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## **Bitcoin - Boom or Bust**

Implementation of cryptocurrencies in diversified portfolios

Daniel Alexandre Luís Dias

*Master in Finance*

Supervisor:

Pedro Manuel de Sousa Leite Inácio, Assistant Professor,  
ISCTE Business School  
Department of Finance

October, 2020



BUSINESS  
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## **Abstract**

Bitcoin stands as one of the most revolutionizing tools of the last decade. Its incredible and eventful run makes it highly valued by some and equally criticized by others. Many defend it as the future of banking and the entire financial industry, while others call it a bubble waiting to burst.

Despite this duality the biggest of the cryptocurrencies has been gaining a following with more and more investors willing to invest in it. As of October 2020, the Director of Research of Fidelity said that portfolio managers should consider allocating up to 5% of their value to Bitcoin, with the cryptocurrency's current market value being just a "drop in the bucket" of its potential, alerting that the move of retail investors towards Bitcoin is inevitable.

Thus, we studied how viable is the implementation of Bitcoin on an investor's portfolio, making mainly use of well diversified index replicators of some of the biggest and most used benchmarks.

The results have shown that despite concerns mostly of volatility there is relevance in considering adding the cryptocurrency to one's portfolio. Retail and institutional investors alike should not sleep on the emergence of this new asset class and should take measures to ensure that they study and comprehend it in order to assess if it fits their risk profile and not miss on the potential diversification effect and returns that one could have with Bitcoin.

The main theoretical contribution of this work comes from the in depth analysis of Bitcoin and its diversification capabilities when faced to more common investment vehicles, while the main practical contribution lies on the application of the individual portfolio weights to an Exchange-traded fund (ETF) adept investor looking to diversify into Bitcoin.

Journal of Economic Literature (JEL) classification: G11, G12

Key-Words: Bitcoin, Portfolio, Performance, Risk

## Resumo

O Bitcoin mostra-se como uma das mais revolucionárias ferramentas da última década. O seu incrível caminho cheio de eventos faz com que seja altamente valorizado por uns e altamente criticado por outros. Muitos defendem que será o futuro da banca e do sistema financeiro, enquanto que outros afirmam que não passa de uma bolha à espera de rebentar.

Apesar desta dualidade, a maior criptomoeda tem vindo a ganhar cada vez mais seguidores e investidores dispostos a investir nela. Em outubro de 2020, o Diretor de Research da Fidelity afirmou que os gestores de portfolios deveriam considerar alocar até 5% do valor do portfolio ao Bitcoin, com o valor da criptomoeda sendo apenas uma “gota num oceano” do seu potencial e alertando que a mudança de investidores particulares para o Bitcoin é inevitável.

Assim, estudou-se a viabilidade da implementação de Bitcoin no portfolio de um investidor, fazendo uso de índices replicadores de mercado já bastante diversificados que são muitas vezes usados como termos de comparação para outros portfolios.

Os resultados mostraram que, apesar de maioritariamente preocupações com a volatilidade, há relevância em considerar a adição de criptomoedas ao portfolio de um investidor. Investidores tanto particulares como institucionais devem estar atentos à emergência desta nova classe de ativos e devem tomar medidas para garantir que a estudam e compreendem para assim poderem ver se será ou não indicada para o seu portfolio, de modo a não perder os ganhos e efeitos de diversificação que podem advir do Bitcoin.

A contribuição teórica principal deste trabalho vem da análise detalhada ao Bitcoin e do seu potencial de diversificação quando comparado aos instrumentos de investimento mais frequentemente usados. Enquanto que a maior contribuição prática está na aplicação dos pesos de carteira de Bitcoin e do ativo complementar para um investidor adepto de fundos Exchanged-traded (ETF's) que procure diversificar com Bitcoin.

Classificação do Journal of Economic Literature (JEL): G11, G12

Palavras-chave: Bitcoin, Portfolio, Performance, Risco

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# 1. Introduction

Everyone has heard about it, but most do not really know what it is.

Bitcoin is a digital currency, the most valuable one amongst cryptocurrencies. One of its potential strengths, and potential weakness is that it does not exist at all. It is not backed by any bank or government. It is not material in any way, shape or form. It exists in the internet cloud. All transactions done within bitcoin are kept public and are checked and controlled. It has been the trendsetter for other virtual currencies. Its superiority in value and commercial power is so high that those other coins have been dubbed as “altcoins”, or alternative coins.

It is important to notice the similarities between bitcoin and gold, a currency that we will use as term of comparison further ahead and will take relevance in our work. Comparable to gold and as it has a finite supply, bitcoin also has been increasing in amounts on the economy, but it is set to have a finite number of available bitcoins. That number is set to be around 21 million.

