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PROCEEDINGS

# INFINITE SERVERS QUEUE SYSTEMS COMPUTATIONAL SIMULATION

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**Abstract.** For many parameters of interest in the study or application of queuing systems either there are not theoretical results or, existing, they are very complicated what makes their utility doubtful. In this situation it is very common to use simulation methods in order to get useful results. Here a Fortran program to simulate infinite servers queuing systems is presented. Through it, some experiences are performed and the respective results are used to conjecture about some interesting quantities of those systems, mainly about the busy period.

**Keywords:** Infinite servers queue systems, simulation, busy period

**Mathematics Subject Classification:** 60G99

## 1 Introduction

The absence of theoretical results, often difficult to obtain, or its extreme complexity that makes its utility doubtful, leads to the search for numerical methods, in particular simulation methods, in the study of queue systems.

The objective in this work is to do so for queue systems with infinite servers. The most known and studied of these systems is the $M / G / \infty$ queue. For it a lot of results are known and result very clear and simple to use, see for example (Carrillo, 1981), (Ferreira, 2002) and (Ferreira and Andrade, 2009, 2010, 2010a, 2010b, 2011, 2012).

One of the situations simulated in this work is related to the consideration of non-Poisson arrivals that is with inter-arrival times not exponential for which there are no analytical results.

Other results collected are about the busy period[1], for Poisson and non-Poisson arrivals. In fact any queue system has a sequence of busy periods and idle periods. A busy period followed by an idle period is a busy cycle. In the study of infinite servers systems busy periods very useful information is, for instance, about the maximum number of customers served simultaneously in a busy period. If something is known about this quantity the system may be dimensioned as a finite servers queue. As there are not analytical results the simulation is one of the issues to study it.

[1] A busy period begins with the arrival of a customer to the system, being it empty, ends when a customer abandons the system, letting it empty, and there is always at least one customer in the system.

In the next section, details about the FORTRAN program built to perform the simulations and the experiences performed are presented. The following section consists in the presentation of the results and the respective comments. The paper ends with a conclusions section.

## 2 The Computational Simulation

The simulations were performed using a Fortran program, see Appendix, composed of:

*i)* A main program, in Fortran language, called FILAESP,

*ii)* A subroutine GERASER,

*iii)* A package SSPLIB

*iv)* A function system RAN.

The proceedings are as follows:

*i)* The sequential random generation of 25 000 arrivals instants, being the inter- arrivals mean time $\lambda^{-1} = 0.99600$,

*ii)* The generation of 25 000 service times that are added to the arrivals instants so obtaining the departure instants,

*iii)* The ordination of the arrivals and departure instants, through an ordination algorithm making to correspond to each arrival 1 and to each departure -1,

*iv)* The generation, in fact, of the queue summing by order those values 1 and -1, in correspondence with the instants at which they occur,

*v)* The processing of the information of *iv)* in order to obtain

*a)* Data related to the state of the system[2]:

- Number of the visits to the assumed states,
- Mean sojourn time in each one of those states.

*b)* Data related to the busy period:

- The maximum number of customers served simultaneously in a busy period,
- Total number of customers served in the busy period,
- Length of the busy period.

The arrivals instant generation is performed in the program FILAESP and the service times in the GERASER subroutine. The arrivals and departures instants ordination is performed in the program FILAESP through the SSPLIB package. The construction of the queue and the processing of the information occur also in FILAESP.

In the generation of the arrivals and the departures are used sequences of pseudo-random numbers supplied by the system function RAN. In general it is made RAN(E*J), being E constant in each experience and assuming J the values from 1 to 25 000. E was chosen to be an integer with four digits.

[2] The state of the system, in a given instant, is the number of customers that are being served in that instant.

To the arrivals process one or two sequences of pseudo-random numbers are needed as considering M, exponential inter-arrival times, or E2, Erlang with parameter 2 inter-arrival times. In the first case it must be made an option for an integer with four digits, E, and in the second for two integers with four digits that will be designated by E and by F. The same happens with the service distribution, considering so G or G and H, as working with M or E2.

The experiences performed are described below, being $\mu^{-1}$ the mean service time and $\rho = \lambda\mu^{-1}$ the traffic intensity.

- $\boldsymbol{M/M/\infty}$
  E = 7 528
  F = 7 548
  $\mu^{-1} = 4$
  $\rho = 4.016$
  *Number of observed busy periods*: 208

- $\boldsymbol{M/M/\infty}$
  E = 7 529
  F = 7 549
  $\mu^{-1} = 5$
  $\rho = 5.020$
  *Number of observed busy periods*: 28

- $\boldsymbol{M/E_2/\infty}$
  E = 7 528
  G = 7 552
  H = 6 666
  $\mu^{-1} = 4$
  $\rho = 4.016$
  *Number of observed busy periods*: 337

- $\boldsymbol{M/E_2/\infty}$
  E = 7 529
  G = 6 552
  H = 6 667
  $\mu^{-1} = 5$
  $\rho = 5.020$
  *Number of observed busy periods*: 69

- $\boldsymbol{E_2/E_2/\infty}$
  E = 4 536
  F = 4 537
  G = 5 224
  H = 6 225
  $\mu^{-1} = 4$
  $\rho = 4.016$
  *Number of observed busy periods*: 804

- ***E₂ / E₂ / ∞***
  E = 4 538
  F = 4 539
  G = 5 228
  H = 6 229
  $\mu^{-1} = 5$
  $\rho = 5.020$
  *Number of observed busy periods*: 208

The mean service times considered, 4 and 5, were those for which a reasonable number of busy periods was obtained, among the highest. In fact, increasing the mean service time the observed busy periods decrease very quickly.

