Appendix A – Notations of the Section 2.5

A – Acquisition cost per customer
A' – Average acquisition cost per customer
$A_t^{''}$ – Variable cost of acquisition the customer
$A_t^{'''}$ – fixed cost of acquisition the customer
a – Acquisition rate, given a specific level of acquisition costs A
a' – Probability the firm convinces a prospect to become a customer
$B(t)$ – Potential benefit from the customer at period t
C – Annual costs
C' – Costs per period
C_{jt} – Cost of sales per transaction j in period t
C_t – Total cost of generating the revenue R_t in period t
$C_t^{'}$ – Costs from the total of customers in period t
$C_t^{''}$ – Direct cost of servicing the customer at time t
$C_t^{'''}$ – Cost of sales in period t
$C_t^{''''}$ – Unit cost of goods sold to the customer in year t
$C_t^{'''''}$ – Variable cost per unit sold to the customer in year t
$C_t^{''''''}$ – Fixed cost associated with the customer in year t
c – Promotion costs per customer per year
c' – Retention cost per customer per year
c'' – Mailing costs
c''' – Average marketing costs per customer
c_{kt} – Variable marketing cost, k, in period t
$c_{m,t}$ – Unit marketing cost for the customer in channel m in year t
c_s – Promotion costs per customer per sales cycle
c_t – Retention costs per customer in period t
$c_t^{'}$ – Retention spending between t and t+1 (present value at time t)
$c_t^{''}$ – Mailing cost in month t for the customer
$c_t^{'''}$ – Termination costs for the relationship with the customer
$c_t^{''''}$ – Variable costs of loyalty programs for the customer in year t
$c_t^{'''''}$ – Fixed costs of loyalty programs for the customer in year t
CF^V – Vector of customer cash flows in any future period
CLV^T – Column vector of expected present value over T periods
$CLV_{r'}$ – Value of a customer with recency r'
CM_{NRT} – Average contribution margin of base transaction
CM_t – Average contribution margin in month t
CM_{Up} – Average contribution margin of upgrading transactions
CM_y – Contribution margin of the customer in transaction y

$CoopV_t$ – Cooperation value of the customer in period t

CP_t – Customer potential = f (predicted sales volume, predicted profit, …)

CQ_t – Customer quality = f (sales per period, profit contributions, number of different products, …)

CS_t – Customer share = $f(SQ_t, SP_t)$

D – Discount factor (the inverse of one plus the discount rate)

$D(t)$ – Discount function

d – Defection rate

f – Predicted purchase frequency for the customer

g – Growth rate

GC – Yearly gross contribution margin per customer

GC' – Average gross contribution

GC_s – Gross contribution margin per customer per sales cycle

GC_t – Gross contribution of the customer in month t

GC'_t – Gross contributions from reference activities of the customer in period t

I – Identity matrix

i – Discount rate of the period

$InfoV_t$ – Information value of the customer in period t

$InnoV_t$ – Innovation value of the customer in period t

m – Margin or profit from a customer per period

m' – Margin from a customer per year

m'' – Average gross margin per period

m''' – Margin per transaction

m_{dt} – Margin of the direct transactions of the customer in period t

m_t – Margin or contribution of each customer in period t

m_{WoM} – Margin generated by referral behaviour of the customer in period t

m_0 – Margin at time 0, or the minimum margin

m_∞ – Margin at infinite time period, or the maximum possible margin

n' – Number of transactions per year

n'' – Number of years between two consecutive sales

n'''' – Number of marketing actions per year

n_0 – Number of customers acquired in base year 0

n_b – Initial customer base at the time of the determination of CLV

n_k – Number of customers in the k^{th} cohort

$n_{m,t}$ – Number of contacts to the customer in channel m in year t

$n_{r',k}$ – Number of transactions generated by a customer of recency r' in period k

n_t – Number of base (i.e., non-upgrade) transactions made by the customer in period t

n'_t – Number of "upgrade" transactions by the customer in period t

n''_t – Number of units purchased by the customer in year t

n_t''' – Number of hours that the salesperson spends attempting to acquire the customer in year t

n_{t-j} – Number of customers in period t-j

p_l – Response probability of a segment of recency l

P_{u-j} – Probability of purchase for the current year u

q_k – Cumulated probability for a customer to be eliminated (purged) after k periods

R – Annual revenues

R' – Revenues per period

R_{jt} – Revenue per transaction j in period t

R_t – Revenue from the customer in period t

$R_t^{'}$ – Revenue from the total of customers in period t

$R_t^{''}$ – Revenue per unit charged to the customer in year t

R_t^A – Autonomous revenue of the customer in period t

R_t^{CS} – Cross selling revenue of the customer in period t

R_t^{US} – Up selling revenue of the customer in period t

r – Yearly retention rate

r' – Recency

r'' – Recency limit beyond which a customer is ticked off the database

r_c – Retention probability as a function of the retention budget

r_j – Probability of customer retention in period j

r_t – Probability of customer retention in period t

r_s – Retention rate per sales cycle

r_0 – Retention at time 0 or the minimum retention

r_∞ – Retention at infinite time period or the maximum possible retention

S – Transition matrix

s – Rate of change of margin from the minimum to maximum

s' – Rate of change of retention from the minimum to maximum

$S(t)$ – Survival probability at time t (survival function)

t – Time period

T – Time horizon for estimating CLV

T' – Total service period of the customer

T'' – Expected service period of the customer

T''' – Period over which the customer is assumed to remain active

u – Year u

$v(t)$ – Expected customer value at time t

$v'(t)$ – Expected value generated by the customer at time t

W – Aggregated weighting for the discounted margins generated by customer the customer as a result of his lead user, reference and option value potential

$W_t^{'}$ – Hourly wage for a salesperson calling on the customer in year t

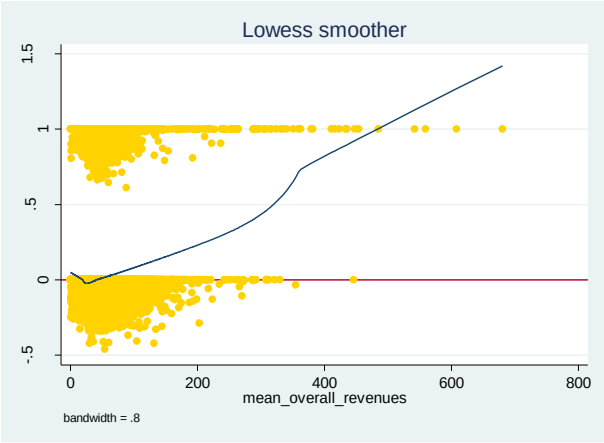
λ – Length of the average repurchase cycle

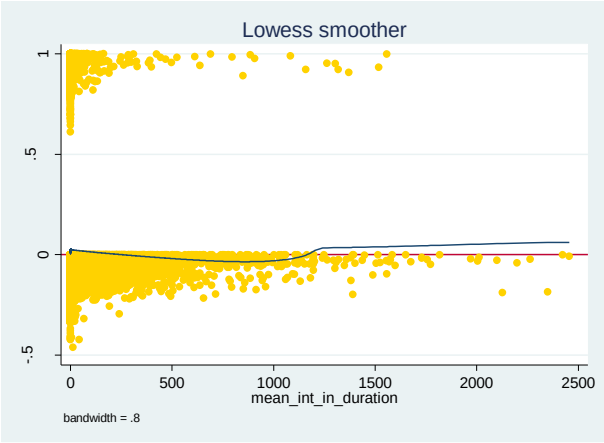
$\pi_f(t)$ – Future profit contribution of the customer at period t

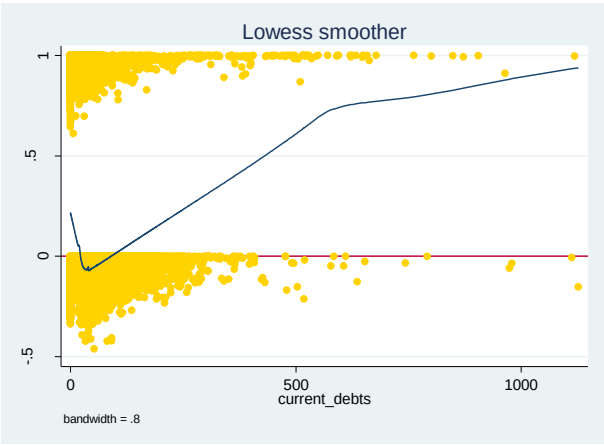
$\pi_p(t)$ – Past profit contribution of the customer at period t

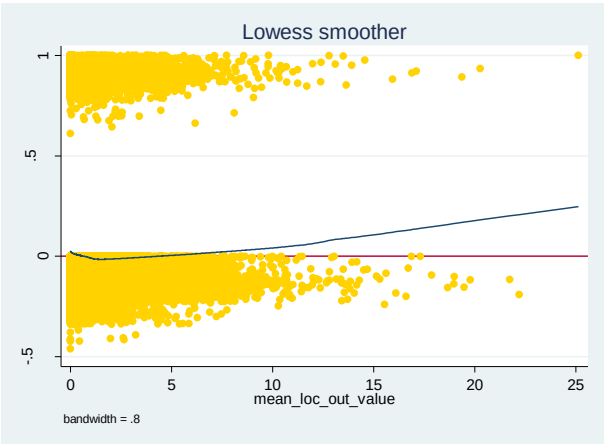
$\pi(u)$ – Profit per customer in year u

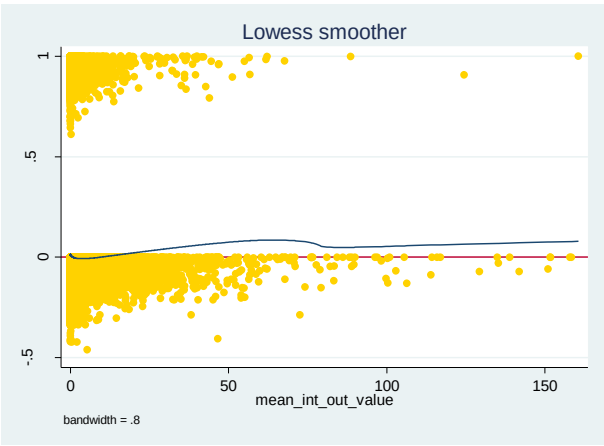
Appendix B – Analysis of the functional form of covariates (fixed-telephone)