It has been enjoying a tremendous amount of attention during the past years, and with it (or because of it?) also gigantic returns. Of course, nothing comes for free, and it has also been a victim of intense volatility and brutal swings over its run, going through incredible price swings that sometimes even occur daily and large market cap fluctuations over notably short periods of time.

After all that nonetheless, Bitcoin still remains. And its strength and notoriety as a portfolio diversification tool, along with those investors that possess a risk tolerance level that allows its insertion on their portfolios have been thriving. Over its inception more than 10 years ago Bitcoin has grown over 157% yearly when we look at the annualized data. It had its largest slump on the space of a year that encompassed December of 2017 and of 2018 where at the end its market value stood almost 6 times lower than it was in the beginning of that year.

Bitcoin endured and rallied back up. While it still has not reached its vertiginous highs it is still going strong. Investors that invested on it near its inception made fortunes. Those that did so before November 2017 have already more than broke even, after all the

downturns. While those that invested more recently, likewise have been enjoying a surplus on their accounts.

As such we feel that it has not received the attention it deserves by the literature as there was a considerable gap on the latter especially on the main topic and aim of this thesis which we finally reach along with the question we will try to get closer to finding the answer to: **Should Bitcoin be included on an investors' portfolio?**

Now, we of course do not claim to know the future. As such a more reasonable goal focus on analyzing how Bitcoin has been behaving over its inception and see what conclusions we can make of it, knowing of course that the future remains unsure and those past findings are in no way a guarantee of future success.

We seek, by the employment of the much well known and regarded Sharpe and Sortino ratios. to have at the end of this study the means to answer with backed up proof if it would make sense or not to have a sustained investment in bitcoin as part of an investment strategy.

For this we aimed at some of the most diversified equity markets. Those seen as benchmarks by many investors and institutions over the world. The notorious S&P500, NASDAQ and MSCI Indexes along with Gold are all broken down and analyzed in this project. Our aim is to see if the addition of Bitcoin to those assets would result in an increase in returns for investors. In order to increase the solidness of our conclusions we decided by the study of three different scenarios. Three timeframes in which Bitcoin's performance has varied between one another.

Our findings serve to widen the favorable light that recent times has shown on the cryptocurrency. With the most promising results being those that imply a great performance over the most recent period analyzed in which Bitcoin soared the portfolio of those investors that employed the cryptocurrency. Notably over the entire inception of the cryptocurrency it also displayed interesting ratios. With the only negative being the scenario that occurred between the years it suffered its biggest downturn.

The results indicate how much market timeliness of entrances and exits was linked with Bitcoin. Not a great characteristic for most risk averse investors, but however our study serves as proof that there is potential for diversification and monetary gains to be had with this cryptocurrency for investors willing to sail with its volatile characteristics.

## **2. Literature Review**

### **2.1.Bitcoin**

#### **2.1.1 General Considerations**

We do not even know who the creator of bitcoin was. A decade ago a mysterious name, “Satoshi Nakamoto” appeared as the publisher of a paper by the name of “Bitcoin: A Peer-to-Peer Electronic Cash System”. We do not know if Nakamoto is/was a person or a group. All we know is that this new release revolutionized the entire world.

Happening amidst all the chaos that accompanied the financial recession of 2008, Nakamoto had an idea. A digital currency. A currency that would be traded only amongst their users and without any hand in it by the government. An entire new way to see our monetary system. One that would not involve the government or any central banks.

Nakamoto’s plan was not one of a financial commodity. Far from that. What he idealized was a transactional system. Something that could be used to keep a trade between the two involved parties, without the meddling or need of intervention of a third party. A system based on cryptographic proof instead of trust, that allows for transactions practically impossible to reverse that protect both parties from fraud (Nakamoto, 2008).

Bitcoin is a digital currency, the most valuable one amongst the cryptocurrencies. One of its potential strengths, but also potential weakness, is that it does not exist at all. It is not backed by any bank or government. It is not material in any way, shape, or form. It exists in the internet cloud. All transactions done within bitcoin are kept public and are checked and controlled. It has been the trendsetter for other virtual currencies. Its superiority in value and commercial power is so high that those other coins have been dubbed as “altcoins”, or alternative coins.

It is important to notice the similarities between bitcoin and gold, a currency that we will use as term of comparison further ahead and will take relevance in our work. Comparable to gold as it has a finite supply, bitcoin also has been increasing in amounts on the economy, but it is set to have a finite number of available bitcoins. That number is set to be around 21 million.