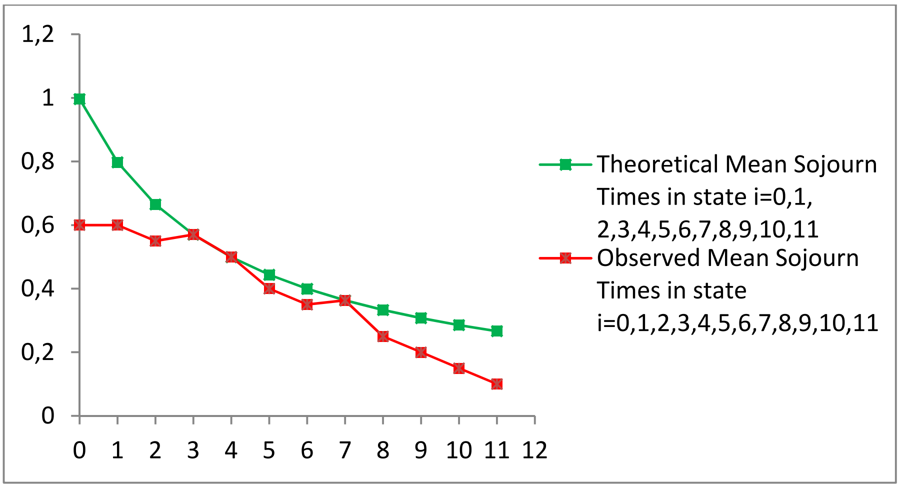
Note that for the systems $M / M / \infty$ and $M / E_2 / \infty$, for the same values of $\rho$ the arrivals instants generated are identical.

## 3 The Results – Presentation and Comments

In Figure 1 and Figure 2 the graphics that represent the mean sojourn times in the various states[3], for the $M / M / \infty$ system, considering ρ = 4.016 and ρ = 5.020, respectively, are presented. Beyond the observed mean values the theoretical values are also presented, see (Ramalhoto, 1983), given by

$$T_{M_i} = \frac{\mu^{-1}}{i+\rho}, \ i = 0, 1, 2, \ldots \qquad (1)$$

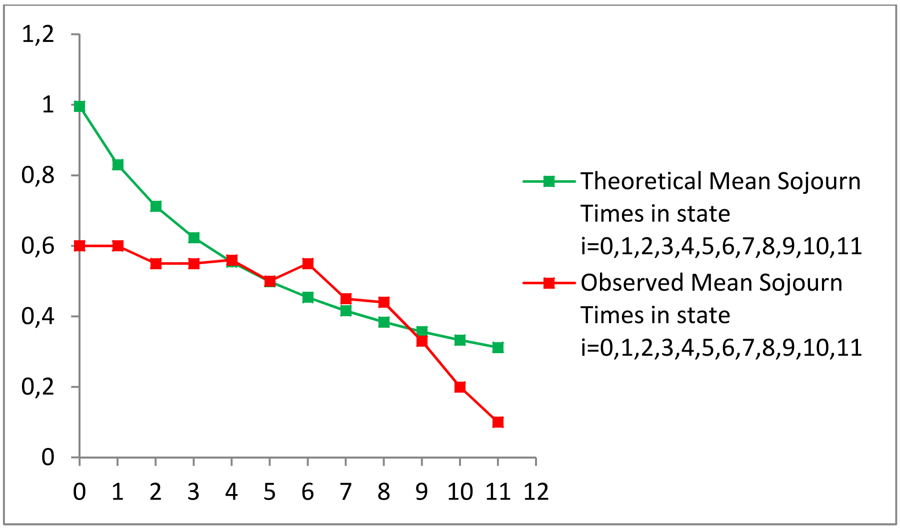
In correspondence with the various states are also indicated the number of times that they were visited, in the right columns.




| state | visits |
|---|---|
| 0 | 247 |
| 1 | 2018 |
| 2 | 5970 |
| 3 | 9624 |
| 4 | 10339 |
| 5 | 8809 |
| 6 | 6106 |
| 7 | 3581 |
| 8 | 2132 |
| 9 | 961 |
| 10 | 205 |
| 11 | 7 |

Figure 1. Mean Sojourn Times, in seconds, theoretical and observed, for the M / M / ∞ queue system in states i = 0, 1, …, 11, with ρ = 4.016.

[3] The state of the system in a given instant is the number of costumers that are being served in that instant.




| state | visits |
|---|---|
| 0 | 27 |
| 1 | 441 |
| 2 | 2190 |
| 3 | 5700 |
| 4 | 9267 |
| 5 | 10517 |
| 6 | 9042 |
| 7 | 6207 |
| 8 | 3742 |
| 9 | 2043 |
| 10 | 731 |
| 11 | 92 |

Figure 2. Mean Sojourn Times, in seconds, theoretical and observed, for the M / M / ∞ queue system in states i = 0, 1, …, 11, with ρ = 5.020.