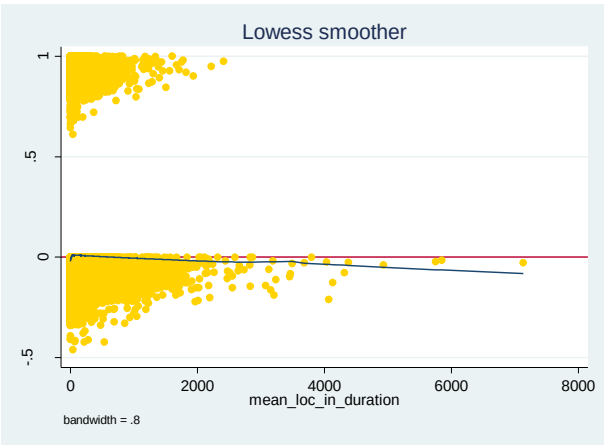


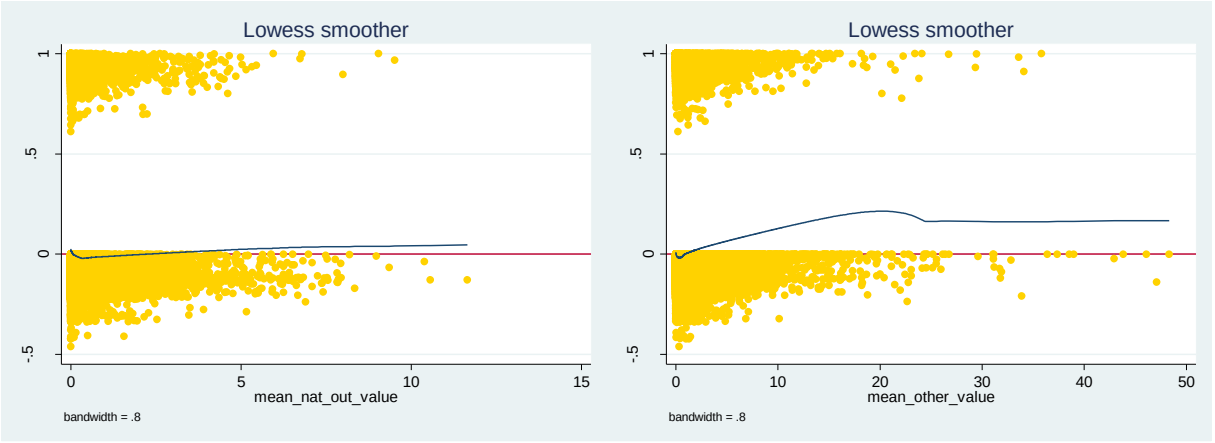








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0
-.5
0
5
10
15
mean_nat_out_value
bandwidth = .8
Lowess smoother
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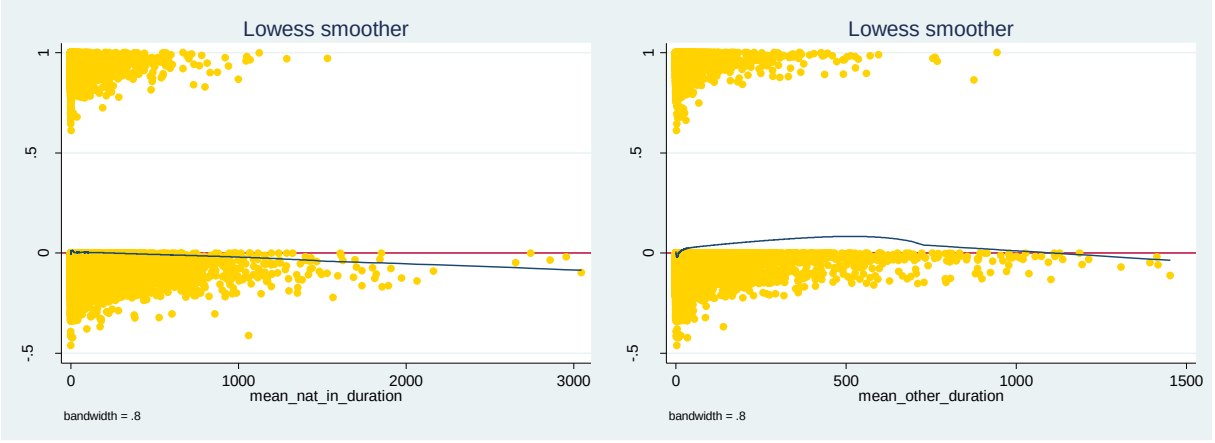
0
-.5
0
10
20
30
40
50
mean_other_value
bandwidth = .8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mean_other_duration
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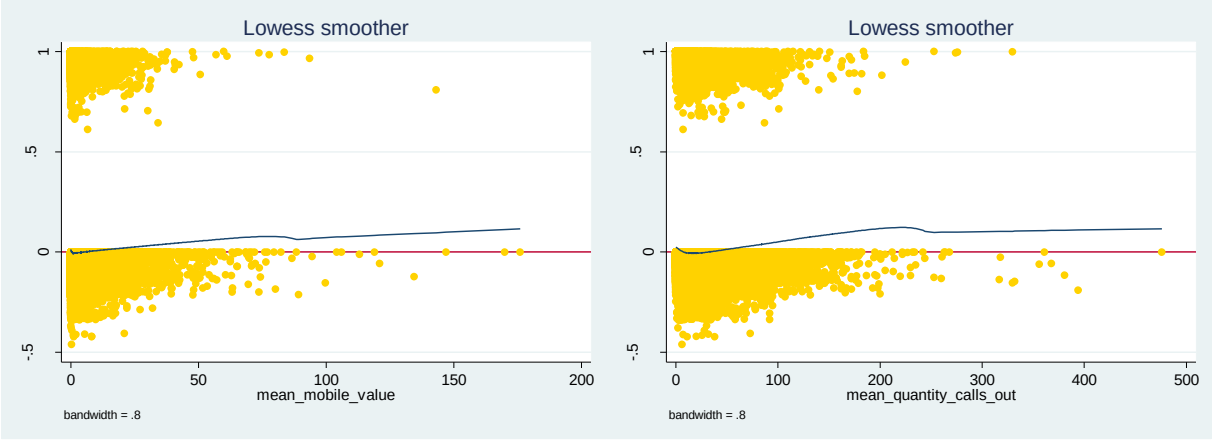
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200
mean_mobile_value
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mean_quantity_calls_out
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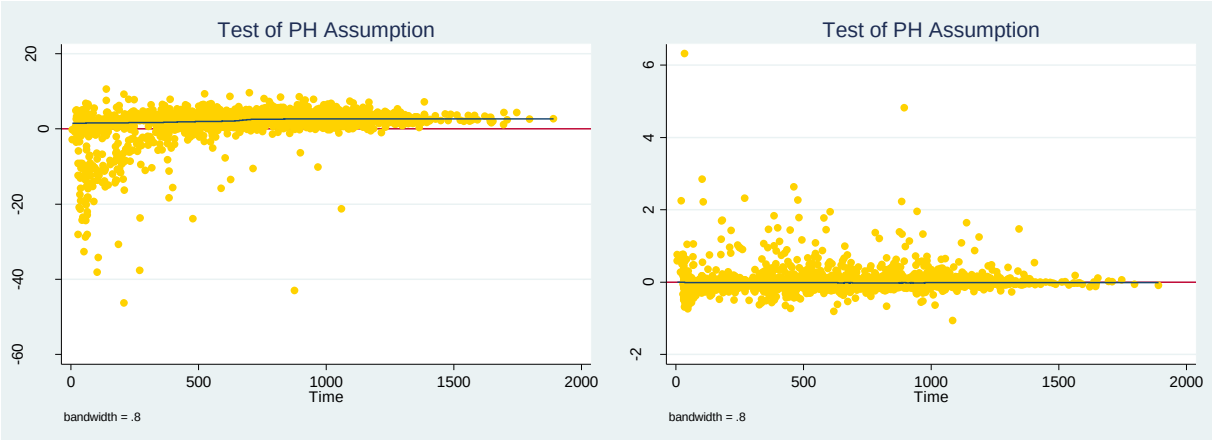
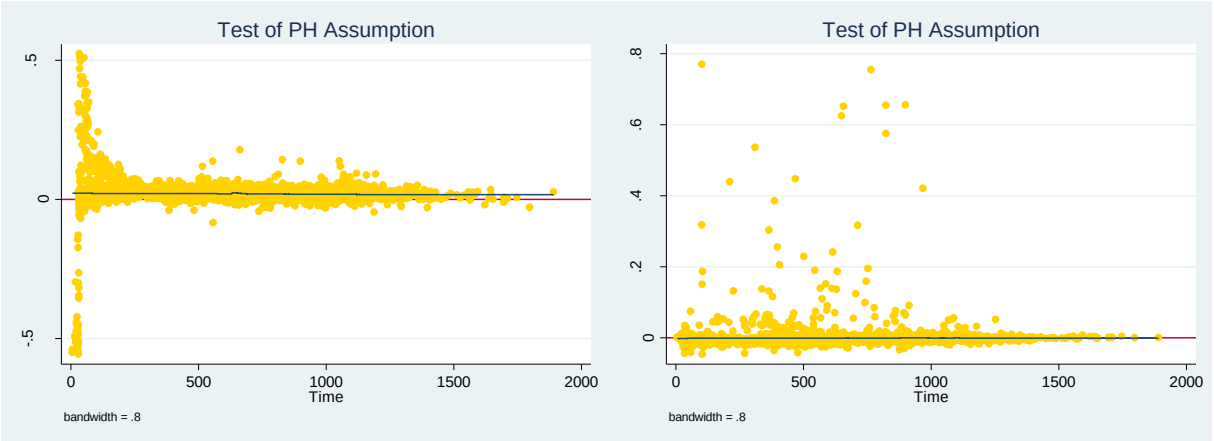
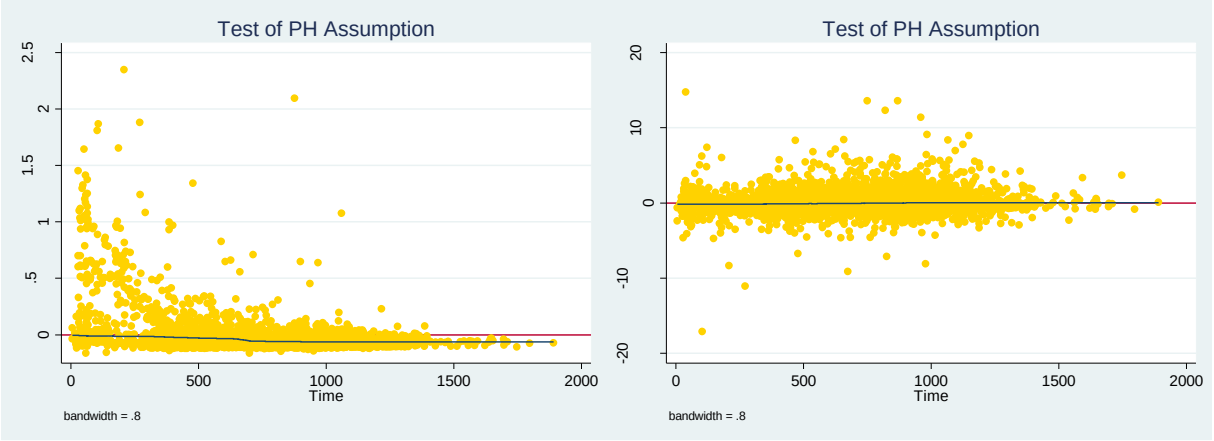
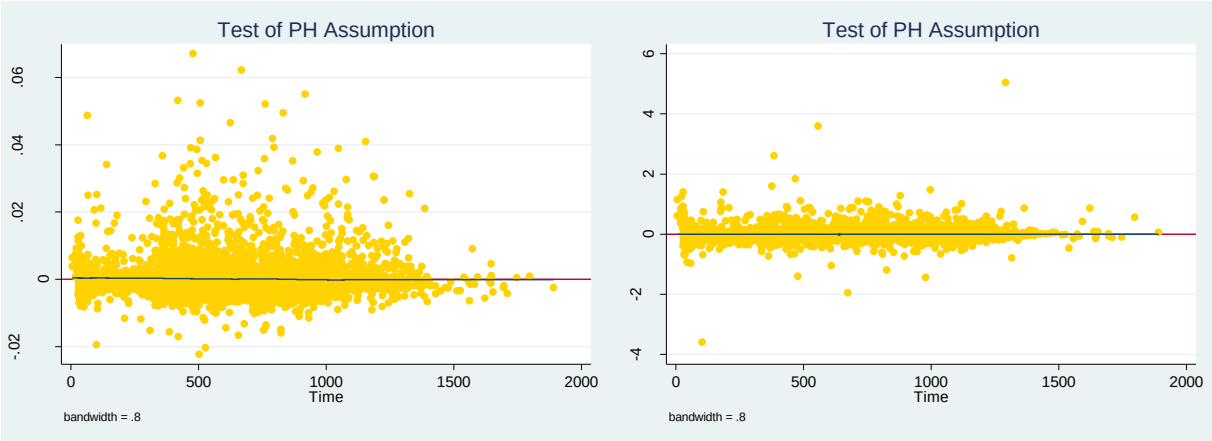
Appendix C – Graphs of Schoenfeld residuals (fixed-telephone)





