one period, we can, using plain vanilla bonds, compute the remaining zero coupon bonds. To illustrate, let´s assume a plain vanilla bond with a coupon of 4%, face value of 100 and a maturity of 2 years. Let´s also assume that its current market price is 97.5 and that the one year zero coupon rate is 5% Assuming that this price represents a fair price, we can compute what is the implicit two years zero coupon rate associated it, considering that the one year zero coupon rate is 5%:

$$97.5 = \frac{4}{(1+0.05)^1} + \frac{104}{(1+r_{(0,2)})^2}$$

$r_{(0,2)}$ =5.36%

Having now the two year zero coupon rate and if we had a three year plain vanilla we could compute the three year zero coupon rate and so on.

These zero coupon rates are usually called the yield curve and if we are dealing with Government bonds it is the reference yield curve of the economy[5].

Still using the previous example let´s compute the ytm of our two years plain vanilla bond:

$$97.5 = \frac{4}{(1+ytm)^1} + \frac{104}{(1+ytm)^2}$$

ytm =5.355%

The ytm tends to be very close to the longer zero coupon rate employed (because the face value has a dominant percentage of the price, especially if the bond

[5] It is the closest proxy of a risk free yield curve.

doesn´t have a very long maturity). In practice, if we want to price a bond using the zero coupon yield curve to discount each cash flow or using, instead, the ytm of a bond with a similar maturity and risk, it doesn´t produce a huge error in the valuation of the bond. However, this error tends to increase for longer maturities and/or very steep zero coupon yield curves.