In Figure 3 and Figure 4 are shown the distributions obtained for the number of customers in the systems M / M / ∞, M / $E_2$ / ∞ and $E_2$ / $E_2$ / ∞ with ρ = 4.016 and ρ = 5.020, respectively. Together is also presented the theoretical distribution, in equilibrium, for the systems M / M / ∞ and M / $E_2$ / ∞, see (Tackács, 1962):

$$p_n = e^{-\rho}\frac{\rho^n}{n!},\quad n = 0, 1, 2, \ldots \tag{2}$$

Performing the direct computations for ρ = 4.016 and ρ = 5.020. E[N] is the mean number of customers in the system.

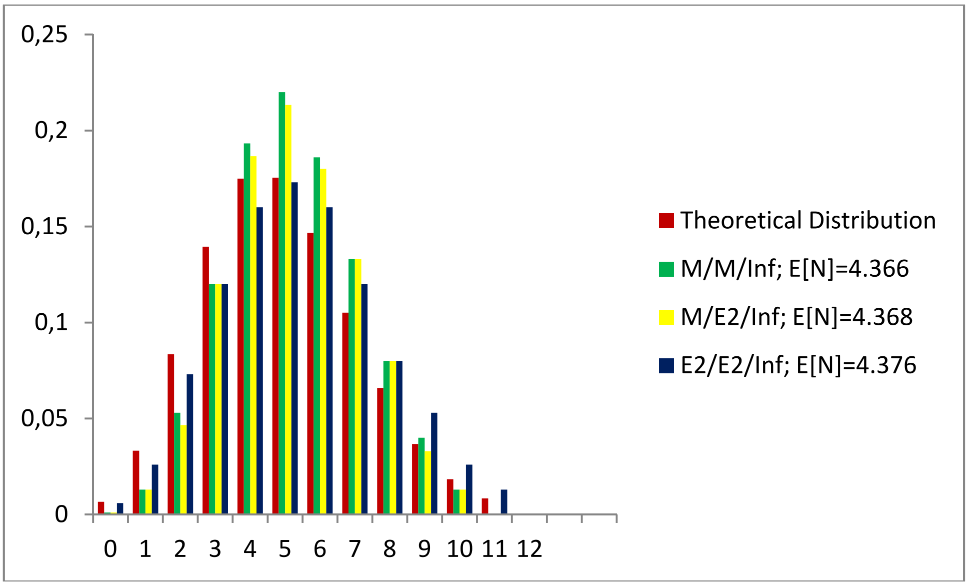



Figure 3. Distribution of the Number of Customers in the System and Theoretical Distribution for the Systems M / M / ∞, M / E2 / ∞ and E2 / E2 / ∞ with ρ = 4.016.

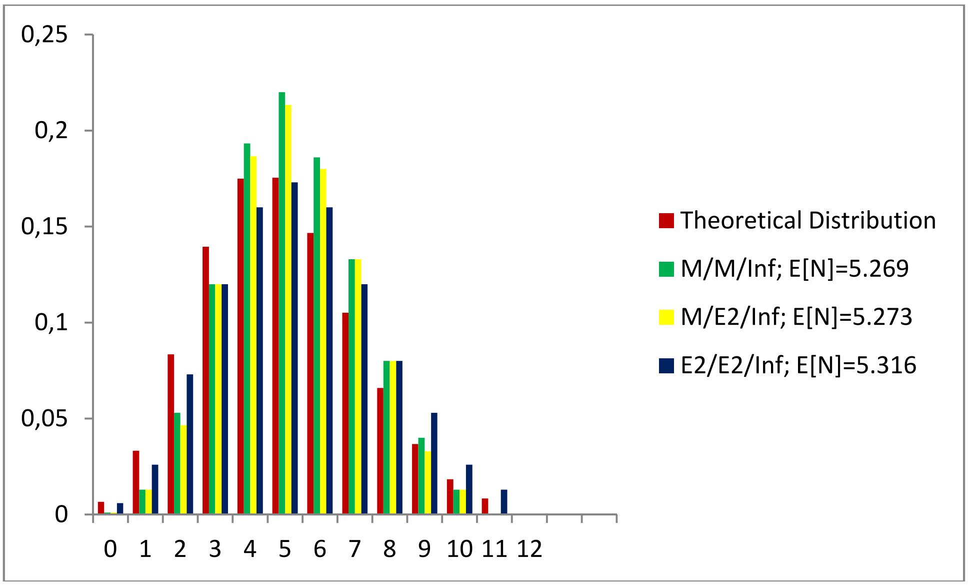



Figure 4. Distribution of the Number of Customers in the System and Theoretical Distribution for the Systems M / M / ∞, M / E2 / ∞ and E2 / E2 / ∞ with ρ = 5.020.

These Figures suggest some similarity of behaviour among the empirical distributions and the theoretical distribution. Particularly, in the whole of them the mode is identical to the one of the theoretical distribution. But although for the systems M / M / ∞ and M / $E_2$ / ∞ the empirical distributions are more concentrated around the mode, in comparison with the theoretical distribution, the opposite happens to the system $E_2$ / $E_2$ / ∞. And, surprisingly because for this system it is not known the theoretical distribution, the empirical distributions obtained for $E_2$ / $E_2$ / ∞ seems closer to the theoretical distribution than the ones of the other systems.