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.06
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1000
1500
2000
Time
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Test of PH Assumption
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4
6
0
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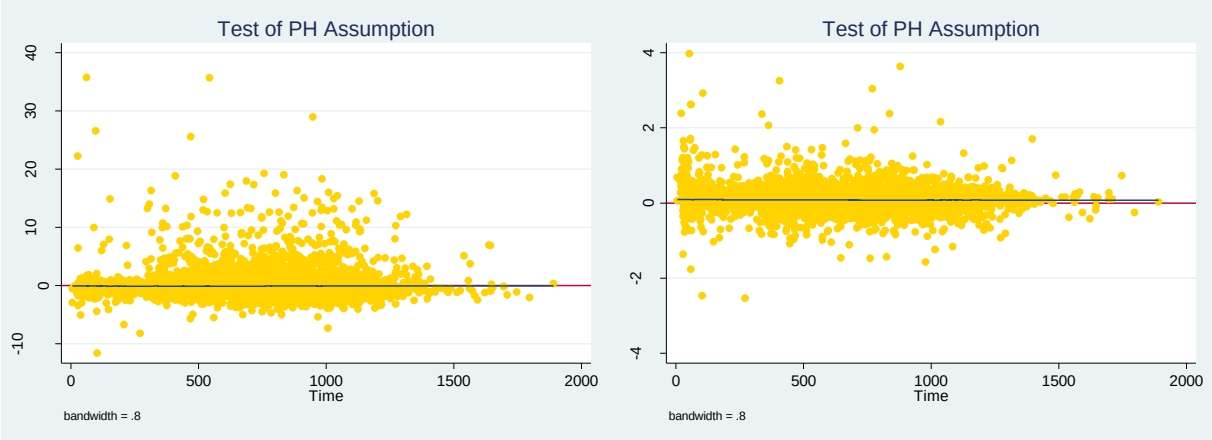
1000
1500
2000
Time
bandwidth = .8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1000
1500
2000
Time
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Test of PH Assumption
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-2
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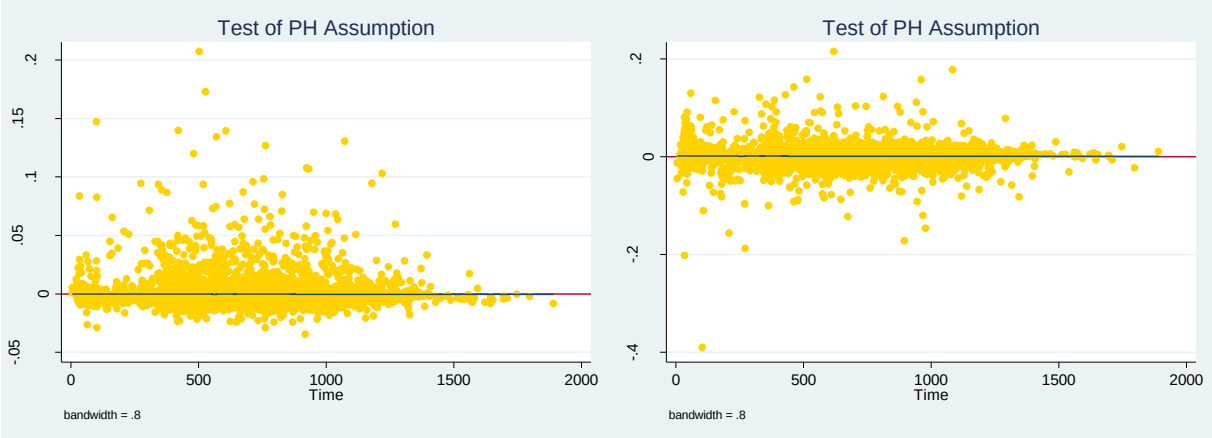
2
4
0
500
1000
1500
2000
Time
bandwidth = .8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.15
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500
1000
1500
2000
Time
bandwidth = .8
Test of PH Assumption
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-.2
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.2
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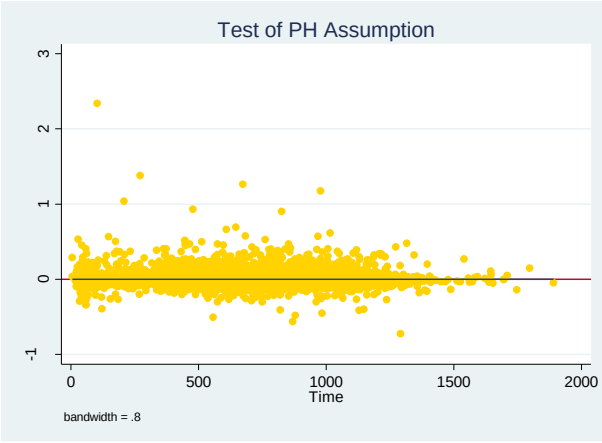
1500
2000
Time
bandwidth = .8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1000
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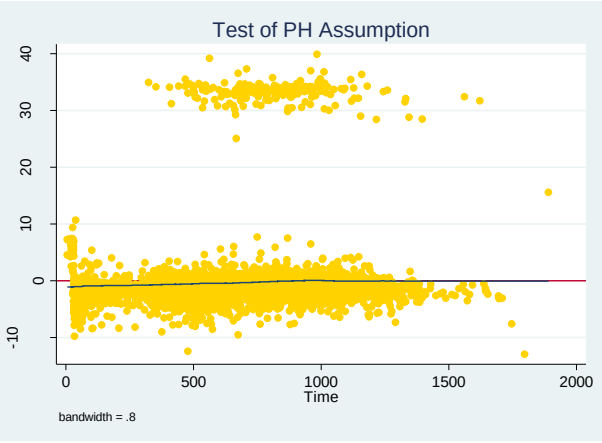
Time
bandwidth = .8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20
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0
-10
0
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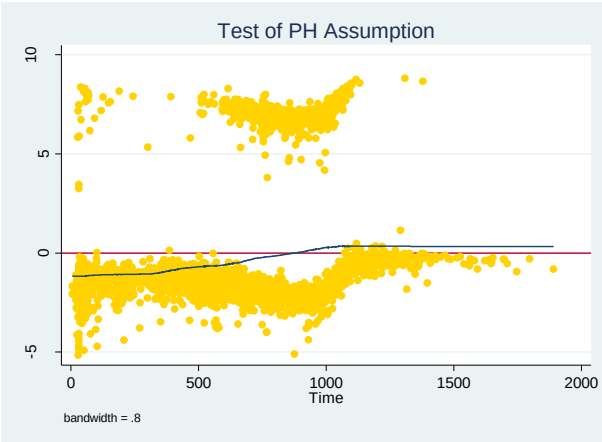
1000
1500
2000
Time
bandwidth = .8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Time
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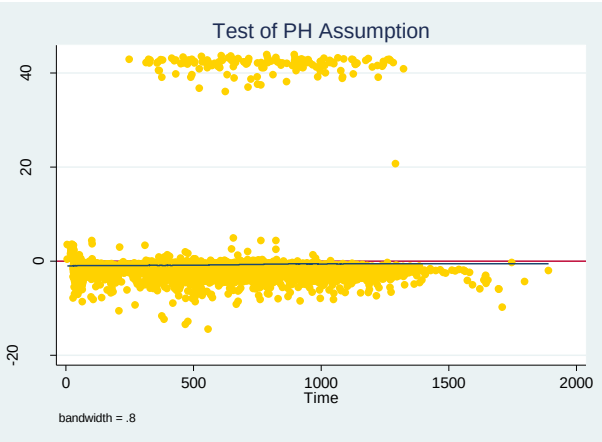
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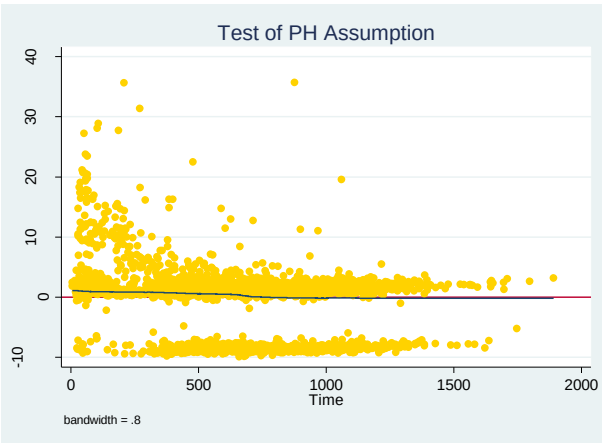
2000
Time
bandwidth = .8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-10
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1500
2000
Time
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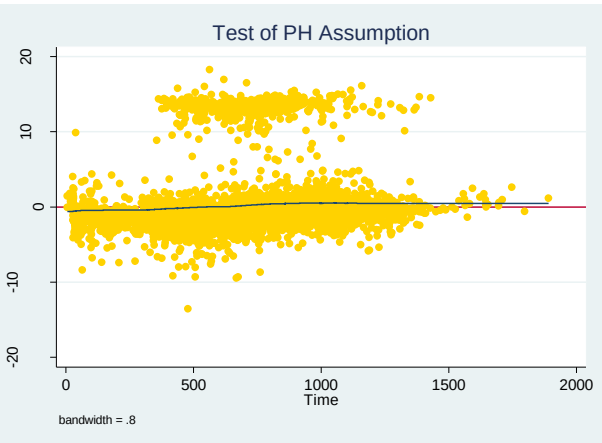
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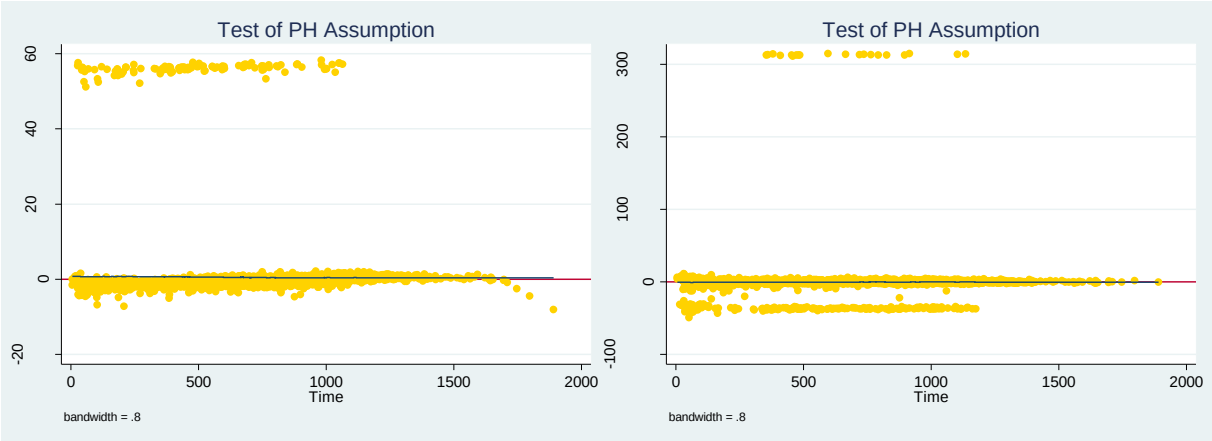
bandwidth = .8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Test of PH Assumption
Time
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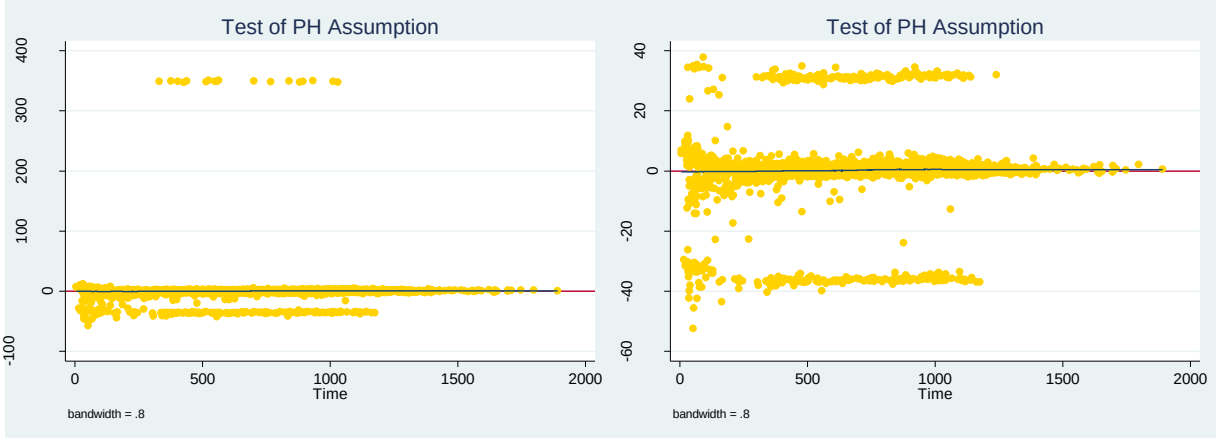
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Test of PH Assumption
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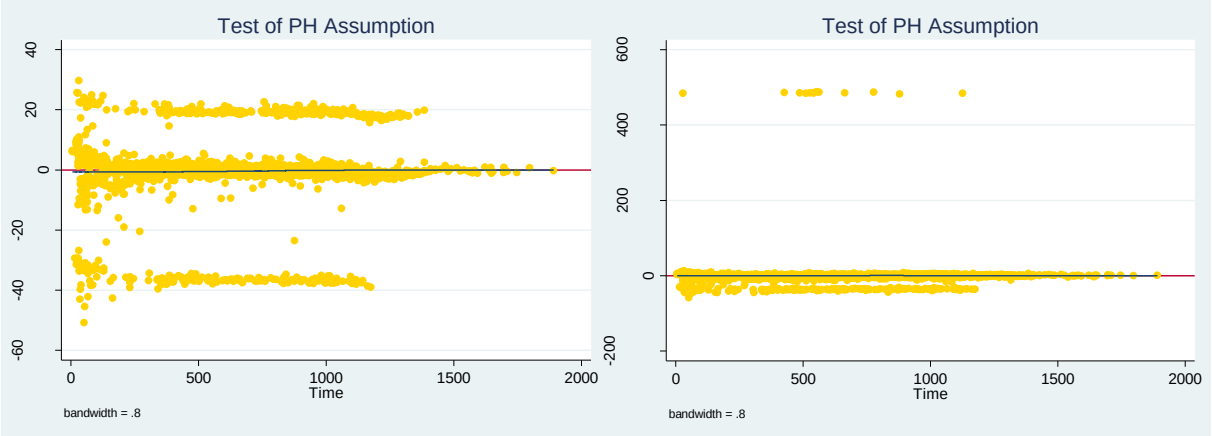
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Test of PH Assumption
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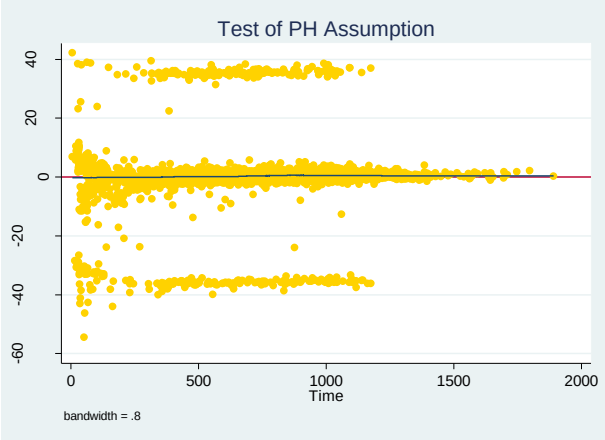
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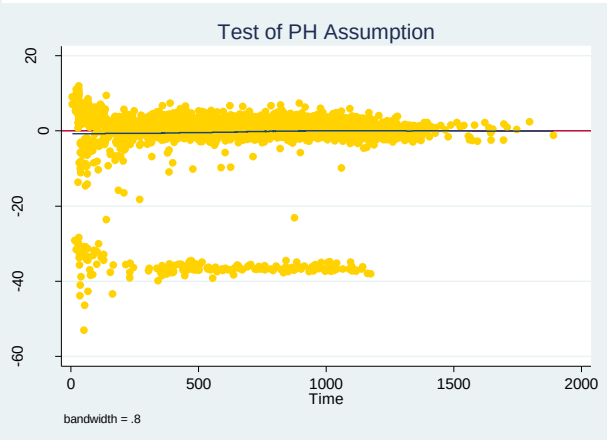
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2000
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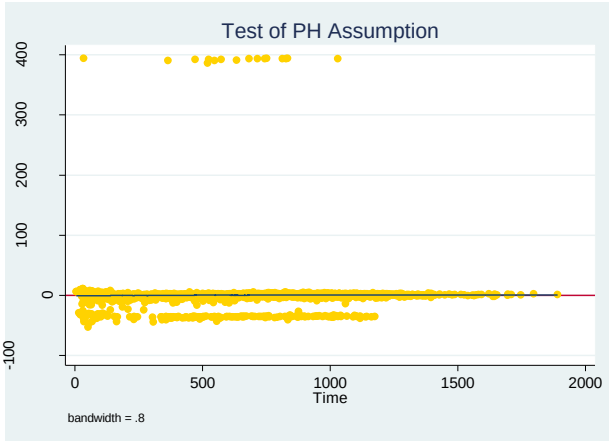
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2000
Time
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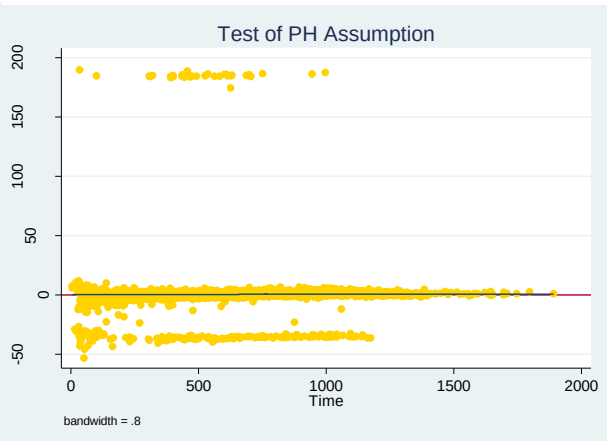
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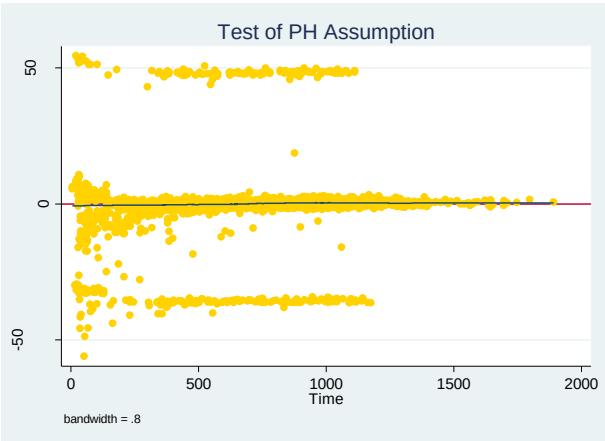
Time
bandwidth = .8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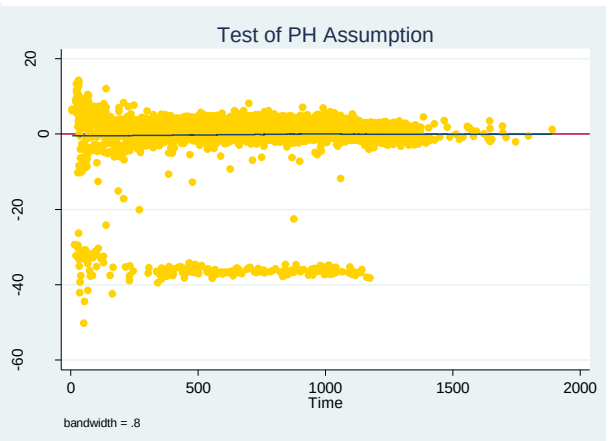
Time
bandwidth = .8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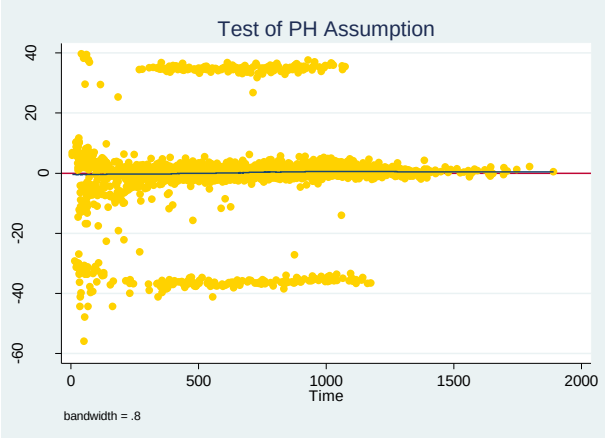
2000
Time
bandwidth = .8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-20
-40
-60
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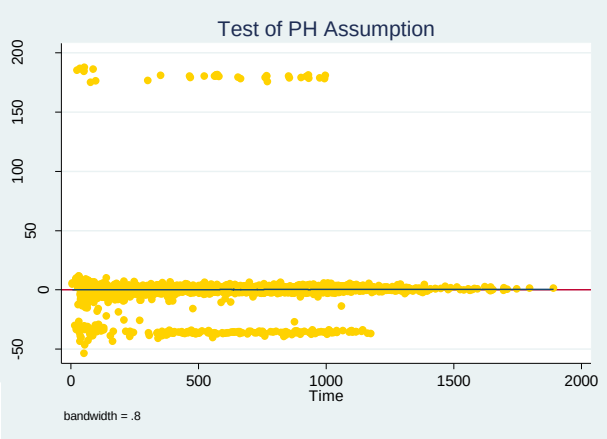
500
1000
1500
2000
Time
bandwidth = .8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1000
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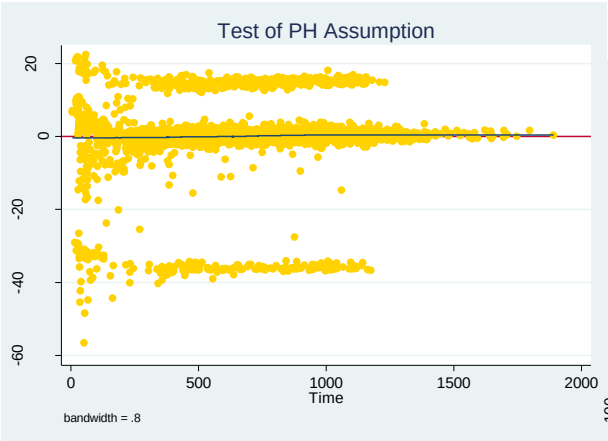
2000
Time
bandwidth = .8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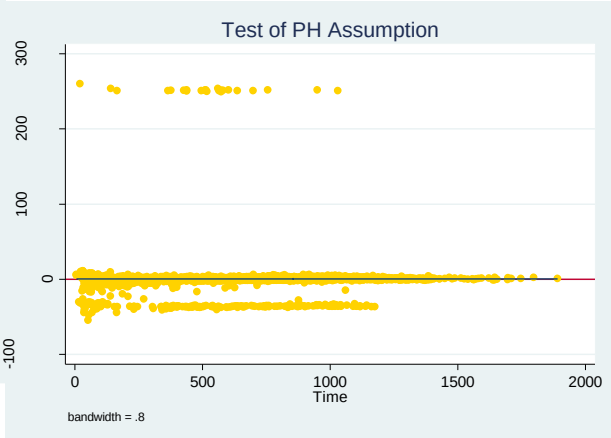
2000
Time
bandwidth = .8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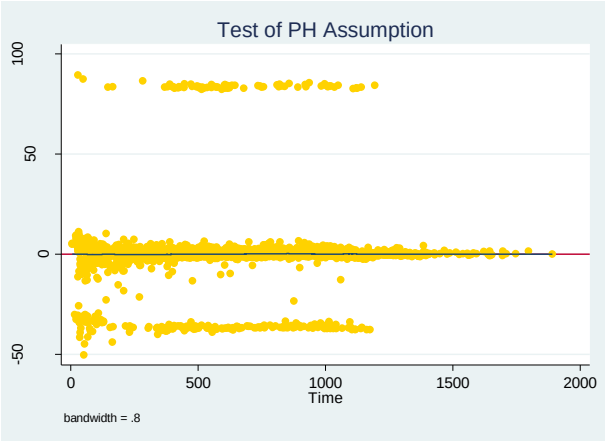
500
1000
1500
2000
Time
bandwidth = .8


