An Agent-Based Model of Civil Violence with Imprisonment Delay and Legitimacy Feedback

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Abstract — **Epstein's Agent-Based model of civil violence has been very successful due to its simplicity and explanatory power, but does not represent important phenomena, such as processes operating at multiple scales and feedback mechanisms. In this work, we present an extension of Epstein's model that includes the effects of imprisonment delay, media coverage and feedback of rebellion bursts of the government's legitimacy. These innovations are relevant for a more realistic modeling of the complex and path-dependent effect of protests and violent confrontations on the evolution of the social context. The resulting simulations showed punctuated equilibrium as in Epstein's model, but the violence bursts lasted longer and displayed more complicated structure and interdependence on previous events. The rebellion peaks lead to drops and lowering of the time-averaged value of the government's legitimacy.**

Keywords – Agent-Based model; Epstein's model; civil violence; complexity; social simulation

I. Introduction

The study of social conflict phenomena is an important and timely topic in social sciences and an active area of research in social simulation studies. Such phenomena are extremely heterogeneous and varied and there is no generally accepted classification. Fig. 1 shows a qualitative classification of social conflict phenomena using intensity as a criterion, together with present-day examples of each type and a reference to the disciplines in which those phenomena are studied [1].

The time evolution of social conflict is a very complex and path-dependent process, involving phenomena and events with different scales (in terms of time, space and proportion of agents involved) and multiple interactions, such as provided by formal media and widespread access to information and communication technologies (ICT) and social networks (SN) (see for instance [2], [3] or [4]). Social context variables and possibly external influences can trigger events such as large protests, which in turn change the social context. This process may lead to a stable or unstable evolution, as sketched in Fig. 2. This evolution may be gradual (escalation of tension or violence) or sudden (revolution, or outbreak of insurgence or war) [5], [6]. Understanding these complicated mechanisms and if possible anticipate or control the evolution of these processes for different situations, is a problem of practical importance for the state's purposes.

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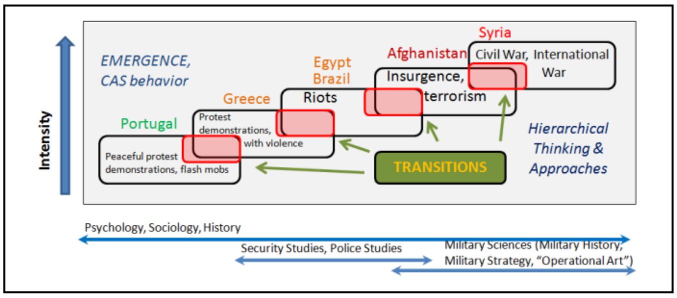

Figure 1. Qualitative classification of social conflict phenomena based on the intensity (or level of violence). Transition between different types of phenomena is represented by red rectangles.

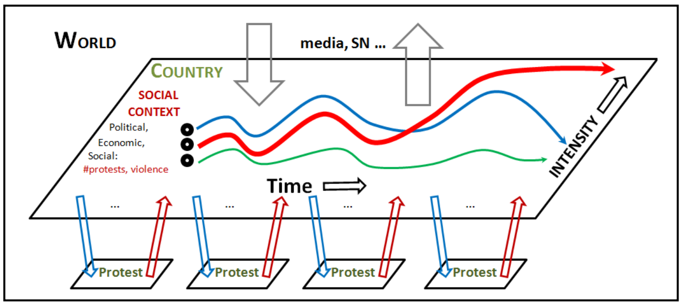

Figure 2. Evolution of the intensity of social conflict viewed as a complex and path-dependent process with micro-macro and feedback links.

Although the study of classical warfare (military strategy, operational art and tactics) often uses mechanistic, analytic and hierarchical models of thinking, the phenomena mentioned in Fig. 1 can be considered as emergent properties in a complex adaptive system and studied using ABM. A review of existing ABM for the simulation of several distinct types of social conflict manifestations (civil violence [7], [8], [9], worker protest [10], riots [11], revolution [12], urban crime [13], and guerrilla/insurgence warfare [14], [15]) and some guidelines for possible improvement of these models can be found in [1].