The potential use of bitcoin and other cryptocurrencies as money is still a highly debatable topic, as are most of those surrounding this new financial and technological

asset. More so than ever on the current days, money can be considered as many things, but it still retains its main function as a medium of exchange for individuals. For something to qualify as money, it needs to have some characteristics such as being generally accepted as payment for goods and services, being a unit of account, which enables the quantification of wealth and enabling agents to store the mentioned. We are of the opinion that bitcoin accounts for all the above.

Over its initial release phase, bitcoin was mainly used and seen as an instrument for anarchists, futurists, and criminals. Hardly what you would call respectable investors. However, instead of plunging down, bitcoin kept riding on its solid technologic background and managed to catch the attention of big shark venture capitalists. Governments did not shut them down or regulate them out of existence, and so more and more recognized investors started to appear and jump in the waters.

Those can be divided into four big groups (Marsh&McLennan Companies, FireEye, Circle, 2019):

- Alternative Investors
- Government Skeptics
- Privacy Seekers
- Speculators

Building on each; we can see **alternative investors** groups and individuals such as professional traders, small hedge funds and venture capital firms, whose investment beliefs in crypto assets as a new asset class, combined with the desire for an early access and for diversification, are strong.

On the other hand, **government skeptics** can be defined as those that resonate heavier with the decentralized aspect of the currency. This distrust has been bred and grown a lot since the last financial crisis of 2008 and these investors believe that governments will mismanage the economy and are in general skeptic of them.

**Privacy seekers** englobe those that place value on the privacy aspect implied on the blockchain and by following up on bitcoin. Their bet on the asset lies on this characteristic. This has been raising some concerns and has created the stigma that bitcoin is associated with criminals in the past and raised some concerns with law enforcement.



However, it is important to note that while being private, blockchain still provides a public record of every transaction.

Finally, we have the **speculators** which are attracted by the volatility associated with this new form of asset and whose interest lies mainly not on its underlying potential but on short-time trading and making a quick profit.

At the peak of its value in early 2018, the combined market cap value of all the cryptocurrencies equated to 1/10 of the value of gold in the entire world. This was a great feat for an asset that existed for not even 10 years at that time.

It has challenged and given headaches to investors, policymakers, governments, corporations, and executives over the recent years. It has changed the perception of how the world can work and how technology and cryptocurrencies might enable decentralized transactions.

Market Cap of the Top Three Crypto-Assets (US \$B)

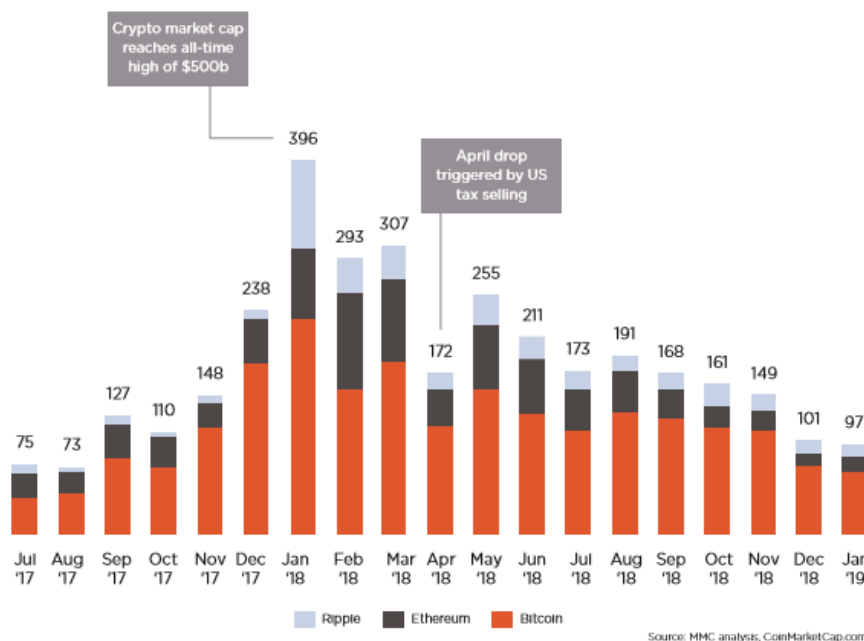


Figure 1. 1: Market cap of the three biggest cryptocurrencies

Source: Marsh&McLennan Companies, FireEye, Circle, 2019

Above we can see how the market cap of just the three biggest cryptocurrencies has been evolving over since June 2017 all the way to January of 2019, covering the peak that occurred on January 2018. We can already see here how volatile the market and

specially bitcoin has been over the period. A trend that we will explore further below this project.