As for the differences observed between the systems M / M / ∞ and M / $E_2$ / ∞, for the number of the customers in the system, the adequate interpretation may be as follows: although the systems reach certainly the equilibrium, since the number of the simulated arrivals is quite large there is a strong presence of an initial transitory trend that must last a long time. Note, observing Figure 1 and Figure 2, that the mean sojourn times observed and theoretical, given by (1), for the system M / M / ∞ are quite close. This closeness is better for the states to which corresponds greater frequency.

In Figure 5 and Figure 6, about the maximum number of customers served simultaneously in the busy period, it is remarked great diversity in the distributions form. It is always observed a great frequency for the state 1. In the $E_2$ / $E_2$ / ∞ infinite systems it is always the mode. Curiously, these systems being able to serve any number of customers, present, in these simulations, few customers being served simultaneously: never above the number 14, only assumed by the $E_2$ / $E_2$ / ∞ infinite systems. This fact is acceptable, in terms of the theoretical distribution, since in the Poisson distribution the values greater than the mode, far away from it, are little probable. Note still that, excluding from this analysis the state 1, the distributions of maximum number of customers served simultaneously in the $E_2$ / $E_2$ / ∞ infinite systems busy period are more scattered than those of the others. This is in accordance with the fact that, for the same number of arrivals, much more busy periods are observed for the $E_2$ / $E_2$ / ∞ infinite systems.

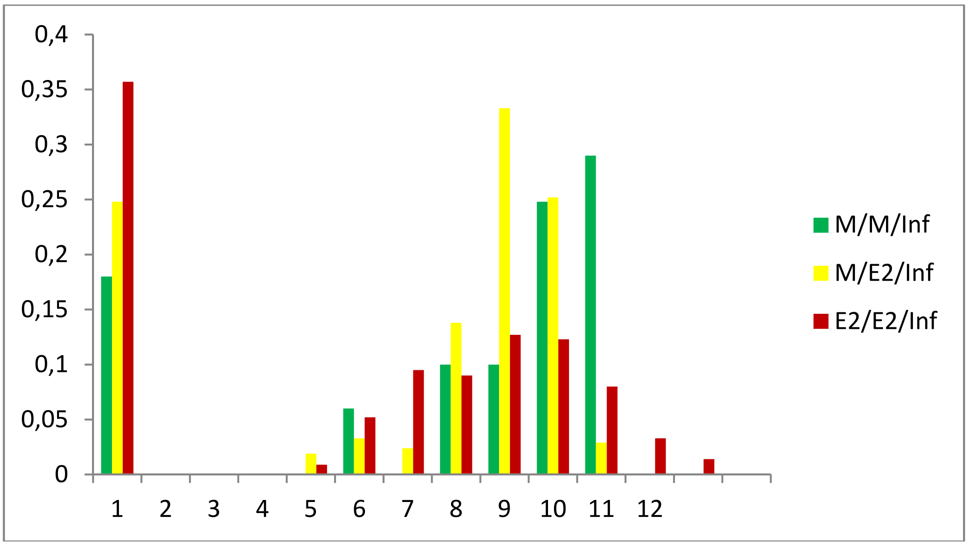



Figure 5. Distribution of the Maximum Number of Customers Served Simultaneously in a Busy Period for the Systems M / M / ∞, M / $E_2$ / ∞ and $E_2$ / $E_2$ / ∞ with ρ = 4.016.

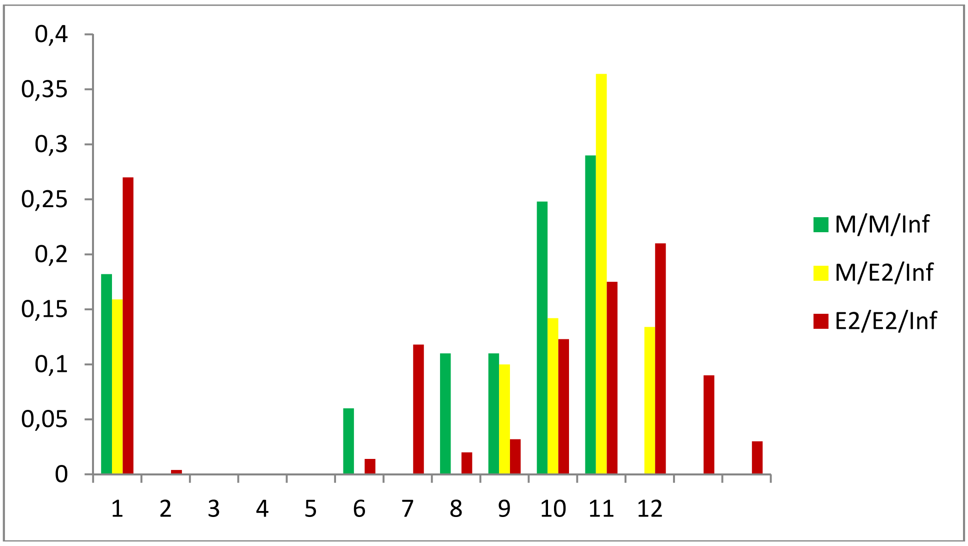



Figure 6. Distribution of the Maximum Number of Customers Served Simultaneously in a Busy Period for the Systems M / M / ∞, M / E2 / ∞ and E2 / E2 / ∞ with ρ = 5.020.