Epstein [7], [8] introduced a very successful and popular ABM of civil violence for simulation of rebellion against a central authority (Model I), or violence between two rival groups (ethnic violence) mediated by a central authority

(Model II), in an artificial society with two types of agents: citizens (called 'agents' in [7] and [8]) and cops. In Model I citizens may be 'Quiet' or 'Active' depending on their internal state and the number of other actives and cops within their 'vision radius', while cops arrest active citizens within their 'vision radius'. In Model II there are two groups of citizens (Blue and Green) that can turn 'active'; active citizens try to kill one citizen of the other group and cops try to arrest active citizens. Arrested (jailed) citizens are removed from the simulation space for a certain number of time cycles (jail term), which introduces a memory effect in the system.

The strength of Epstein's model lies in its simplicity (only two types of reactive agents with two simple rules for each agent type), the relevance of the variables chosen (legitimacy, grievance, hardship, risk aversion) and its explanatory power. For certain combinations of agents' attributes and global variables, this model produces simulations that show many typical characteristics of civil violence processes, such as intermittent bursts of violence (Model I) and safe havens in peace keeping (Model II). Epstein's model has been extended and refined in several ways by different authors (see [1] for more details). However, the model also has significant limitations. For instance, the space and time scales are indefinite, the agents' movements are random instead of purposeful, the effects of media coverage are not considered and short-term feedback of violence bursts on legitimacy are not considered in the formulation of Model I. Jager, Popping and van de Sande [16] introduced an ABM for simulating the fighting between two rival groups focused on small-scale processes. In this model the agents' state and movement is determined by their aggressiveness and by the number of nearby agents of each group. Fights between agents are represented by immobilizing fighting agents for a certain number of cycles. The model described in [16] can represent features such as localized clusters of fighting agents but does not apply to conflict against an authority (no police agents) and does not consider social context variables (such as legitimacy).

In this work, we present an extension of Epstein's Model I that includes: *i*) a time delay for imprisonment, which simulates a fight prior to one arrest; *ii*) media agents that seek to locate, record and publish episodes of violence (fights) between 'Active' citizens and cops; and *iii*) a feedback mechanism for varying the legitimacy as a function of the number of arrests and violent episodes registered by media agents. This extension is a first step towards the inclusion of small time-scale effects (imprisonment delay) and endogenous legitimacy feedback, to better represent the path-dependent process sketched in Fig. 2 with Epstein-type ABM.

The rest of this paper is organized as follows. In section II, the extension of Epstein's Model I is described. Section III contains a description of the test matrix of the model parameters used in the simulations and some representative results, followed by a discussion (section IV). In section V, the conclusions are presented, together with a reference to other developments in progress. It was found that the ensuing dynamics is more complicated than obtained with Epstein's original model, due to the interaction between processes with different time scales (fight duration and jail term) and the feedback caused by the action of media and variations of legitimacy.

II. Model Description

In this section, we present a general description of the ABM developed, including an overview, the agents' description, the environment (simulation space) and the global (exogenous) variables. The model description closely follows that in [7] and [8]. The implementation was done in NetLogo, using the "Rebellion" NetLogo library model example [17] as the starting point.

A. *Synopsis*

The "Overview, Design Concepts and Details" (ODD) protocol [18] is a popular and useful method for describing ABM, but due to space limitations full compliance with the ODD specification is unfeasible in the present paper. Therefore, a simplified description is shown in Table 1.

B. *Class Diagram of Agents' Description*

Fig. 3 shows the class diagram for the global variables (defined by the 'Observer' agent in NetLogo) and the three types of agents included in the present model. The Citizen, Cop and Media agents are reactive and behave according to two simple rules that are implemented in the two methods shown in the diagram.