Test of PH Assumption
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0
-50
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1000
1500
2000
Time
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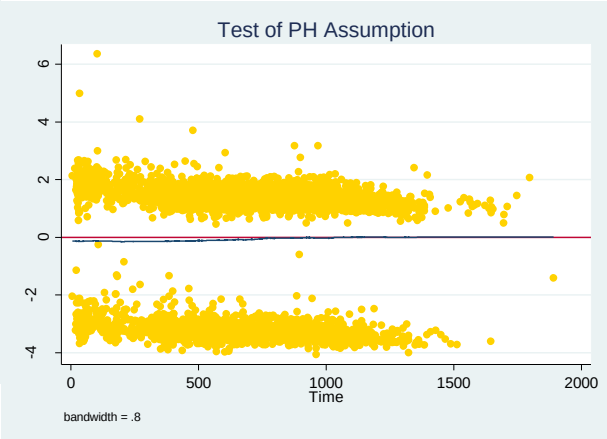
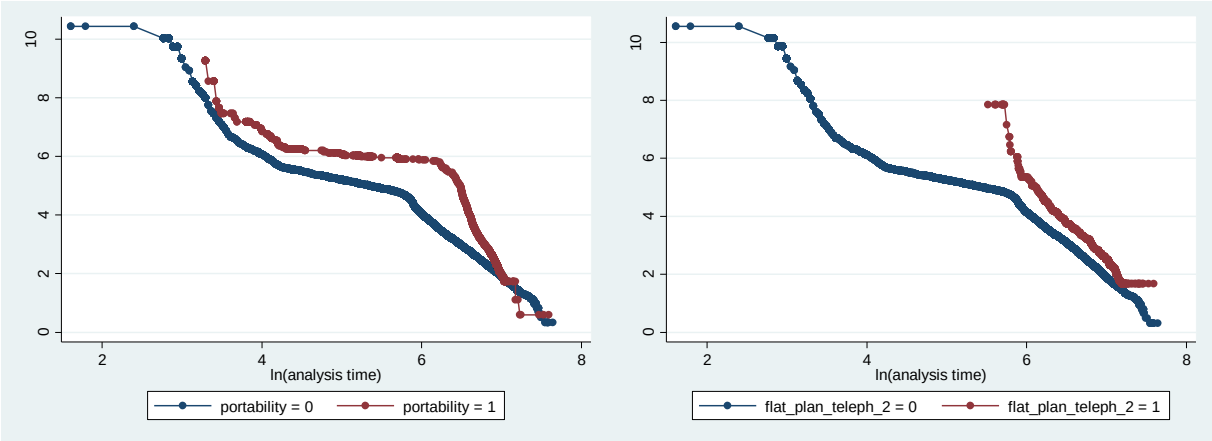
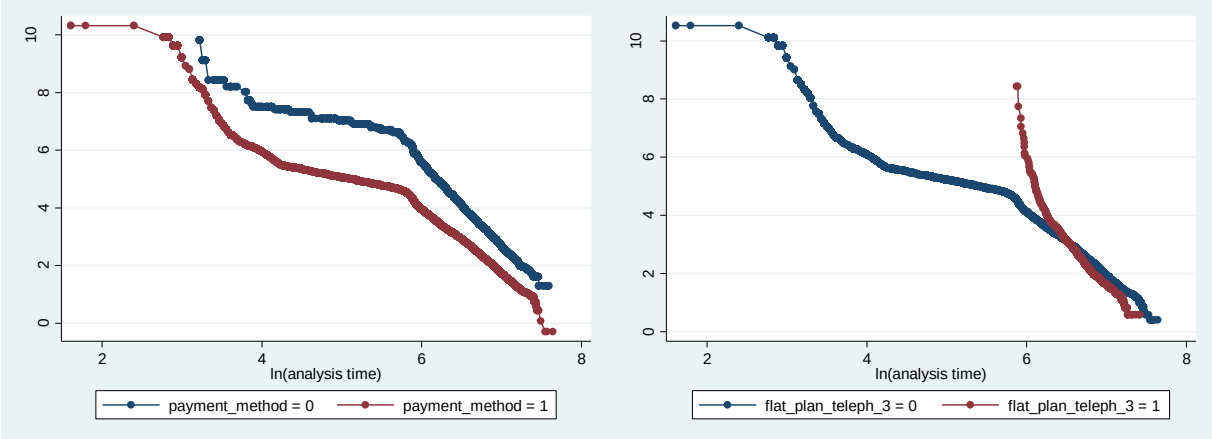
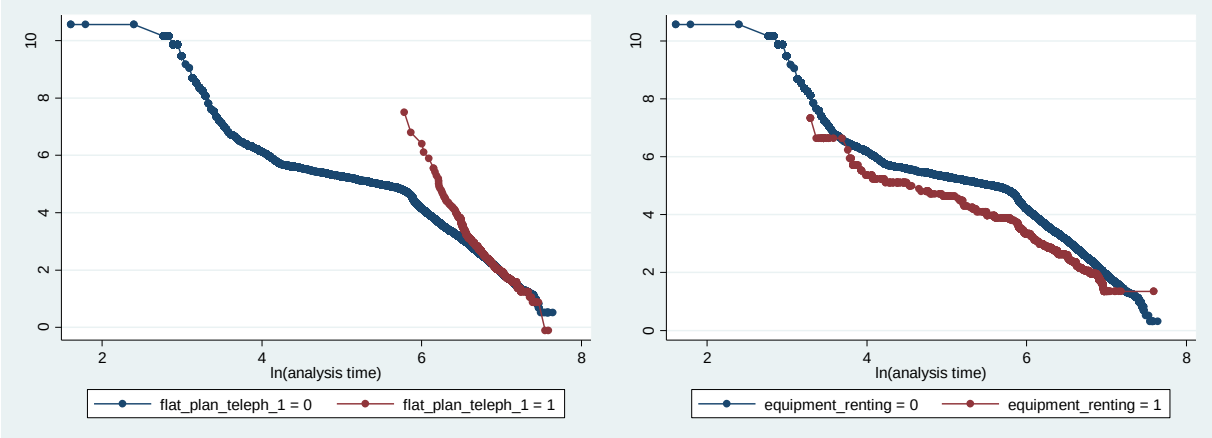
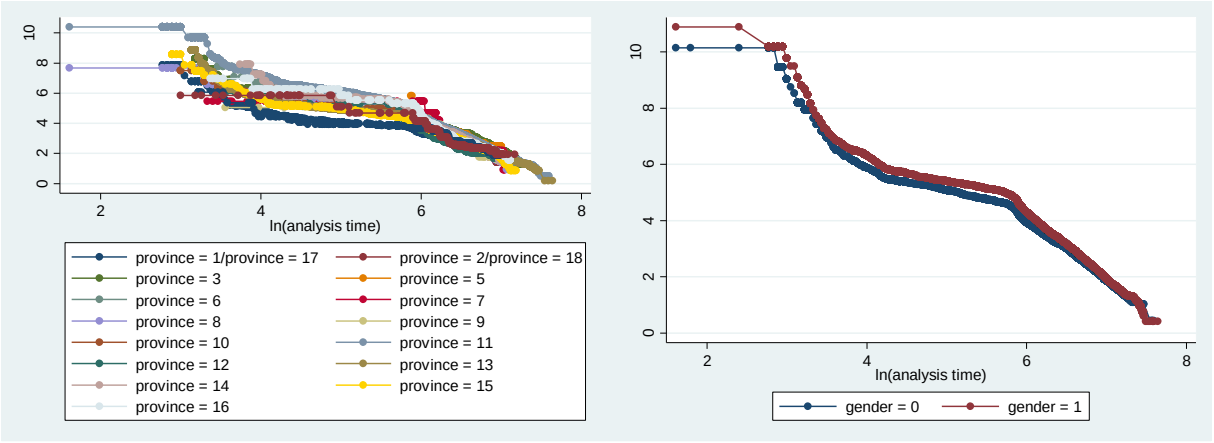

Test of PH Assumption
6
4
2
0
-2
-4
0
500
1000
1500
2000
Time
bandwidth = .8

Appendix D – Graphs of $-\ln\left\{-\ln\left[\hat{S}(t)\right]\right\}$ against $\ln(t)$ (fixed-telephone)

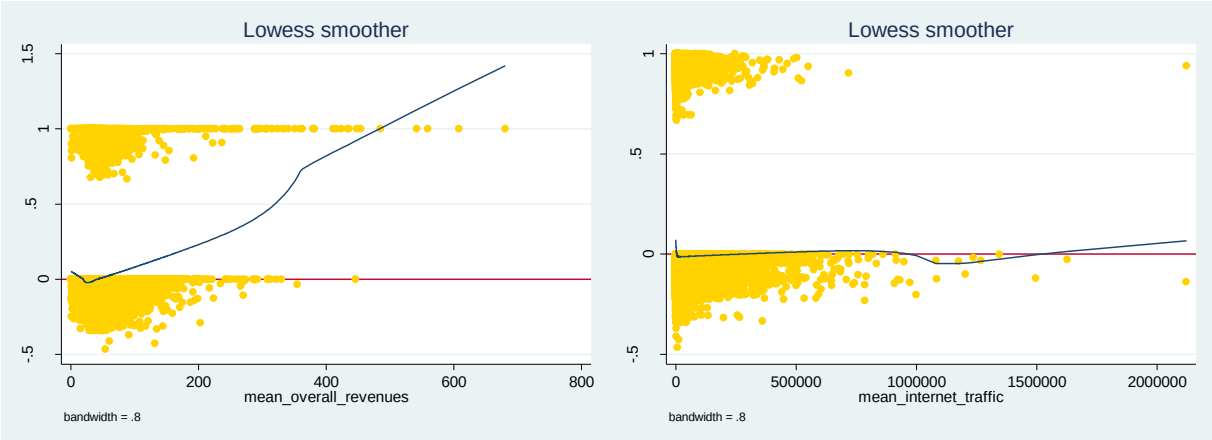


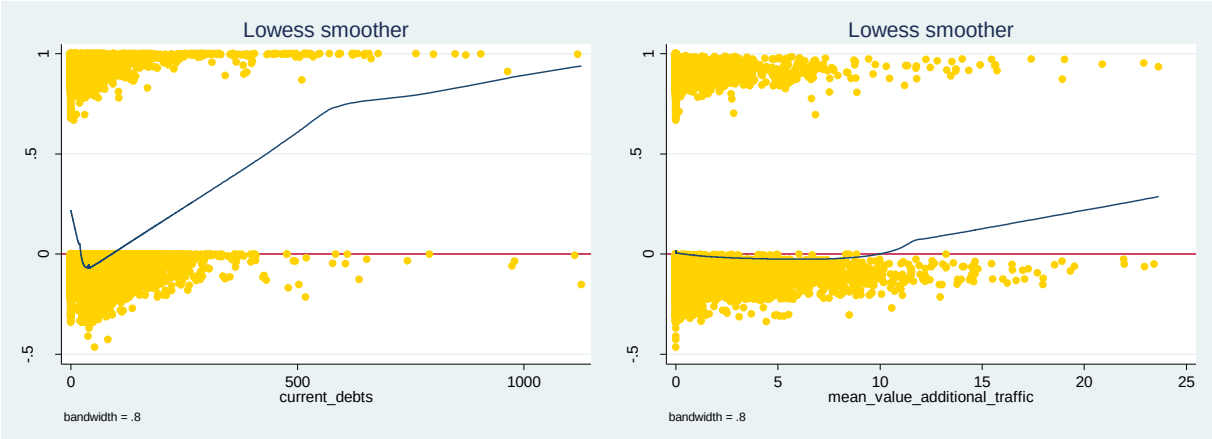




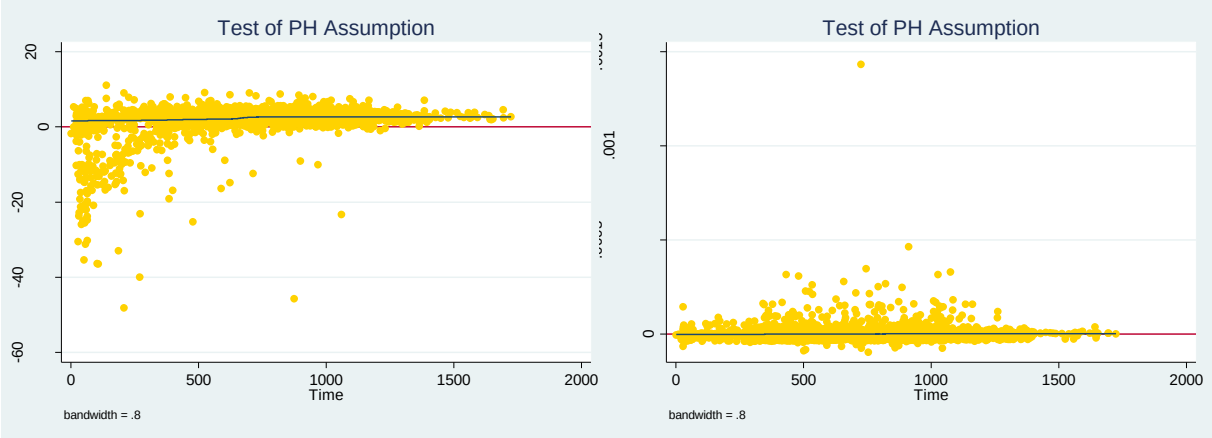

0
2
4
6
8
10
ln(analysis time)
province = 1/province = 17
province = 2/province = 18
province = 3
province = 5
province = 6
province = 7
province = 8
province = 9
province = 10
province = 11
province = 12
province = 13
province = 14
province = 15
province = 16
gender = 0
gender = 1

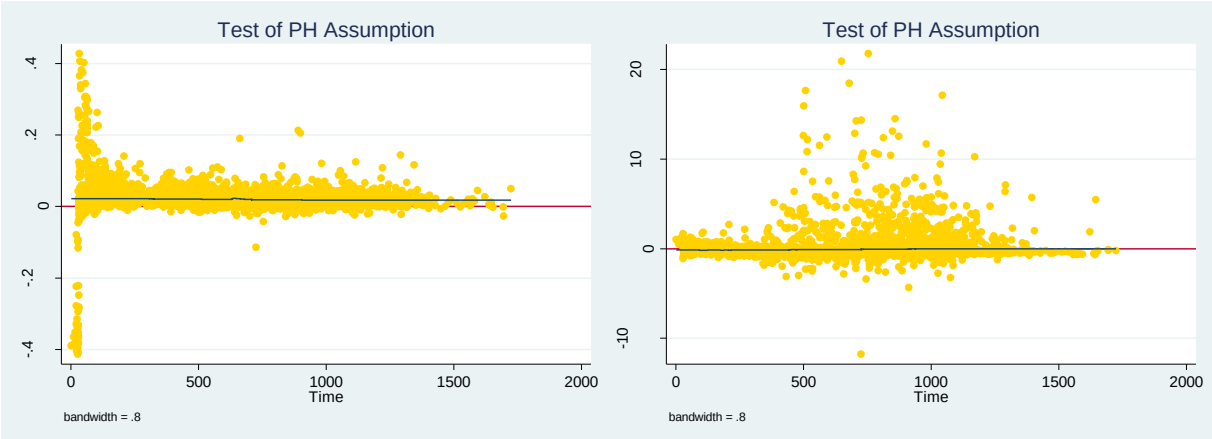
Appendix E - Analysis of the functional form of covariates (ADSL)