Figure 7 suggests a more and more smooth behaviour of the busy period lengths distribution frequency curve when going from M / M / ∞ system for the M / $E_2$ / ∞ system and then for the $E_2$ / $E_2$ / ∞ system. The whole of them present a great frequency concentration for the lowest values of the busy period lengths but, in the case of M / M / ∞ system, the interval along which the observations spread has more than the double of the length of the other systems. Remember, again, that for the M / M / ∞ system much less busy periods are observed than for the M / $E_2$ / ∞ system and, for this one less than for the $E_2$ / $E_2$ / ∞ system. Then the curve of frequencies for the M / M / ∞ system spreads along that interval with two deep valleys. The one of the M / $E_2$ / ∞ system presents also two valleys, but less deep, and in $E_2$ / $E_2$ / ∞ system practically they are not observed. Note, also that those valleys occur for different values in the M / M / ∞ and M / $E_2$ / ∞ systems.

**ρ = 4.016**

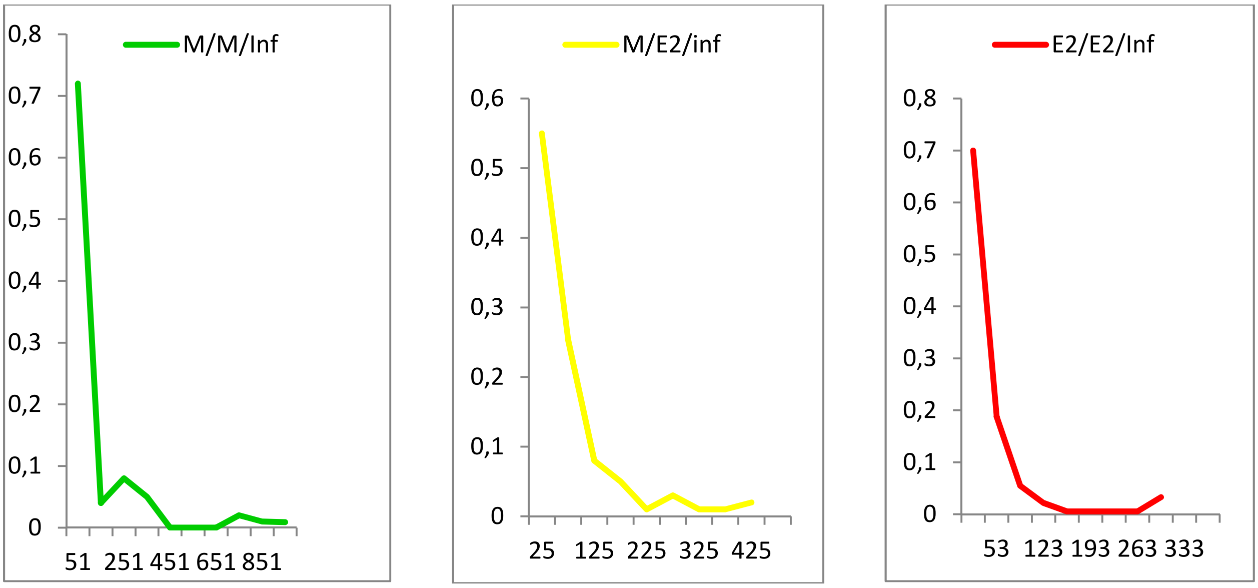



Figure 7. Distribution of Observed Busy Period Lengths, in seconds, for the Systems M / M / ∞, M / $E_2$ / ∞ and $E_2$ / $E_2$ / ∞ with ρ = 4.016.