TABLE I. Simplified ODD Description of the Extended Epstein Model I ABM of civil Violence

ODD item	**Description**
Purpose	ABM of decentralized upheaval/civil violence in a global artificial society; extend Epstein's Model I by adding two time scales (fighting duration, event memory decay) and media coverage effects
Entities	3 types of reactive agents – citizens, cops, media – with one' move' and one 'behave' rule for each type
Time cycle	All agents activated once per period in random order
Model results	Long term behavior can be equilibrium with random fluctuations or intermittent bursts of generalized rebellion; long term behavior and duration of rebellion peaks depends on the ratio between fight duration and jail term; media coverage induces legitimacy drops after each larger burst and decreases the time-averaged legitimacy, but without overall negative trend.
Strengths & weaknesses	Strengths: inclusion of additional processes operating at different time scales in Epstein's model produces richer dynamics Weaknesses: lack of detailed representation of micro processes (more actions such as shouting, fighting, escaping, etc., small group dynamics; and purposeful movements) and environment features (attraction and repulsion points); need for improved modeling of legitimacy variations

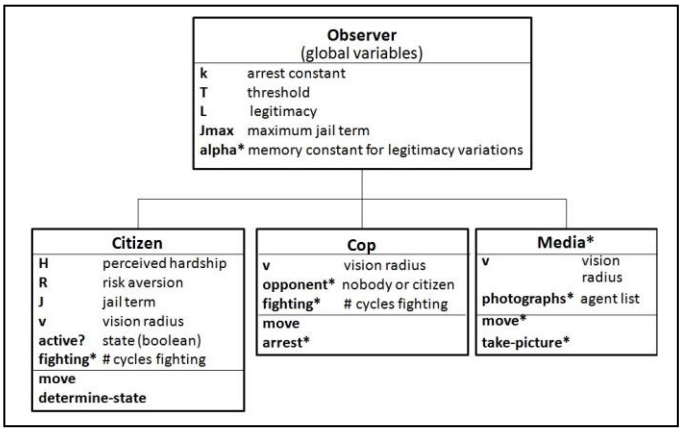

Figure 3. Class diagram of the Observer, Citizen, Cop and Media agents in the NetLogo implementation. The agents' attributes and methods that result from extensions to Epstein's Model I formulation are marked by an asterisc.

C. Citizen Agent Specification

Like in Epstein's Model I specification, Citizen agents[1] can be 'Active' of 'Quiet', and their behavior (state) is determined according to the following rule: if $G - N > T$ be 'Active', otherwise be 'Quiet', where $G = H \cdot (1 - L)$ is the level of grievance, $N = R{\cdot}P$ is the net risk perception, T (constant exogenous variable) is a threshold, $H \sim$ U(0,1) is the (endogenous) perceived hardship, $L \in [0,1]$ is the "perceived government legitimacy", $R \sim$ U(0,1) is the (endogenous) risk aversion, and P is the estimated arrest probability given by the expression

$$P = 1 - \exp(- k \lfloor C/(A+1) \rfloor_v)\,, \qquad (1)$$

in which k is a constant and C and A are the number of active citizens and cops within the vision radius v. The denominator is A+1 because the citizen always counts itself as 'Active' when estimating the arrest probability, which also avoids divisions by zero. For k = 2.3 (as suggested in [7]) this expression leads to a sudden drop of the arrest probability from 90% to zero when the number of 'Active' citizens equals the number of cops within the vision radius. If the arrest probability function does not have this property (which in [13] is called "irrationality"), intermittent bursts of rebellion do not occur in Epstein's model.

In our model, Citizen agents that are not fighting (fighting = 0) move to a random empty site within their vision radius, if they are not in the 'Fighting' or 'Jailed' condition. 'Fighting' citizens are immobilized for a user-specified fight-duration before they are arrested, after which they become jailed and are temporarily removed from the simulation space.