### **2.1.2 Blockchain**

The technology that was the backing of bitcoin, the blockchain, proved itself to be quite strong. Surviving the initial distrust and lack of faith of the masses. Over time it gained strength and it is today one of the pivotal selling points of bitcoin and one of the characteristics that more attracts investors to it.

While in practice quite complicated and hard to understand, the motivation and the concept behind it is quite simple. It is no more than a digital platform, a cloud, where transactions from all over the world are registered and verified. We can in truth see it as a ledger of sort, and that is the most common explanation used when someone attempts to talk about it. It is exactly the same as any other money ledger, such as a bank statement or a PayPal account transaction, with the difference being that it does not rely on any centralized authority such as central banks or the government to maintain it. It is, as said before, entirely peer-to-peer. All transactions are made in a public and secure environment.

It is a continuously growing list of data, the blocks from where it gained its name, that are linked together by means of cryptographic techniques. It then becomes a specialized method of organizing that information, not too different from a Lego's building, where those organized blocks are put together on top of one another and as such consolidate the entire system.

Each transaction is associated with a code that is unique to it. That code will contain information such as the time when the transaction occurred, who paid for it, who received that payment, and at last a mean to link it and connect it to the previous block in the chain.

It then passes by other nodes in the complicated system that it composes where it is validated or not. If it does happen to be validated, it gets forwarded to the entire blockchain system and after being in it for a while only then will it be considered finalized.

There are some nodes of the chain that are the miner nodes. Those are responsible specifically for implementing and connecting those new blocks into the system. Like

already mentioned this is a peer-to-peer system, and as such, those miners are people that through extremely powerful computationally hardware implement complex algorithms, which will link the block to their previous and make the system flow. For that work they are awarded coins of the cryptocurrency associated with the transaction.

Satoshi (Year?) simplifies well the idea behind the process on his paper, by means of 6 steps:

1. New transactions are broadcast to all nodes
2. Each node collects new transactions into a block
3. Each node collects new transactions into a block
4. When a node finds a proof-of-work, it broadcasts the block to all nodes
5. Nodes accept the block only if all transactions in it are valid and not already spent
6. Nodes express their acceptance of the block by working on creating the next block in the chain, using the hash of the accepted block as the previous hash

Bitcoin supporters defend that this decentralized system is one of the key advantages of the system. Making those investors and adopters feel that they truly control their wealth and are not in the hands of other enterprises.

To own their assets most crypto investors, own digital assets where they can store their digital assets. These can be in software form or hardware form. Software wallets are common as the first dip in the water for new investors. Facilitated by websites such as Coinbase and Kraken those are simple to use and intuitive for even the least savvy of the investors. Any person can just create an account and have a wallet available to it to invest in.

For this most of those that control a big sum of wealth in bitcoin, or in truth, any other crypto asset or portfolio of them, own hardware wallets for the increase in security they provide. This highly goes in line with the mentality associated with most of the bitcoin investors who want the most secure way possible to own their wealth without leaving it in the hands of external parties.

On the chart below we can see how the overall process works:

How are Blockchain Transactions Executed?

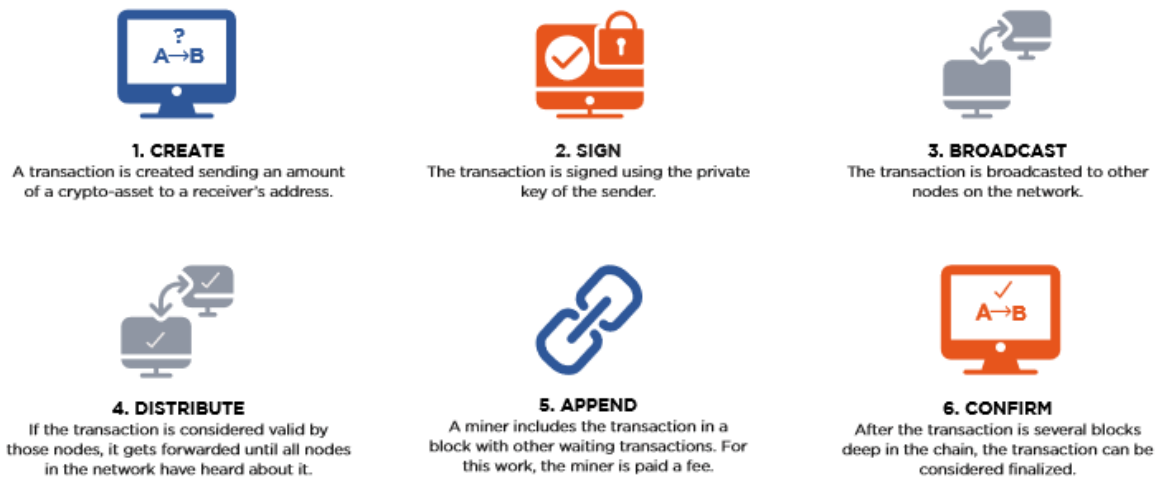


Figure 1. 2: Process of Blockchain Transactions

Source: Marsh&McLennan Companies, FireEye, Circle, 2019

### 2.1.3 Future of bitcoin

Let us remind the challenges mentioned above that would prove pivotal to the success and well development of bitcoin as more than a fad investment.