**ρ = 5.020**

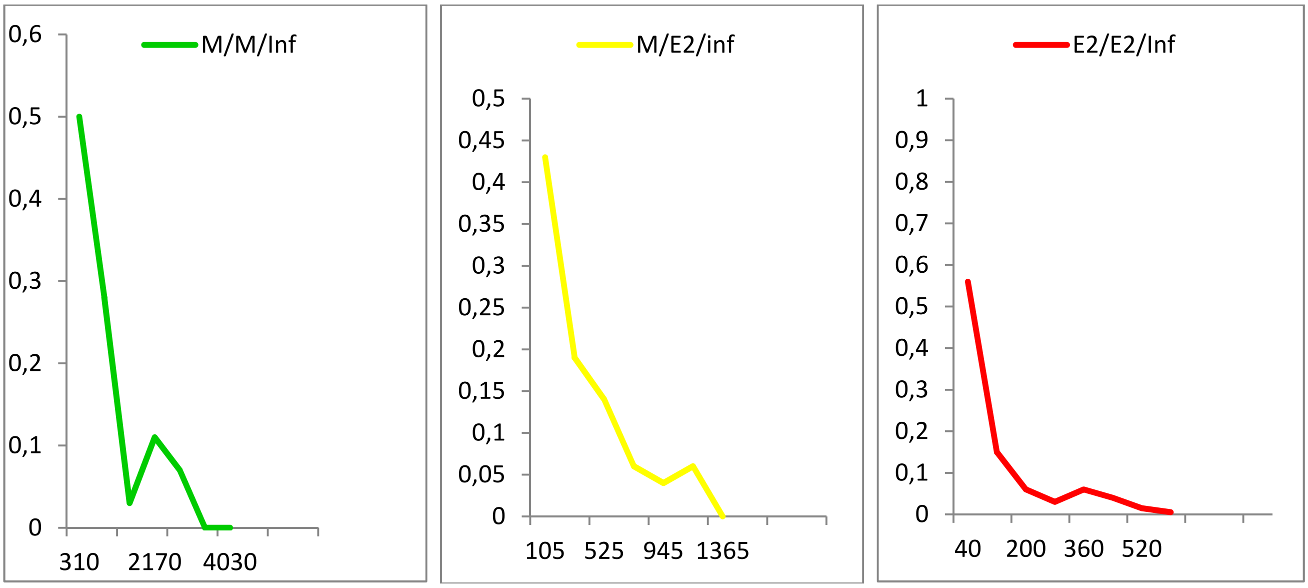


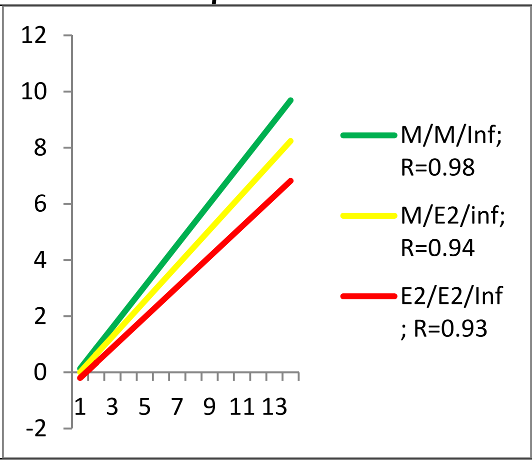

Figure 8. Distribution of Observed Busy Period Lengths, in seconds, for the Systems M / M / ∞, M / $E_2$ / ∞ and $E_2$ / $E_2$ / ∞ with ρ = 5.020.

Figure 8 suggests a greater similarity among the busy period lengths distributions of the various systems. Again it is observed a great concentration in the lowest values although lesser that in Figure 7. But now the whole of them present one valley, although for different values. Still goes on observing a great disparity among the maximum values assumed by the busy period lengths.

It seems evident, either in Figure 7 or in Figure 8, a sharp observations lack in the intermediate values zone for the busy period lengths.

Note also that the Figures 7 and 8 are in accordance with the studies that point in order that the busy period length distribution of the M / G / ∞ system is right asymmetric and leptokurtic see (Ferreira and Ramalhoto, 1994).

**ρ = 4.016**




**ρ = 5.020**

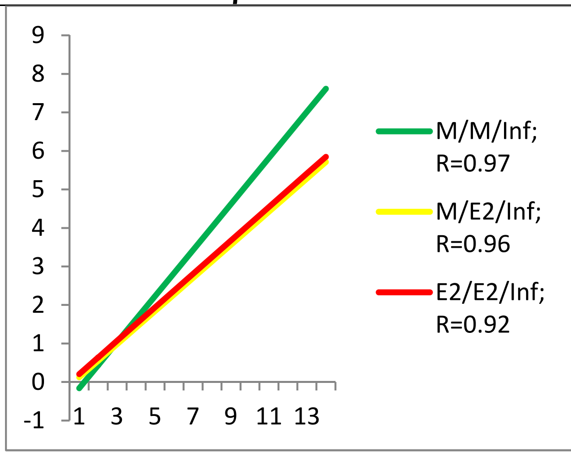



Figure 9. Regression of $Z$ over $X$, for the Systems M / M / ∞, M / $E_2$ / ∞ and $E_2$ / $E_2$ / ∞ with ρ = 4.016 and ρ = 5.020. R is the Linear Coefficient Regression.

Call $X$ and $Y$ the maximum number of customers served simultaneously and the total number of served customers, respectively, in the busy period. Performing the regression of $Z = \ln Y$ on $X$. The results obtained are presented graphically in Figure 9 and the most interesting fact is, maybe the similarity of the lines behaviour, for the various systems, for the two values of ρ considered. The systems for which more busy periods are observed – and, so, also with lesser lengths – are those to which correspond greater values of $\hat{\beta}$. But, for each system there is a great resemblance of the values of $\hat{\alpha}$ and $\hat{\beta}$ for the two values of ρ considered. In fact, it seems natural that the relation between $Z$ and $X$ does not depend on ρ. The values of ρ will only influence the values of $Z$ and $X$ that may occur and not the relation between them. Otherwise, will it be true that the difference observed between the values of $\hat{\alpha}$ and $\hat{\beta}$ for the various systems, not being too great, allows to face the hypothesis of that the relation between $Z$ and $X$ is identical for those systems? Maybe yes if we pay attention to the similarities of observed in its behaviour, namely the ones related with the distribution of the number of customers in the systems.

## 4 Conclusions

It is evident the great waste of these systems when looking to the maximum number of customers present simultaneously in the system.

It is uncontroversial, also, that there is a strong exponential relation between the maximum number of customers served simultaneously in a busy period and the total number of served customers. The question is if either it is always the same or how will it change either with the values of $\rho$ or from system to system.

About the busy period it is important to note also:

*i*) The great occurrence of busy periods with only one served customer,

*ii*) The great amplitude of the interval at which occur the values of the lengths of the busy periods, although with a great irregularity, and a great occurrence of low values.