D. Cop Agent Specification

Cop agents that are not fighting (fighting = 0) randomly select one 'Active' Citizen agent within their vision radius if there is one. If they find one "suspect", they mark it as 'opponent' and start fighting for a user-specified number of cycles. During the fight, both the cop and its 'opponent' are immobilized, which creates an opportunity for media agents to record the episode, if the fight occurs within their vision radius when they are activated (see the specification of Media agents below). At the end of the fight, the 'Active' opponent is jailed for a jail term $J \sim$ U(0, J_{max}). If the user-defined fight duration is set to zero, the model is identical to Epstein's original Model I. This completes the specification of the Cops 'arrest' rule.

Cop agents move at random while searching for 'Active' citizens and occupy the site of the arrested ('opponent') citizen after this latter is jailed.

E. Media Agent Specification

Media agents look for fighting agents (citizens or cops) within their vision radius (that is, citizens or cops with the attribute variable fighting > 0) and take "photographs" of all fighting agents they may find, but retain only one "photograph" of each agent. These rule is a realistic representation of the behavior of field reporters: they take as many records of important events as they can but select the material for publication.

This 'take-picture' behavior is implemented in NetLogo by initializing the 'photographs' attribute variable of the Media agents as an empty list during the model setup. When the Media agent is activated, it scans all sites within its vision radius and builds a list of all fighting agents found. This list is appended to the 'photographs' agent list (which can be tough of as a kind of "roll film"). Duplicate agents are then removed using NetLogo's **remove-duplicates** primitive.

The 'move' rule for Media agents is different from the corresponding rule for citizens or cops, as we considered important to formulate some kind of purposeful motion for such agents. More specifically, Media agents move to the site $(x_m, y_m)_v$ within their vision radius that maximizes the following utility function:

$$(x_m, y_m)_v = \max(5{\cdot}F_{AC}\big|_v + 2{\cdot}M_{AC}\big|_v + A\big|_v + C\big|_v - \tfrac{1}{2} \cdot Q\big|_v) \qquad (2)$$

in which F_{AC} is the number of Citizen and Cops agents that are fighting, M_{AC} is the number of Media agents, A is the number of 'Active' citizens, C is the number of cops, and Q is the number of 'Quiet' citizens. With this utility function, Media agents have the greatest payoff by approaching fighting agents, but are also attracted by other Media agents and local concentrations of 'Active' citizens and cops. They also have a slight incentive to avoid wasting time with uninteresting concentrations of 'Quiet' citizens. Note that if the fight duration is set to zero, Media agents still wander purposefully but the feedback on the legitimacy will not take media coverage into account.

F. Formulation of the Legitimacy Feedback Mechanism

Government legitimacy is a key variable for determining the dynamics of rebellion bursts, but it is widely recognized that the intensity, duration and interval between protests of violent upheavals also influences legitimacy. This is particularly evident when testimony of these events is published by formal media or is spread in SN. In Epstein's exploration of Model I, the effects of sudden and gradual changes in the legitimacy are discussed but the feedback mechanism mentioned here is not considered, although in the Appendix B of [7] the authors suggest a possible way of

[1] In Epstein's model description, 'Citizens' are called 'Agents'.

incorporating endogenous legitimacy variations in Model II (ethnic violence between two rival groups).

In this work, we devised a simple and plausible mechanism for updating the legitimacy for the next time cycle as a function of the number of arrests and number of fights recorded by Media agents in the current time cycle. The number of arrests and recorded fights is used to first compute a "legitimacy drop", which is multiplied by a time-attenuation factor and combined with the value of the 'government-legitimacy' set as input variable to determine the legitimacy for the next time cycle. This mechanism is implemented using the following formulae:

$$L^* = L_t - (Narrests_t/Ncitizens) - Af \cdot (Nfights_t/Ncitizens) \quad (3)$$

$$\Delta L = (L^* - L_0) \cdot \exp(-\alpha \cdot \Delta t) \quad (4)$$

$$L_{t+1} = \max(0, \min(L_0 + \Delta L, 1)) \quad (5)$$

where L_0 is the government legitimacy set as global variable (as in Epstein's model), $Narrests_t$ and $Nfights_t$ are the number of arrests and recorded fights at time t, A_f is a "Media audience factor" and α is a "memory constant" that allows for slower of faster decay of the legitimacy drop due to arrests and fights in the subsequent time cycles. For simplicity, A_f is set to the number of sites within vision radius, to avoid introducing another parameter. This formulation allows for a decay of legitimacy during violent outbursts and for the memory fading from past events published in the "evening news". Eq. (5) ensures that the legitimacy remains in the interval [0,1].