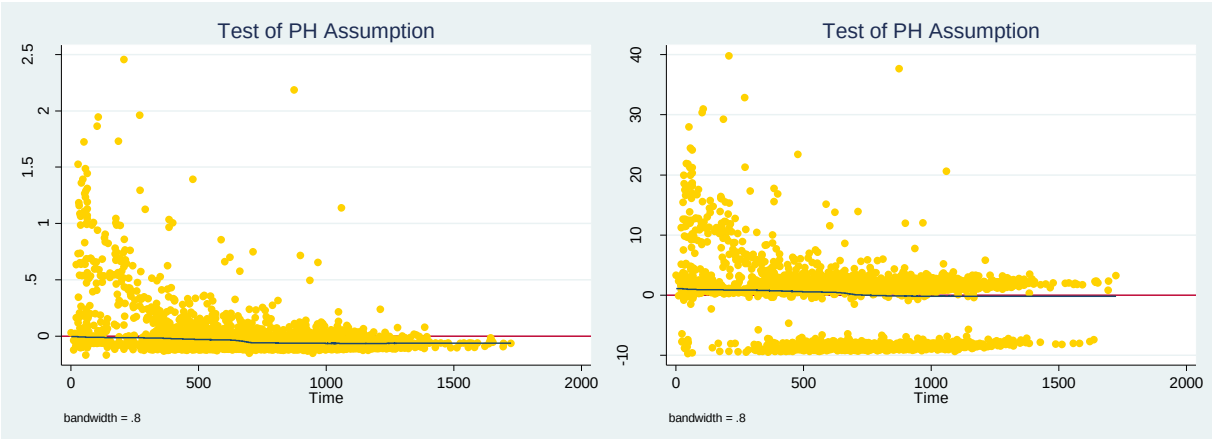


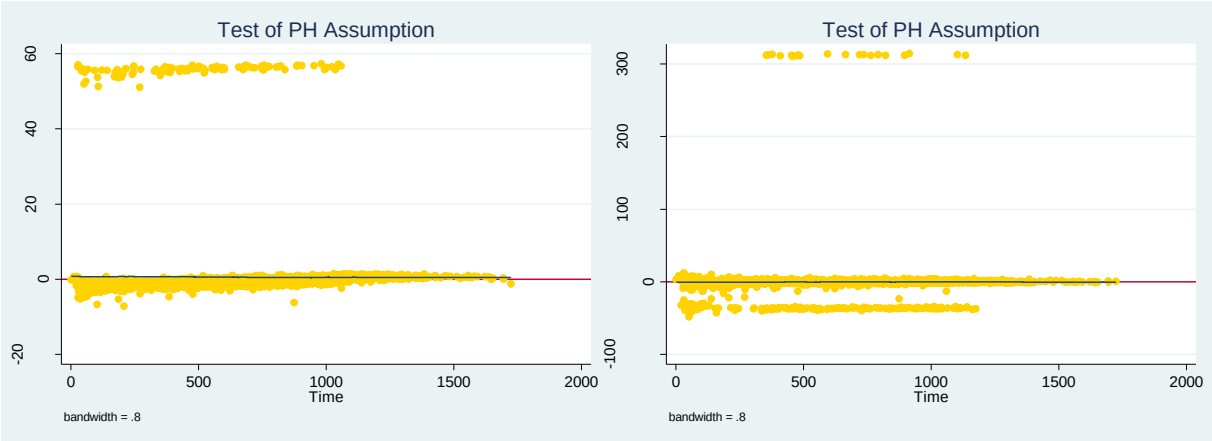


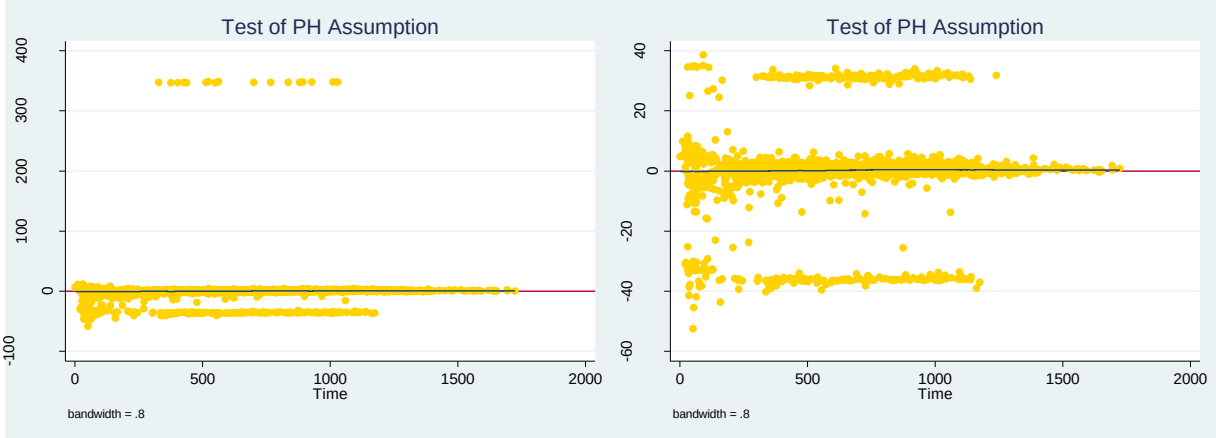
Appendix F - Graphs of Schoenfeld residuals (ADSL)

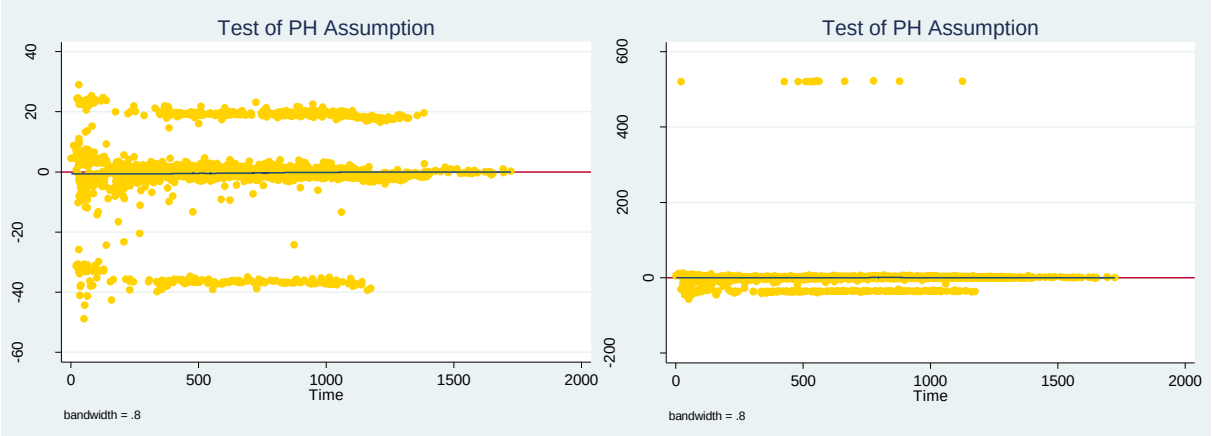


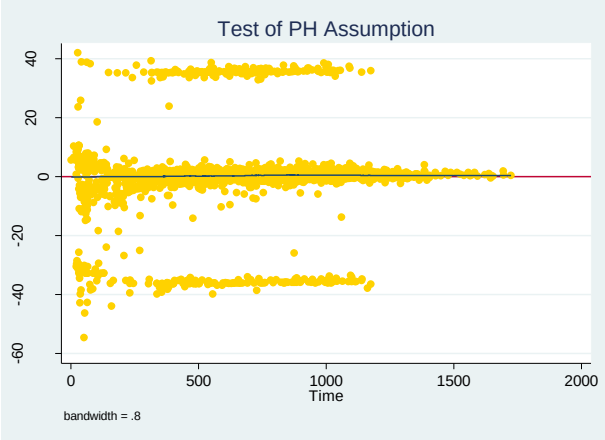


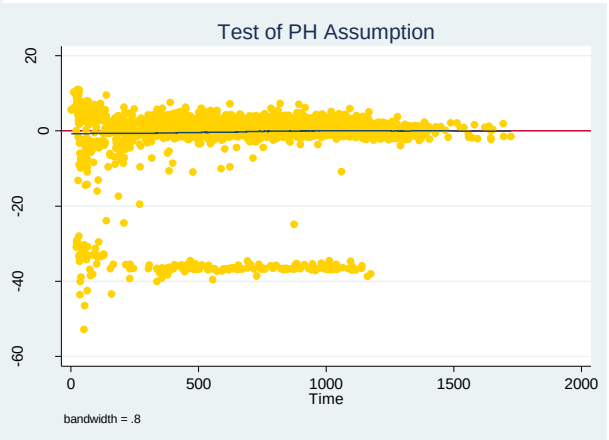


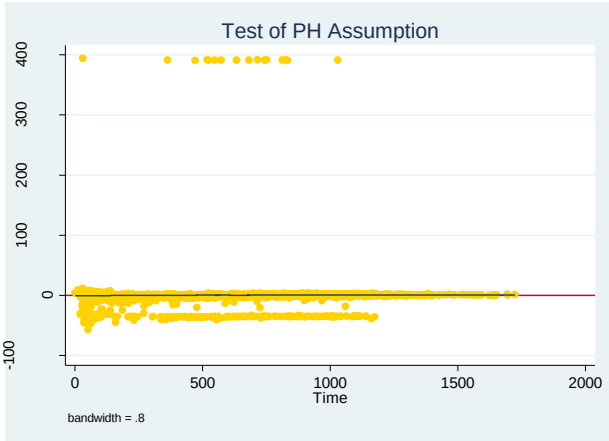

Test of PH Assumption
Time
bandwidth = .8
Test of PH Assumption
Time
bandwidth = .8

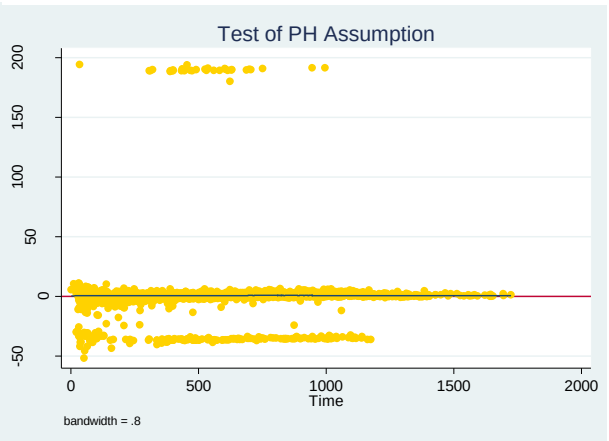

Test of PH Assumption
Time
bandwidth = .8
Test of PH Assumption
Time
bandwidth = .8

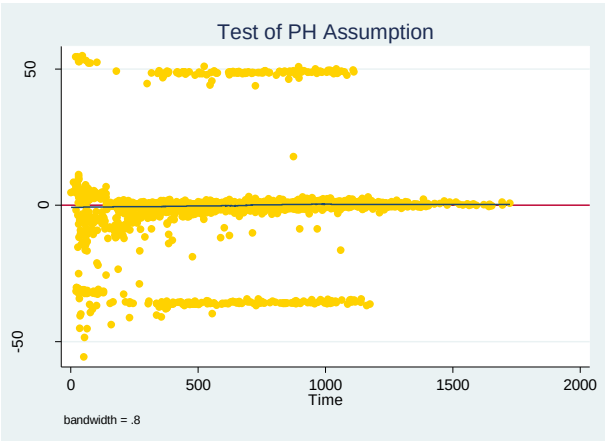

Test of PH Assumption
Time
bandwidth = .8
Test of PH Assumption
Time
bandwidth = .8

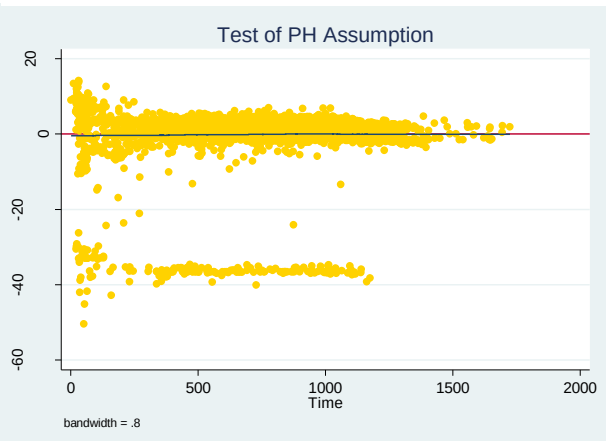

Test of PH Assumption
40
20
0
-20
-40
-60
0
500
1000
1500
2000
Time
bandwidth = .8

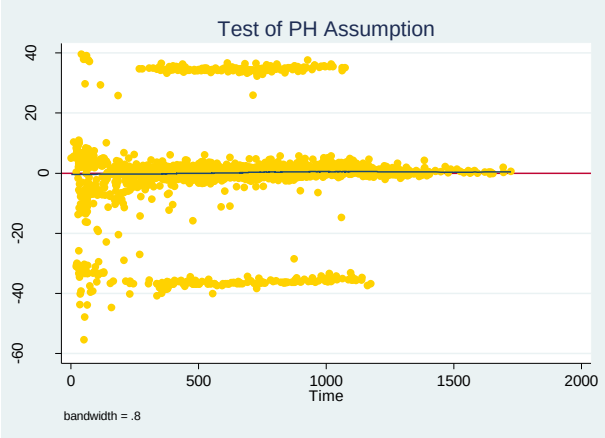

Test of PH Assumption
20
0
-20
-40
-60
0
500
1000
1500
2000
Time
bandwidth = .8

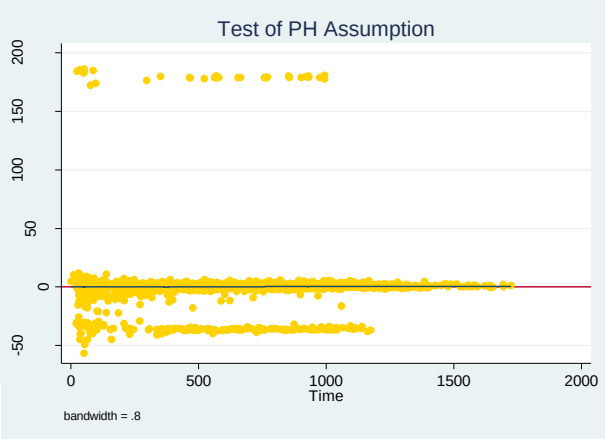

Test of PH Assumption
400
300
200
100
0
-100
0
500
1000
1500
2000
Time
bandwidth = .8

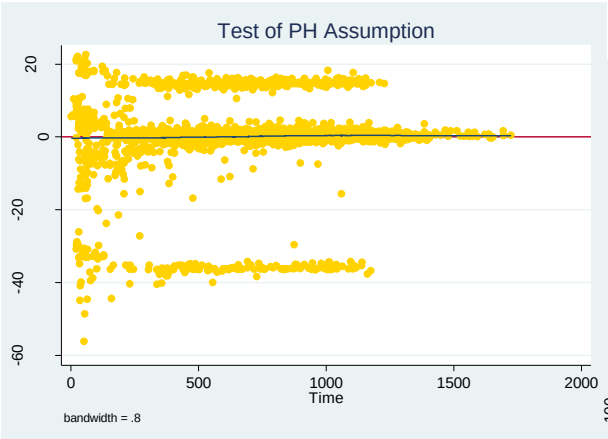

Test of PH Assumption
200
150
100
50
0
-50
0
500
1000
1500
2000
Time
bandwidth = .8