The results of these simulations seem to suggest also that the systems GI /G / ∞ may be quite well approximated by systems M / G / ∞, at least when the GI process possesses a regularity not very far from the one of the Poisson process.

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## Appendix

```
            PROGRAM FILAESP
C       APAGUE 666 OU 555 CONFORME QUEIRA TEMPO INTERCHEGADAS
C        EXPONENCIAL OU ERLANG DE PARAMETRO 2
        DIMENSION V(25000),Y1(25000),Y2(25000),TEPSER(25000)
        DIMENSION C(25000),AUX(50000),P(25000),N(50000),F(50000)
        DIMENSION Z(50000),VAL(50000),ARG(50000)
        DIMENSION NO(0:50000),T(0:25000),TM(0:25000)
        DIMENSION TTR(0:25000),TMR(0:25000)
        DIMENSION BUPE(0:25000),NA(0:25000)
        WRITE(*,*)'O CODIGO DOS SERVICOS E: 0 PARA A PARETO, 1 PARA A'
        WRITE(*,*)'EXPONENCIAL, 2 PARA A ERLANG, 3 PARA A LOGNORMAL,'
        WRITE(*,*)'4 PARA A MISTURA DE EXPONENCIAIS COM RPARAMETRO,'
        WRITE(*,*)'5 PARA A MISTURA DE ERLANG.'
        WRITE(*,*)' '
        WRITE(*,*)' QUAL E O CODIGO DA DISTRIBUICAO DE SERVICO?'
        READ(*,*) ICOD
        WRITE(*,*)' '
        WRITE(*,*)' BOA SORTE NA VIAGEM AO MUNDO DA SIMULACAO '
        WRITE(*,*)' '
        WRITE(*,*)' '
        U=0.99600
        DO 100 I=1,25000
555             V(I)=ALOG(RAN(E*I))*(-U/2.0)
666             V(I)=ALOG(RAN(E*I))*(-U/2.0)+ALOG(RAN(F*I))*(-U/2.0)
100      CONTINUE
        CALL GERASER(TEPSER)
        C(1)=V(1)
        Z(1)=C(1)
        F(1)=1
        Z(25001)=V(1)+TEPSER(1)
        F(25001)=-1
        DO 500 I=2,25000
            C(I)=C(I-1)+V(I)
            P(I)=C(I)+TEPSER(I)
            Z(I)=C(I)
            F(I)=1
            Z(I+25000)=P(I)
            F(I+25000)=-1
500      CONTINUE
        X=0.0
        ICOL=1
        IROW=50000
        NDIM=50000
        CALL ATSG(X,Z,F,AUX,IROW,ICOL,ARG,VAL,NDIM)
        N(1)=1
        DO 600 I=2,50000
            N(I)=N(I-1)+VAL(I)
600     CONTINUE
        MAX=1
        DO 650 I=2,50000
            IF (N(I).GE.MAX)MAX=N(I)
650     CONTINUE
        WRITE(10,*)' SIMULACAO FILA DE ESPERA M|G|∞ '
        DO 700 K=0,MAX
```

```
      WRITE(10,*)' TEMPOS DE RECORRENCIA DO ESTADO ',K
      J=1
      NO(K)=0
      T(K)=0.0
      TM(K)=0.0
      DO 660 I=1,49999
            IF(N(I).EQ.K) THEN
            NO(K)=NO(K)+1
            T(K)=T(K)+(ARG(I+1)-ARG(I))
            J=J+1
            ENDIF
660   CONTINUE
      DO 1000 J=1,NO(K)-1
            TTR(K)=TTR(K)
1000  CONTINUE
      IF(NO(K).NE.0)TM(K)=T(K)/NO(K)
      IF(NO(K).GT.1)TMR(K)=TTR(K)/(NO(K)-1)
      IF(NO(K).EQ.1)TMR(K)=0
      WRITE(10,*)'ESTADO',K
      WRITE(10,*)'NUMERO DE VISITAS =',NO(K)
      WRITE(10,*)'TEMPO DE PERMANENCIA =',T(K)
      WRITE(10,*)'TEMPO MEDIO DE PERMANENCIA =',TM(K)
      WRITE(10,*)'TEMPO TOTAL DE RECORRENCIA =',TTR(K)
      WRITE(10,*)'TEMPO MEDIO DE RECORRENCIA =',TMR(K)
700   CONTINUE
      TTBUPE=ARG(50000)-ARG(1)-T(0)
      NTBUPE=1+N0(0)
      TMBUPE=TTBUPE/NTBUPE
      TTIDP=T(0)
      NTIDP=NO(0)
      TMIDP=TM(0)
      WRITE(10,*)'NO TOTAL DE PERIODOS DE OCUPACAO =',NTBUPE
      WRITE(10,*)'TEMPO TOTAL DE BUSY PERIOD =',TTBUPE
      WRITE(10,*)'TEMPO MEDIO DE BUSY PERIOD =',TMBUPE
      WRITE(10,*)'NO TOTAL DE PERIODOS DE DESOCUPACAO =',NTIDP
      WRITE(10,*)'TEMPO TOTAL DE IDLE PERIOD =',TTIDP