III. RESULTS

Starting from the input data of Run 2 described [7] (a well documented case with punctuated equilibrium), we performed several experiments to study the effect of the introduction of new processes on the model's behavior. Table II shows the relevant parameters used in the tests. The purpose of the E-experiments was to evaluate the influence of delayed imprisonment and legitimacy feedback with respect to Epstein's Run 2. In the F-experiments we studied the influence of the memory decay constant on the legitimacy feedback, and in the G-experiments we investigated the influence of the fight duration on the solutions, keeping the maximum jail term constant (60 cycles).

TABLE II. TABLE OF MODEL PARAMETERS USED IN THE MODEL TESTS

Run	Jmax	Fight duration	Legitimacy feedback	α
E1	30	0	No	-
E2	30	1	No	0.5
E3	30	1	Yes	0.5
F1	60	1	Yes	1.0
F2	60	1	Yes	0.5
F3	60	1	Yes	0.25
F4	60	1	Yes	0.125
G1	60	3	Yes	0.5
G2	60	6	Yes	0.5

Note that our Run E1 is Epstein's Run 2 and that Run F2 is also related to Runs G1 and G2. In all tests, we introduced five media agents and the model was run for 2500 time steps.

Table III shows the summary of results. From experiments E2 and E3, it can be concluded that the introduction of delayed imprisonment with no legitimacy feedback increased the duration of the rebellion bursts and the average numbers of 'Active' and 'Jailed' citizens with respect to Epstein's Run 2, but further introduction of legitimacy feedback changes the long-term behavior. In the F-experiments, the jail term was doubled, and the model produced solutions with punctuated equilibrium as in Epstein's model, but the bursts lasted longer due to imprisonment delay, and had more complex dynamics showing the effect of individual fights on progressive suppression of the rebellion bursts.

The decrease of the α constant, associated with higher decay times of the legitimacy drops (greater latency due to previous events), lead to an increase of the average number of rebellious and jailed agents, and to a decrease of the intensity of rebellion peaks (higher average level of conflict, smaller fluctuations).

Fig. 4 shows a snapshot of the simulation space obtained in Run F2, just before the suppression of one rebellion burst. This figure shows all types of agents in all possible states and it can be observed that 'Media' agents are indeed at favorable positions to capture fights and tend to avoid "uninteresting" areas in the simulation space.

Fig. 5 shows the time history of the number of 'Quiet', 'Active', 'Fighting' and 'Jailed' citizens for part of Run F2. It can be observed that the computed solution displays large rebellion bursts with periods of calm (punctuated equilibrium), like those documented in [7] and [8], but the longer jail term in combination with the two additional processes (imprisonment delay and legitimacy feedback) leads to longer bursts with more complicated time fluctuations (due to fights), shorter waiting times and higher average number of jailed agents that for example in Epstein's Run 2 simulation (see Table III).

TABLE III. SUMMARY OF SIMULATION RESULTS

Run	Average $N_{actives}$	Average N_{jailed}	Average N_{quiet}	Av. burst duration	Long-term behavior
E1	42	245	833	7	Rebellion bursts
E2	86	362	672	24	Rebellion bursts
E3	267	431	422	-	Equilibrium, high $N_{actives}$ and N_{jailed}
F1	62	432	626	11	Rebellion bursts
F2	83	545	492	14	Rebellion bursts
F3	94	652	374	16	Rebellion bursts, peaks merging, high N_{jailed}
F4	92	782	246	20	70% of citizens jailed
G1	246	451	423	-	Equilibrium, rebellion
G2	416	257	447	-	Equilibrium, rebellion