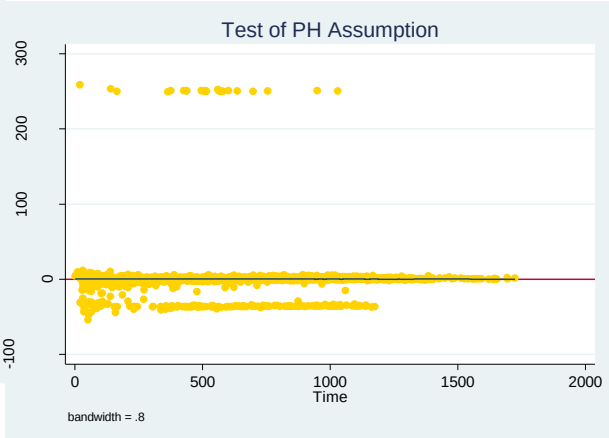

Test of PH Assumption
50
0
-50
0
500
1000
1500
2000
Time
bandwidth = .8

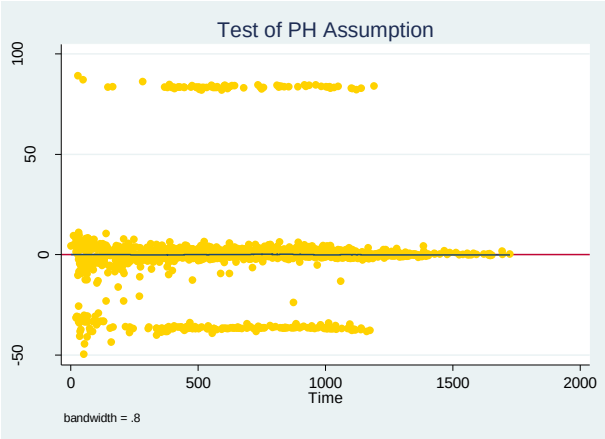

Test of PH Assumption
20
0
-20
-40
-60
0
500
1000
1500
2000
Time
bandwidth = .8

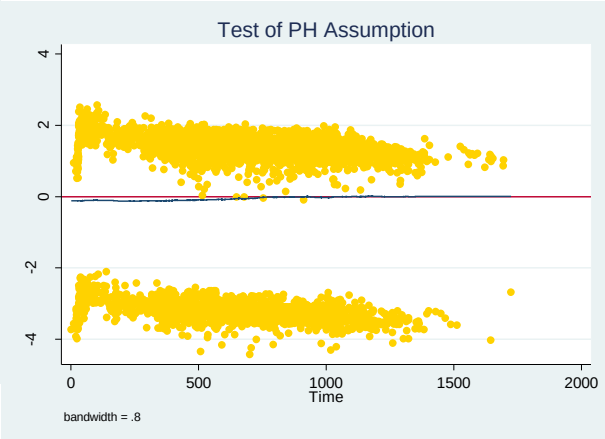

Test of PH Assumption
40
20
0
-20
-40
-60
0
500
1000
1500
2000
Time
bandwidth = .8


Test of PH Assumption
200
150
100
50
0
-50
0
500
1000
1500
2000
Time
bandwidth = .8

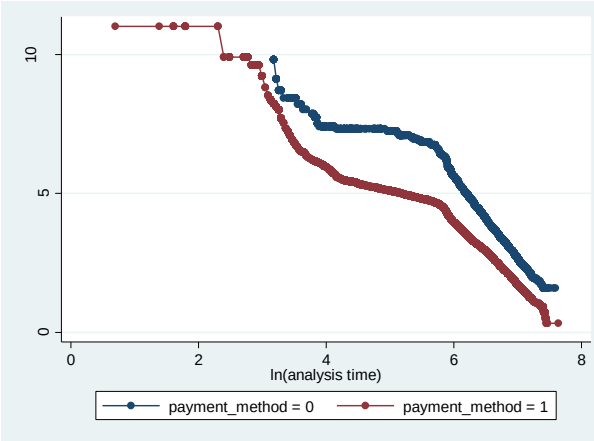

Test of PH Assumption
20
0
-20
-40
-60
0
500
1000
1500
2000
Time
bandwidth = .8

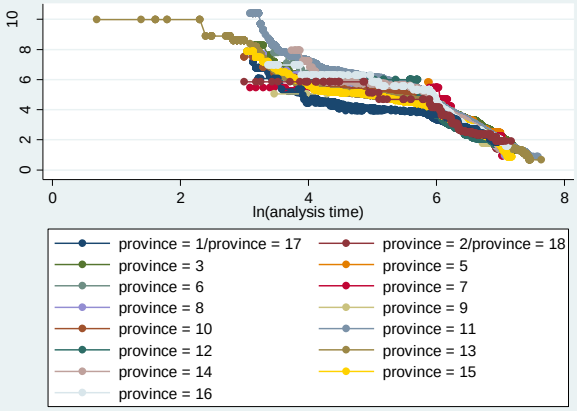

Test of PH Assumption
300
200
100
0
-100
0
500
1000
1500
2000
Time
bandwidth = .8

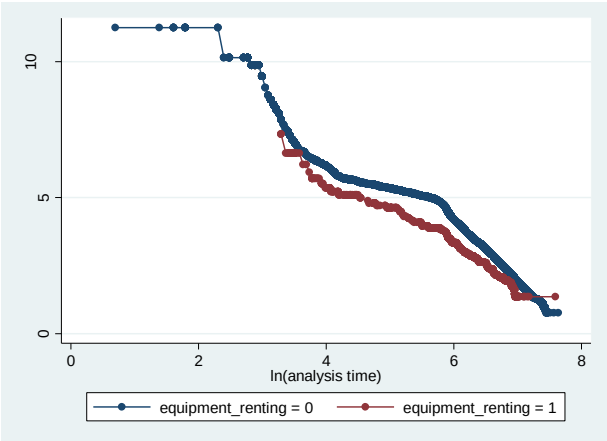

Test of PH Assumption
100
50
0
-50
0
500
1000
1500
2000
Time
bandwidth = .8

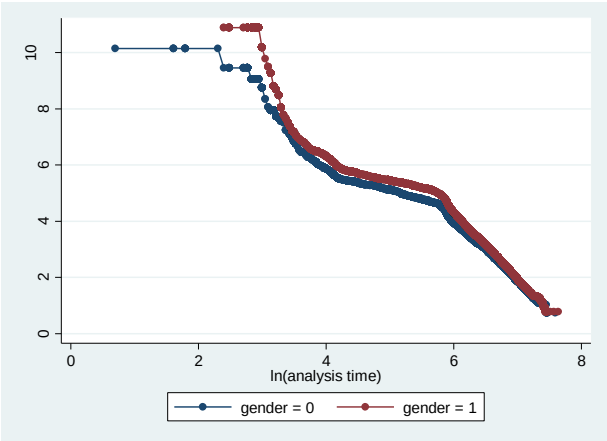

Test of PH Assumption
4
2
0
-2
-4
0
500
1000
1500
2000
Time
bandwidth = .8

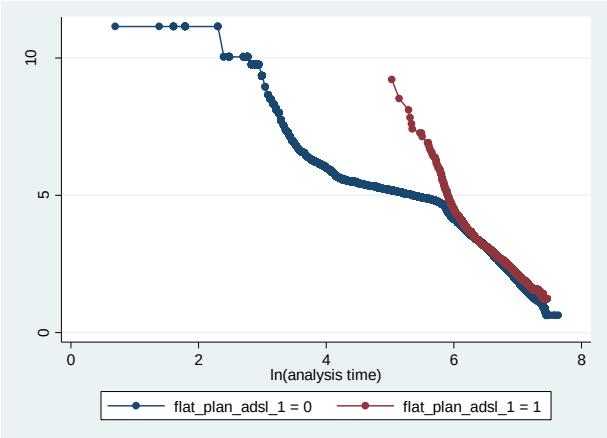
Appendix G - Graphs of $-\ln\left\{-\ln\left[\hat{S}(t)\right]\right\}$ against $\ln(t)$ (ADSL)



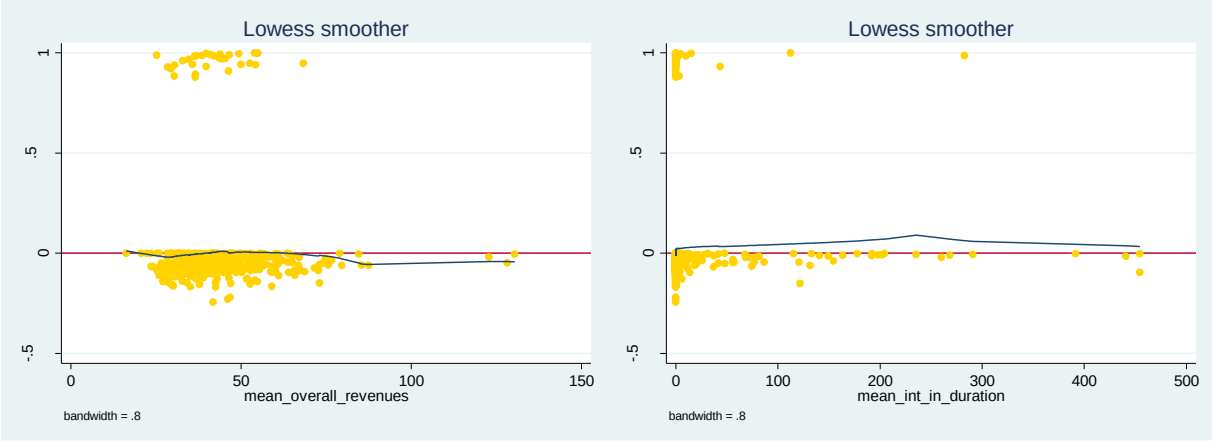


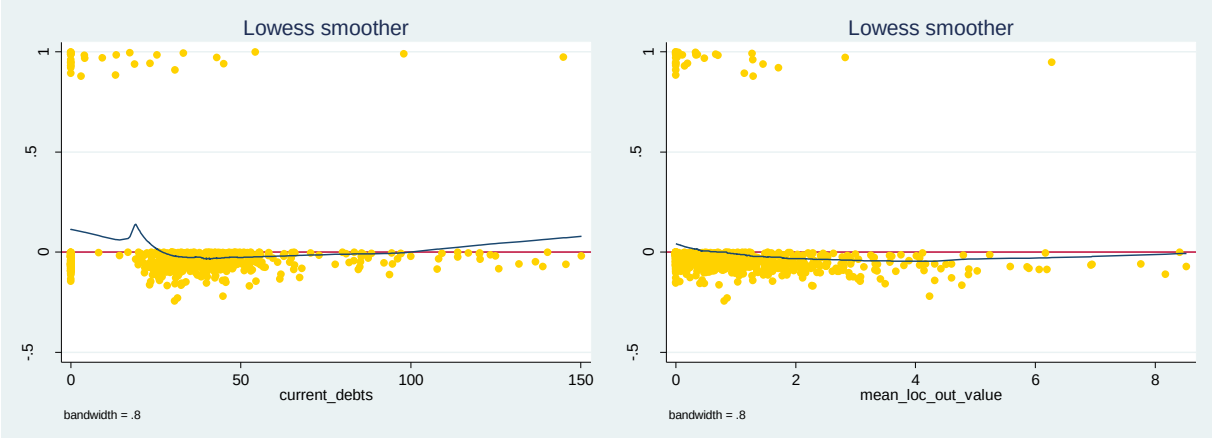


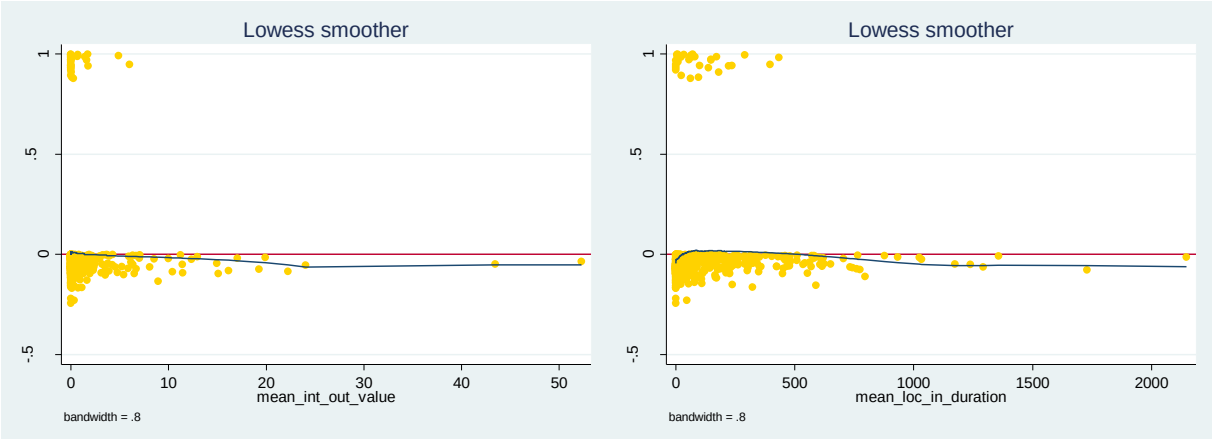


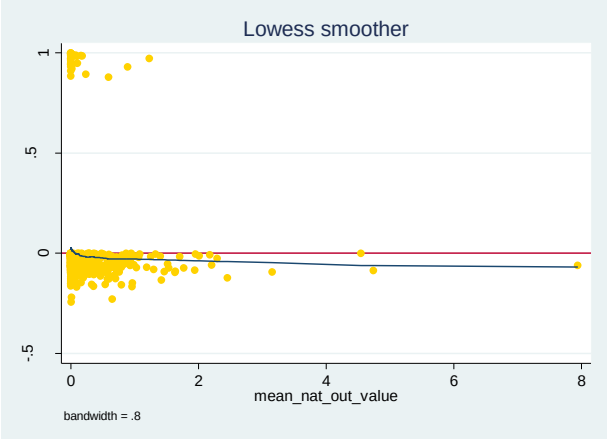


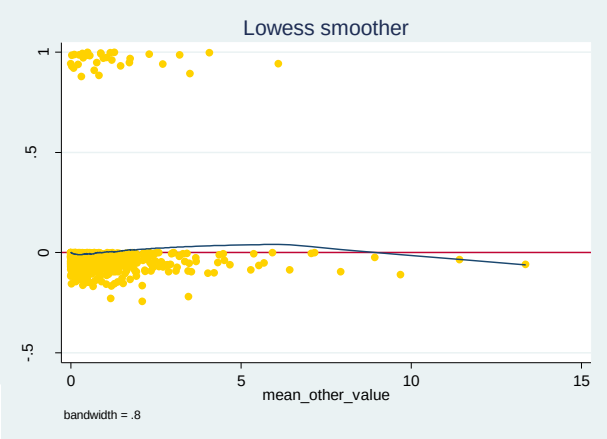
Appendix H - Analysis of the functional form of covariates (fixed-telephone) (with satisfaction level)