      WRITE(10,*)'TEMPO MEDIO DE IDLE PERIOD =',TMIDP
      BUPE(0)=0
      NA(0)=1
      NP=0
      DO 5000 I=1,49999
            IF(N(I).EQ.0) THEN
            NP=NP+1
            WRITE(10,*)'BUSY PERIOD NUMERO',NP
            NA(NP)=I
            NB=0
      DO 4000 J=NA(NP-1),NA(NP)-1
            IF(N(J+1).GT.N(J))NB=NB+1
4000   CONTINUE
      IF(NP.EQ.1)NB=NB+1
      WRITE(10,*)'NUMERO DE CLIENTES ATENDIDOS=',NB
      MAXI=1
      DO 3900 J=NA(NP-1),NA(NP)
            IF(N(J).GE.MAXI) MAXI=N(J)
3900  CONTINUE
      WRITE(10,*)'NUMERO MAXIMO DE CLIENTES ATENDIDOS SIMULTANEAMENTE=',MAXI
      BUPE(NP)=ARG(I+1)
      RBUPE=BUPE(NP)-BUPE(NP-1)-ARG(I+1)+ARG(I)
```

```
      IF(NP.EQ.1)RBUPE=BUPE(1)-ARG(I+1)+ARG(I)-ARG(1)
      WRITE(10,*)'COMPRIMENTO=',RBUPE
      ENDIF
5000  CONTINUE
      NP=NP+1
      WRITE(10,*)'BUSY PERIOD NUMERO=',NP
      NB=0
      DO 4001 J=NA(NP-1),49999
            IF(N(J+1).GT.N(J))NB=NB+1
4001   CONTINUE
      IF(NP.EQ.1)NB=NB+1
      WRITE(10,*)'NUMERO DE CLIENTES ATENDIDOS='NB
      MA=1
      DO 4002 J=NA(NP-1),50000
            IF(N(J).GE.MA)MA=N(J)
4002  CONTINUE
      WRITE(10,*)'NUMERO MAXIMO DE CLIENTES ATENDIDOS SIMULTANEAMENTE='MA
      RBUPE=ARG(50000)-BUPE(NP-1)
      IF(NP.EQ.1)RBUPE=ARG(50000)-ARG(1)
      WRITE(10,*)'COMPRIMENTO= ',RBUPE
      END


            SUBROUTINE GERASER (T)

      DIMENSION T(25000)
            ICOD=2
      IF (ICOD.EQ.0) THEN
      PRINT*, 'INTRODUZA O VALOR DO COEFICINTE DE VARIACAO'
      READ*,GAMA
      ALFA=2*GAMA/(GAMA-1.0)
      RK=E*(GAMA+1.0)/(2*GAMA)
      ELSEIF (ICOD.EQ.4) THEN
      PRINT*,'INTRODUZA O VALOR DO PARAMETRO DA MISTURA'
      READ(*,*)RPARAMETRO
      ENDIF
      IX=35
      IY=43
      IIX=9
      IJX=5
      IKX=11
      ILX=13
      DO 10 I=1,250000
            CALL RANDU(IX,IY,FL)
            IF(ICOD.EQ.O) THEN
            T(I)=RK/(1-FL)**(1.0/ALFA)
            ELSE IF (ICOD.EQ.1) THEN
            T(I)=-7.0*ALOG(RAN(G*I))
            ELSE IF (ICOD.EQ.2) THEN
      T(I)=-(4.0/2.0)*ALOG(RAN(G*I))-(4.0/2.0)*ALOG(RAN(H*I))
            ELSE IF (ICOD.EQ.3) THEN
            CALL RANDU(JX,IJY,YFFL)
            IJX=IJY
            T(I)=EXP((-2*ALOG(YFFL))**(0.5)*COS(8*ATAN(1.0)*FL))
            ELSE IF (ICOD.EQ.4) THEN
            CALL RANDU (IKX,IKY,RFL)
            IKX=IKY
            CALL RANDU (ILX,ILY,SFL)
```

```
            ILX=ILY
            IF (FL.LE.RPARAMETRO) THEN
            T(I)=-(3.45/2.0)*(1.0/RPARAMETRO)*LOG(RFL)
            ELSE
            T(I)=-(3.45/2.0)*(1.0/1.0-RPARAMETRO)*LOG(SFL)
            ENDIF
            ELSE IF (ICOD.EQ.5) THEN
            CALL RANDU (IKX,IKY,RFL)
            IKX=IKY
            CALL RANDU (ILX,ILY,SFL)
            ILX=ILY
            CALL RANDU (IMX,IMY,TFL)
            IMX=IMY
            CALL RANDU (INX,INY,UFL)
            INX=INY
            CALL RANDU (IPX,IPY,VFL)
            IPX=IPY
            CALL RANDU (IQX,IQY,WFL)
            IQX=IQY
            IF(RFL.LT.0.400) THEN
      T(I)=-((10.0/2.7)/4.0)*(LOG(UFL)+LOG(SFL)+LOG(TFL)+LOG(WFL))
            ELSE IF ((RFL.GT.0.4000).AND.(RFL.LT.0.75)) THEN
            T(I)=-((10.0/4.2)/2.0)*(LOG(VFL)+LOG(WFL))
            ELSE IF (RFL.GT.0.75) THEN
            T(I)=-((10.0/3.6)/3.0)*(LOG(VFL)+LOG(WFL)+LOG(SFL))
                  ENDIF
            ENDIF
10     CONTINUE
      END
```