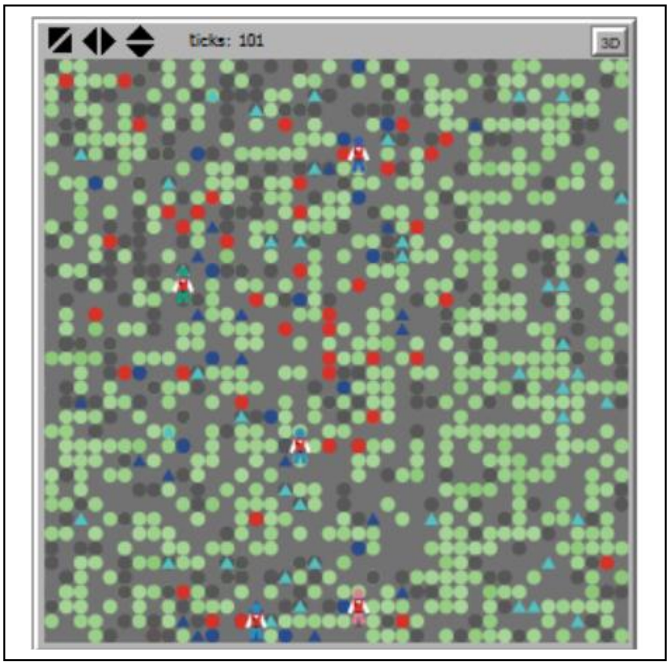

Figure 4. Snapshot of the simulation space obtained in Run F2. 'Citizen' agents are marked by circles, red if 'Active', blue if 'Fighting' and green if 'Quiet'. 'Cop' agents are marked by triangles, blue if 'Fighting', cyan if searching for rebellious citizens. 'Media' agents are shown as little figures with jacket.

Fig. 6 shows the time history of the legitimacy, number of arrests and number of 'photographs' taken by media agents in one time cycle, during part of Run F2. It can be observed that the feedback mechanism behaves as expected, in that arrests and media coverage of fights lead to legitimacy drops, but in the absence of new arrests or violent episodes registered by media it recovers to the default (user-specified) government legitimacy. Although the number of media photographs per cycle (barely visible in Fig. 6) is much smaller than the number of arrests per cycle, the influence of media coverage is clearly detectable by the correlation between legitimacy fluctuations and number of pictures. The time-averaged value of the legitimacy was 0.76, with an average legitimacy drop of 0.06 relative to the input value (0.82, as in Epstein's Run 2).

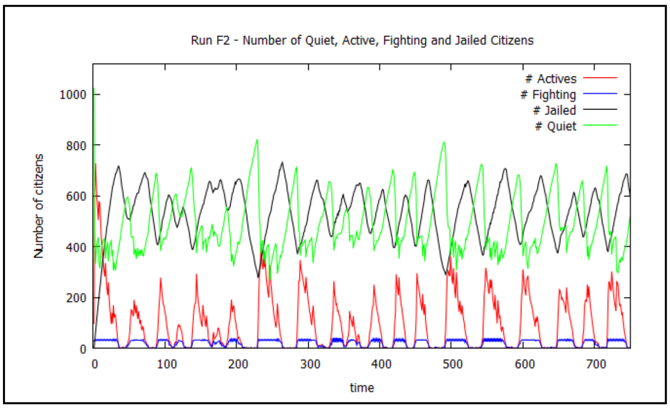

Figure 5. Time history of the number of 'Quiet', 'Active', 'Fighting' and 'Jailed' citizens obtained in Run F2.

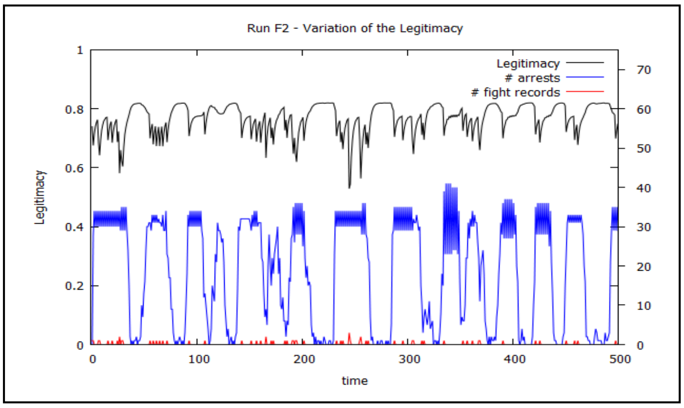

Figure 6. Time history of the legitimacy, number of arrests per cycle and number of pictures taken by media agents during one time cycle, obtained in Run F2.