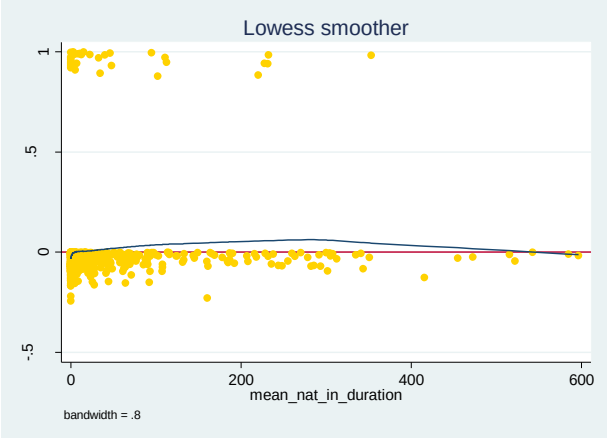


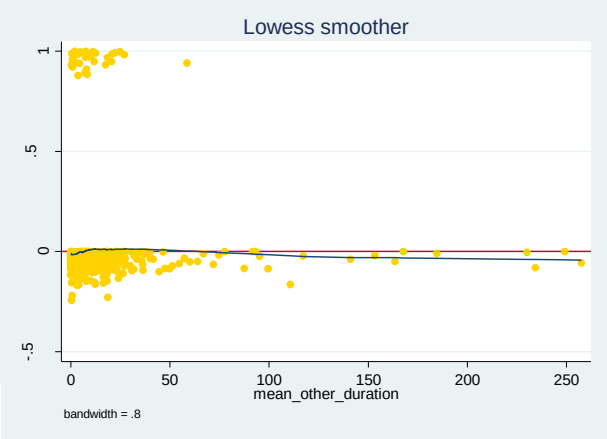


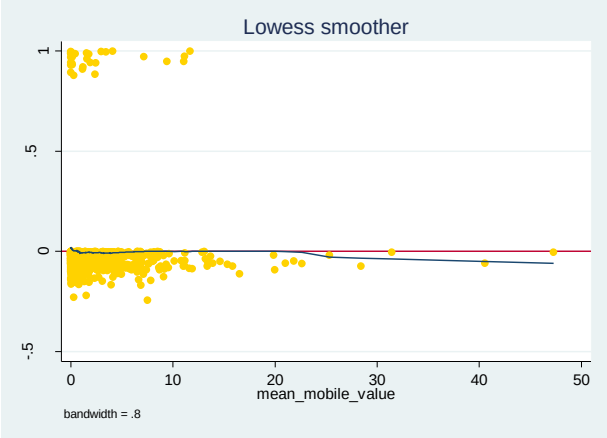


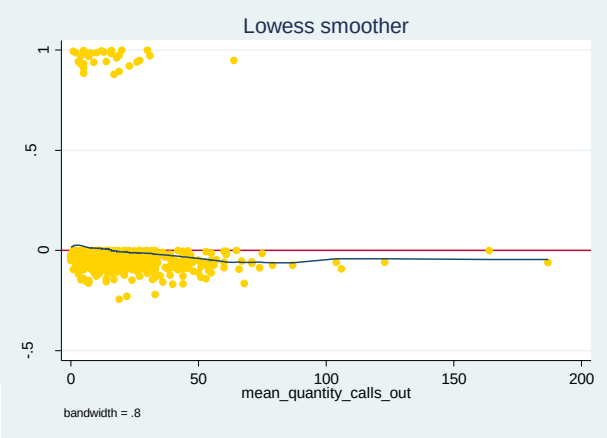




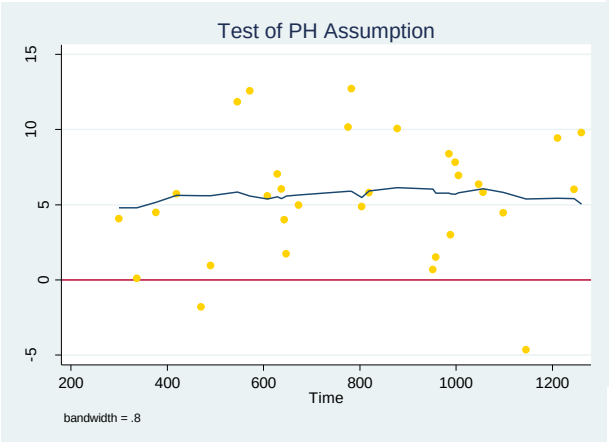


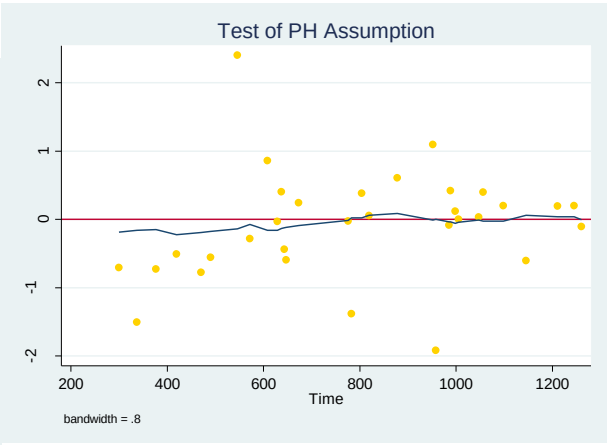


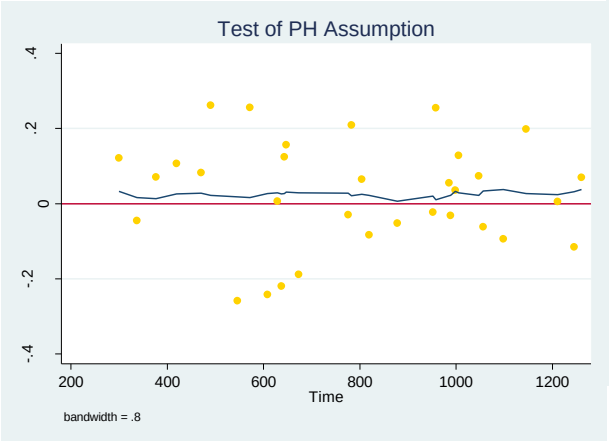


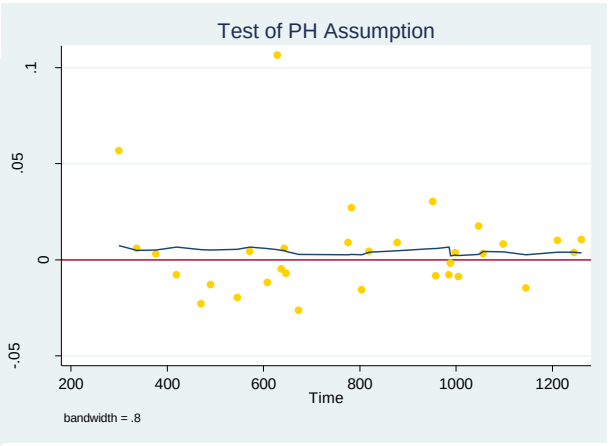


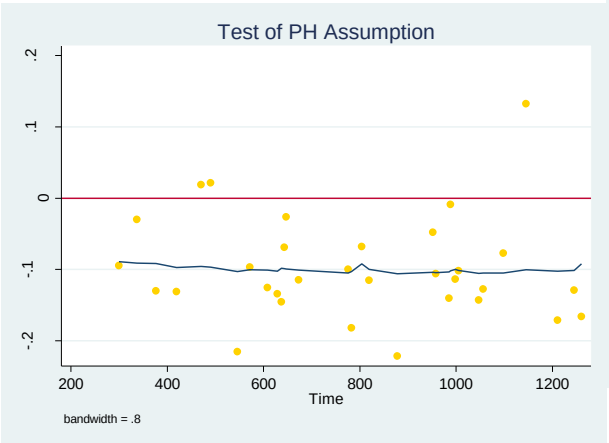
Appendix I – Graphs of Schoenfeld residuals (fixed-telephone) (with satisfaction level)

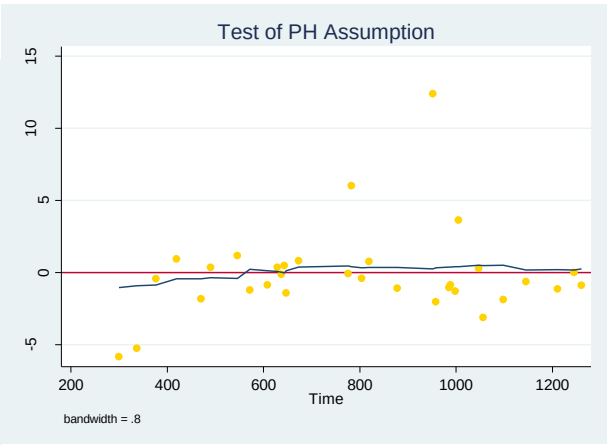

Test of PH Assumption
15
10
5
0
-5
200
400
600
800
1000
1200
Time
bandwidth = .8

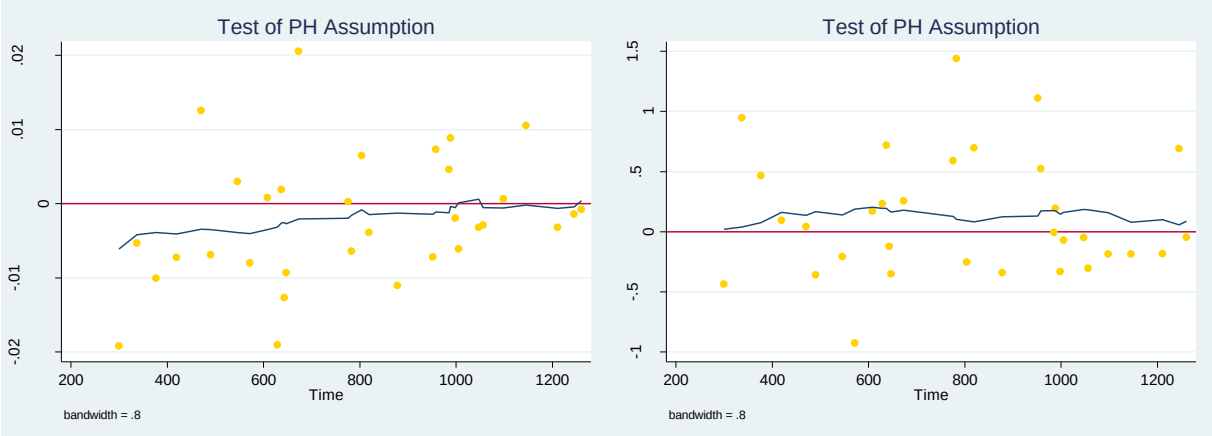

Test of PH Assumption
2
1
0
-1
-2
200
400
600
800
1000
1200
Time
bandwidth = .8

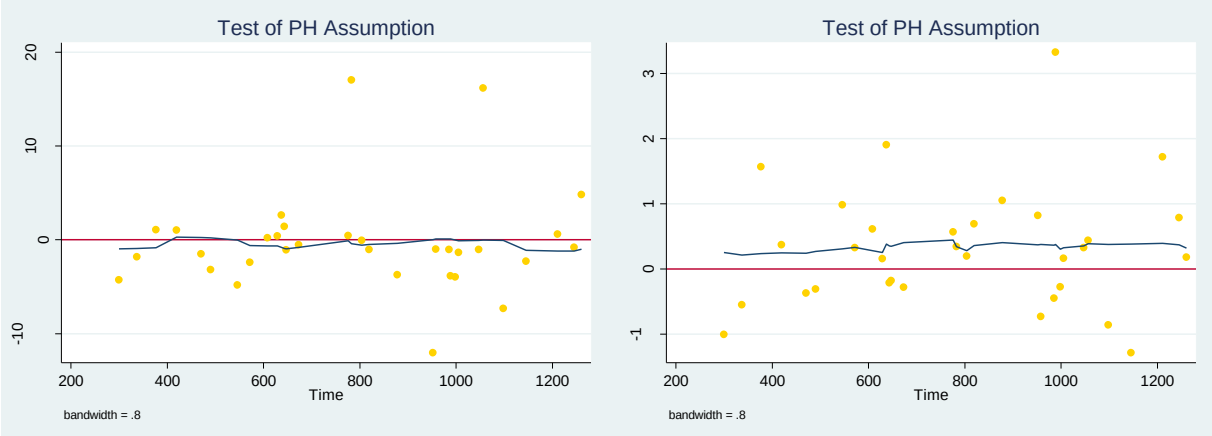

Test of PH Assumption
.4
.2
0
-.2
-.4
200
400
600
800
1000
1200
Time
bandwidth = .8

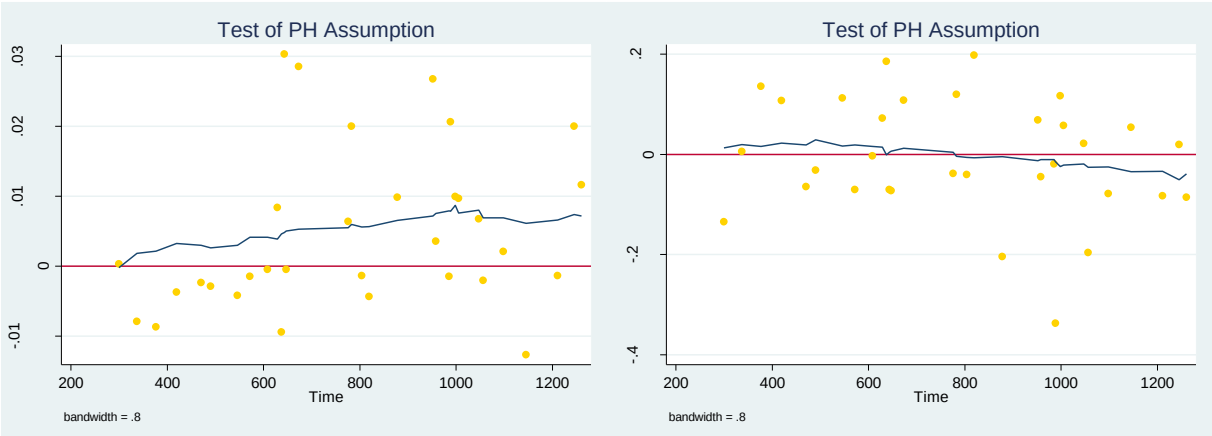

Test of PH Assumption
.1
.05
0
-.05
200
400
600
800
1000
1200
Time
bandwidth = .8

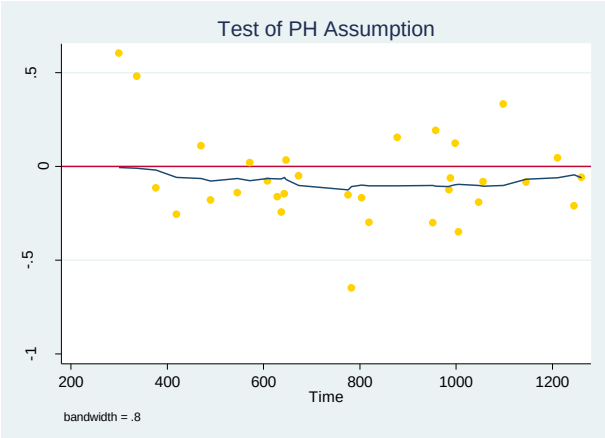

Test of PH Assumption
.2
.1
0
-.1
-.2
200
400
600
800
1000
1200
Time
bandwidth = .8

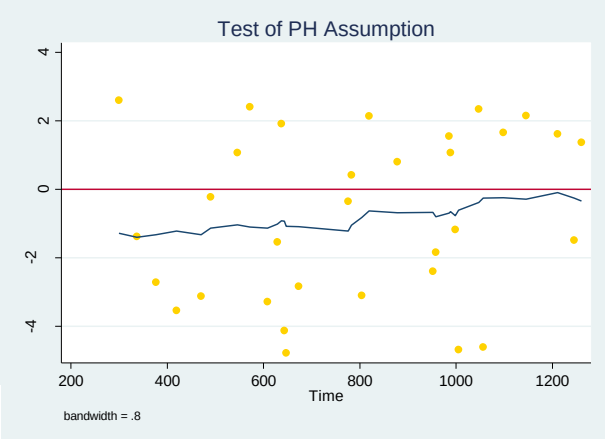

Test of PH Assumption
15
10
5
0
-5
200
400
600
800
1000
1200
Time
bandwidth = .8

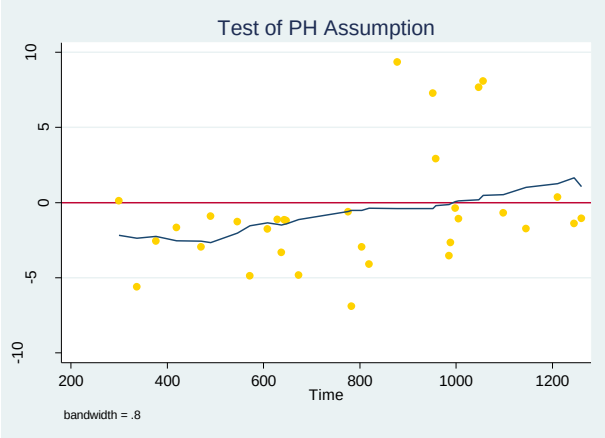

Test of PH Assumption
.02
.01
0
-.01
-.02
200
400
600
800
1000
1200
Time
bandwidth = .8
Test of PH Assumption
1.5
1
.5
0
-.5
-1
200
400
600
800
1000
1200
Time
bandwidth = .8