IV. DISCUSSION

The introduction of delayed imprisonment and legitimacy feedback effects in Epstein's Model I produces more complicated dynamics than the original model. In some cases, the system's long-term behavior is changed from punctuated equilibrium (intermittent bursts of violence) to equilibrium with random fluctuations. In other cases, if the imprisonment delay is considerably smaller than the jail term, our extended ABM produces solutions with intermittent bursts of rebellion, which take a longer time to be dominated by the cops than with the original model and show fine structure fluctuations due to individual fights. As the ratio between fight duration and maximum jail term increases, the system's long term behavior changes from intermittent bursts to a condition of equilibrium with random fluctuations.

The 'Media' agents introduced in our ABM provided a simple, yet realistic, representation of the effects of media coverage on the dynamics of violent outbursts or large protests, via the connection between the records of fighting agents and the legitimacy, which is a key factor in the path-dependent process in Fig. 2. The formulation of purposeful movement for media agents in Eq. (2) can also be used to devise 'move' rules for the citizens and cops that are more realistic than random movement for simulating small-scale processes (with 'Actives' and cops trying to achieve local superiority).

Legitimacy feedback is a key aspect in the evolution of social conflict phenomena. The introduction of this mechanism in the model had a significant impact in the results of the simulations, namely by increasing the time-averaged numbers of 'Active' and 'Jailed' citizens and even changing the system's long-term behavior from Run E2 to Run E3. Our formulation is based on the hypotheses that arrests and violent episodes published by media induce legitimacy drops, and that the latency of these drops is attenuated according to an exponential law. This is different from the formulation suggested in [7] for extending Model II, which computes a legitimacy drop factor as a function of the system's states in a few previous cycles and behaves like a truncated exponential, so that both schemes lead to recovery of legitimacy to its quiescent value. In reality, legitimacy is a subjective (endogenous) attribute and its average perceived value can also be considered as an emergent property. The formulation this mechanism requires further research and context (empirical) data for parameterization and validation.

V. Conclusions

In this paper, we presented an extension of Epstein's ABM of civil violence against a central authority with important novel features and mechanisms: imprisonment delay to represent the cost to the police forces posed by arresting rebellious citizens (which we called fights), a new type of agents ('Media') and a feedback mechanism for changing the government's legitimacy as a function of the number of arrests and violent episodes (fights) recorded by 'Media' agents. These features are necessary to model processes operating at different time scales (fights and jailing), media coverage and feedback on the social context (legitimacy).

Introduction of imprisonment delay changes the system's long-term behavior and increases the average numbers of rebellious and jailed citizens. For certain combinations of parameters, the model produced large intermittent peaks of rebellion, which lasted longer and had a more complicated fine structure than those obtained with Epstein's model. The 'Media' agents moved in a realistic way and played a key role in the legitimacy feedback mechanism. Legitimacy feedback can also change the system's long-term behavior, increase the average numbers of 'Active' and 'Jailed' citizens, and decrease the time-averaged legitimacy. Thus, the present model can represent a wider range of dynamic behaviors than Epstein's model and integrates long and short term processes, media coverage and legitimacy feedback in a coherent way.

The model presented herein is part of an ongoing work on ABM of the dynamics of protests, violent confrontation, and their effects on the social context, as described in [1]. Further developments include the refinement of the legitimacy feedback formulation, a model of contagion based on the layered networks, a model of protest dynamics focused on micro-situational processes and data collection in protest events for defining the agents' attributes and model parameters according to real contexts.

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