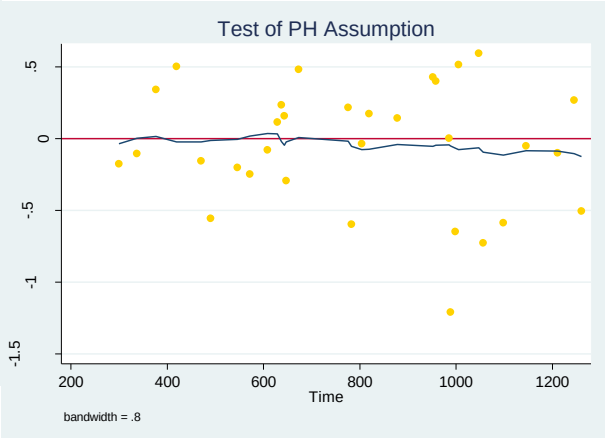

Test of PH Assumption
20
10
0
-10
200
400
600
800
1000
1200
Time
bandwidth = .8
Test of PH Assumption
3
2
1
0
-1
200
400
600
800
1000
1200
Time
bandwidth = .8

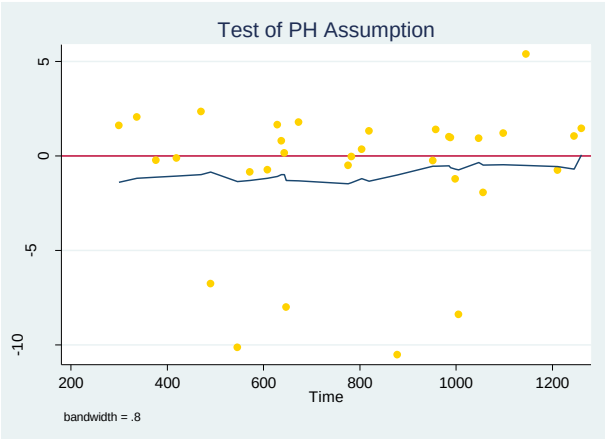

Test of PH Assumption
.03
.02
.01
0
-.01
200
400
600
800
1000
1200
Time
bandwidth = .8
Test of PH Assumption
.2
0
-.2
-.4
200
400
600
800
1000
1200
Time
bandwidth = .8


Test of PH Assumption
.5
0
-.5
-1
200
400
600
800
1000
1200
Time
bandwidth = .8

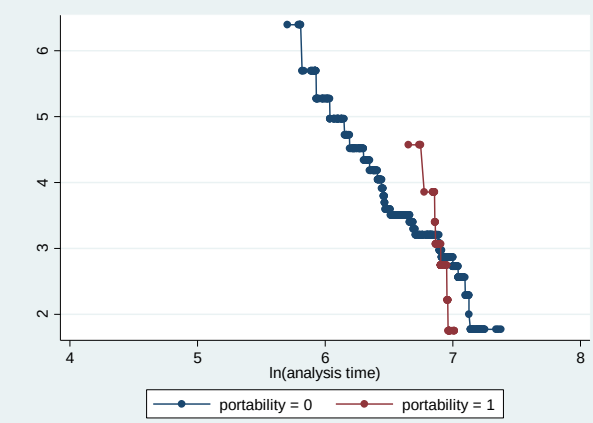

Test of PH Assumption
4
2
0
-2
-4
200
400
600
800
1000
1200
Time
bandwidth = .8

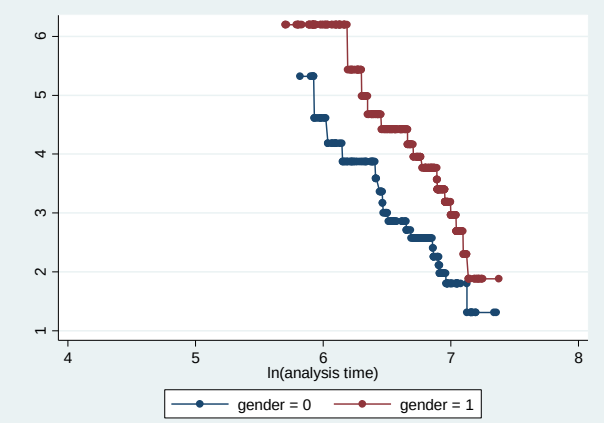

Test of PH Assumption
10
5
0
-5
-10
200
400
600
800
1000
1200
Time
bandwidth = .8

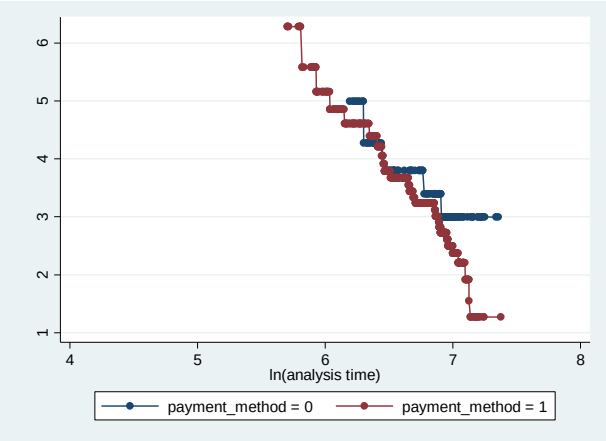

Test of PH Assumption
.5
0
-.5
-1
-1.5
200
400
600
800
1000
1200
Time
bandwidth = .8


Test of PH Assumption
5
0
-5
-10
200
400
600
800
1000
1200
Time
bandwidth = .8

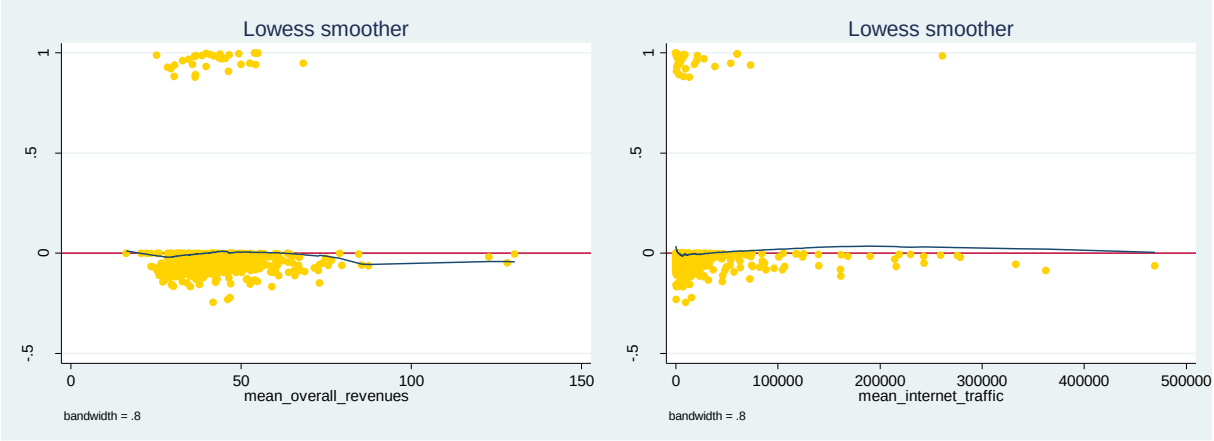
Appendix J – Graphs of $-\ln\left\{-\ln\left[\hat{S}(t)\right]\right\}$ against $\ln(t)$ (fixed-telephone) (with satisfaction level)

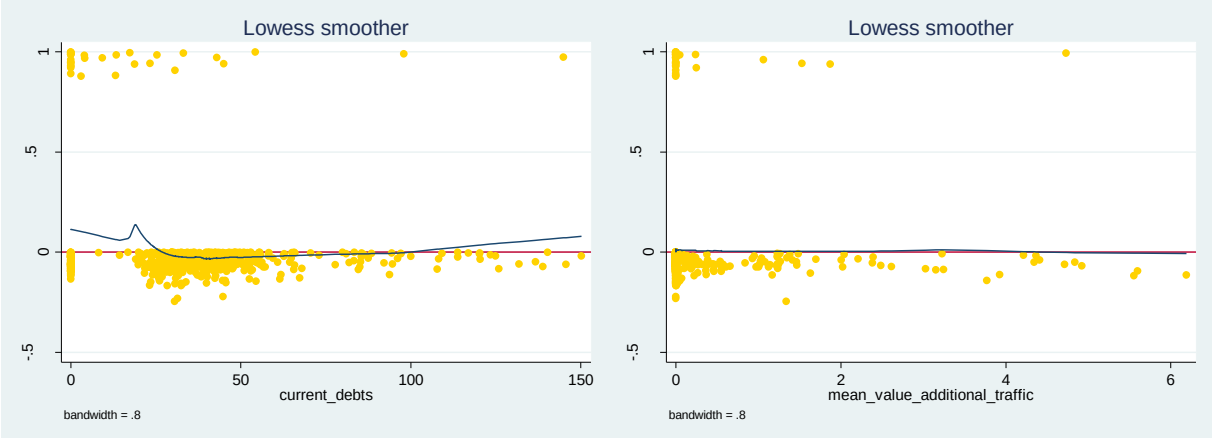




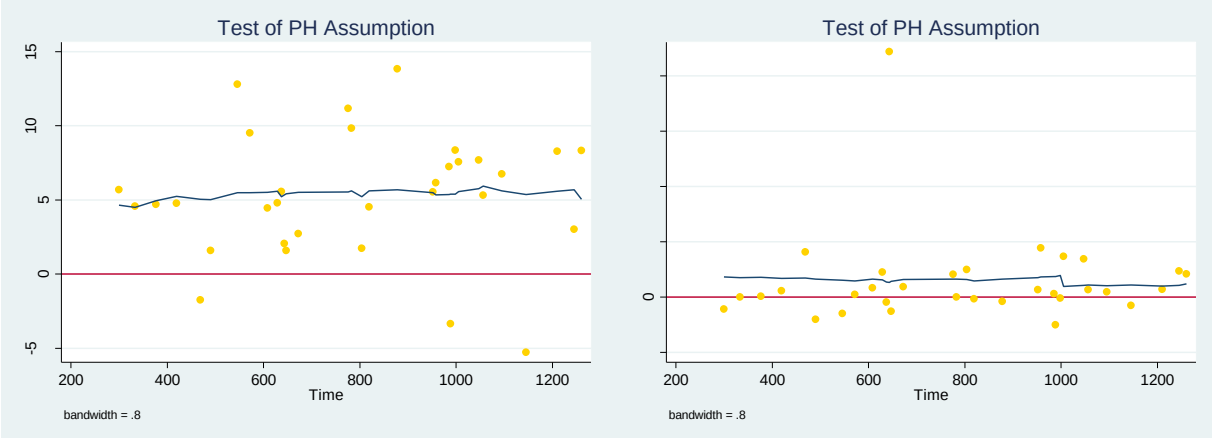


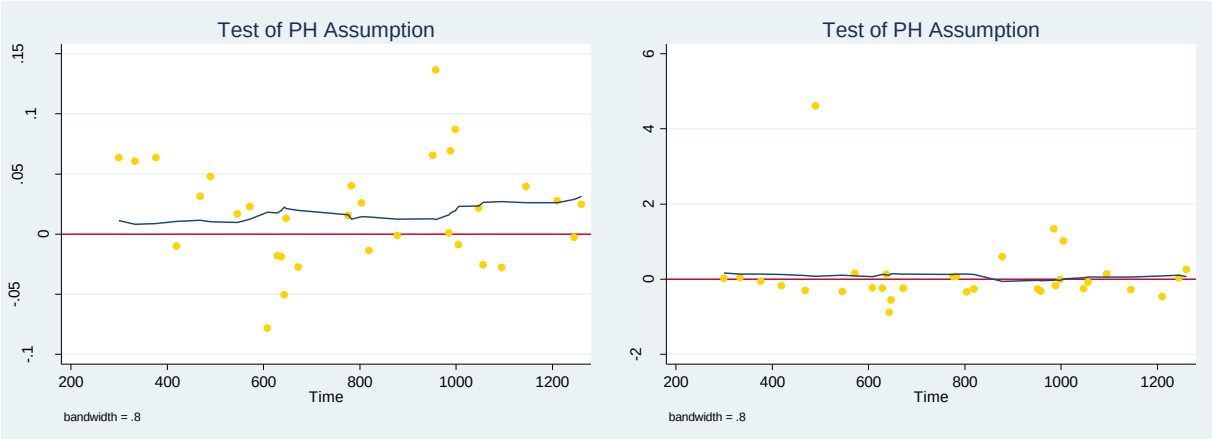
Appendix K - Analysis of the functional form of covariates (ADSL) (with satisfaction level)

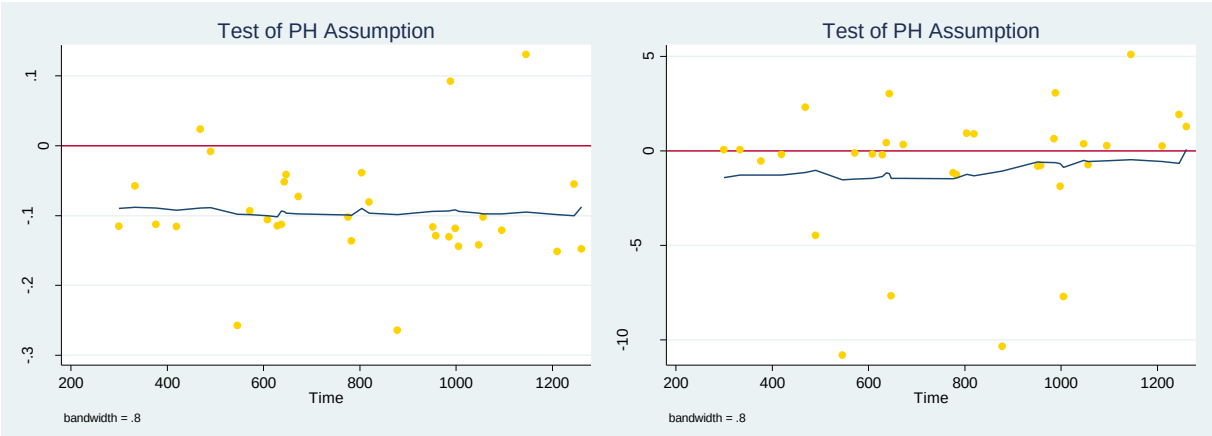


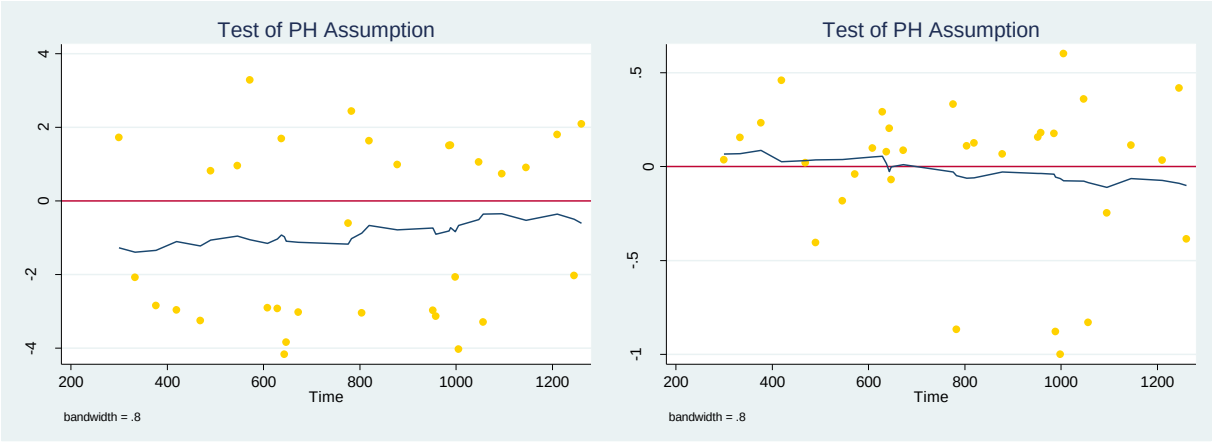


Appendix L – Graphs of Schoenfeld residuals (ADSL) (with satisfaction level)








Test of PH Assumption
4
2
0
-2
-4
200
400
600
800
1000
1200
Time
bandwidth = .8
Test of PH Assumption
.5
0
-.5
-1
200
400
600
800
1000
1200
Time
bandwidth = .8

Appendix M – Graphs of $-\ln\left\{-\ln\left[\hat{S}(t)\right]\right\}$ against $\ln(t)$ (ADSL) (with satisfaction level)