- The creation of a good market infrastructure
- A Government framework that regulates it
- Blockchain proving itself as a secure and trustworthy network

Corbet, et al (2018), defend notion of cryptocurrencies as a new investment class and the implementation of the former on investors' portfolios, with every cryptocurrency being highly interconnected between each other and disconnected from other mainstream assets and with the overall market showing similar patterns to those of other asset classes (Corbet S. , Meegan, Larkin, Lucey, & Yarovaya, 2018).

We think that it is highly relevant and important for a correct future analysis of bitcoins potential as an investment asset further along this project that we understand as most as possible its potential future and where does it stand at this moment. For that reason, we think it is important to analyze each of the three topics above in a more detailed fashion.

### ***2.1.3.1 Market infrastructure***

Building upon the study by Marsh (Marsh&McLennan Companies, FireEye, Circle, 2019), it is clear that lots of progress has been made in the ways of the investors and society in general seeing bitcoin as legitimate money.

The creation of exchanges for cryptocurrencies such as Coinbase, Kraken, Binance and BitGo have had a great impact in the development of the market and on its general accessibility to more and more investors, instead of limiting it to those that have some tech savviness under their belts. This surge in investors and development has attracted the attention of multi-national companies that the ears of investors are more accustomed to their names, such as Bloomberg and Tesla, which have already started to accept bitcoin as a form of payment in transactions.

The irony being that bitcoin, formed in an age where distrust with the mainstream financial institutions, now needs traditional structures to realize its potential is not lost on the authors of the article.

This is needed in order to attract more and more traditional and large financial institutions. While it is true that it has survived this long, and probably would have survived anyway without their investment, no one can deny that this would be a huge step on the right direction for its valuation, not to mention the important step of converting it into a viable alternative asset class.

Blockchain could have a role to play in this. Making redundant many of the current responsibilities of the organized financial systems, being able to execute transactions easily.

Lots of bigger players in the market have been taking the leap into this new thriving and everchanging market of the cryptos such as Fidelity Investments which has announced in October 2018 that it is developing a business to focus on it.

Other big players, especially those on the technological area of business have also been playing their cards. With Samsung announcing the development of an app to work as a wallet for crypto assets for their brand-new phone model. The success of this endeavor is still however to be seen, as most big player crypto investors still give more value to the security assured by owning their wallets in hardware physical form. Microsoft

and Amazon, other two giants, have also been keen in jumping into the new tech and have been using blockchain for some years now.

There has also been some movement regarding the more traditional asset traders. In November of 2017, the Chicago Exchange announced that it was in the process of creating the first ever futures contract for the trading of crypto assets. This was one of the motivators for the big surge of price that bitcoin suffered in the later stages of the year leading up to 2018. One of the big pluses of these contracts was the fact that they could be settled in cash. Allowing for the easy conversion of bitcoin to cash on the spot.

In the end of 2018 while bitcoin was still recovering from the big crash it suffered in the beginning of the year, we saw the Intercontinental Exchange (ICE) announcing that it was in discussions with the Commodity and Futures Trading Commission to launch physically settled bitcoin futures. This means that the ICE would take physical ownership of bitcoins. It requires however an exemption from the latter mentioned institution, as their regulations only allow the former to hold collateral in the form of cash, securities or commodities.

The ICE achieved this goal in September of 2019. This is a great step forward in order to push the organizations into solving the issue that they have been neglecting regarding bitcoin. With the possibility of this working favorably it will also bring some needed light into the currency and maybe relieve some of the skepticism of the markets regarding it.

Outside of the United States, in Europe we have Germany's Boerse Stuttgart Group already has a crypto trading platform up and running. With the potential to be used by other European countries to be opened soon. In Switzerland and still in 2018 the first-ever crypto-asset trade was approved.

These incredible developments have all been surging one after the other in the short period that has passed between late 2017 and early 2019. It is great news for bitcoin investors and people interested in it that it is becoming ever more a thematic that is being acknowledged by the world and financial institutions in general.

Below we can find a simple flux gram with notable developments from how the market developed over the year of 2018:

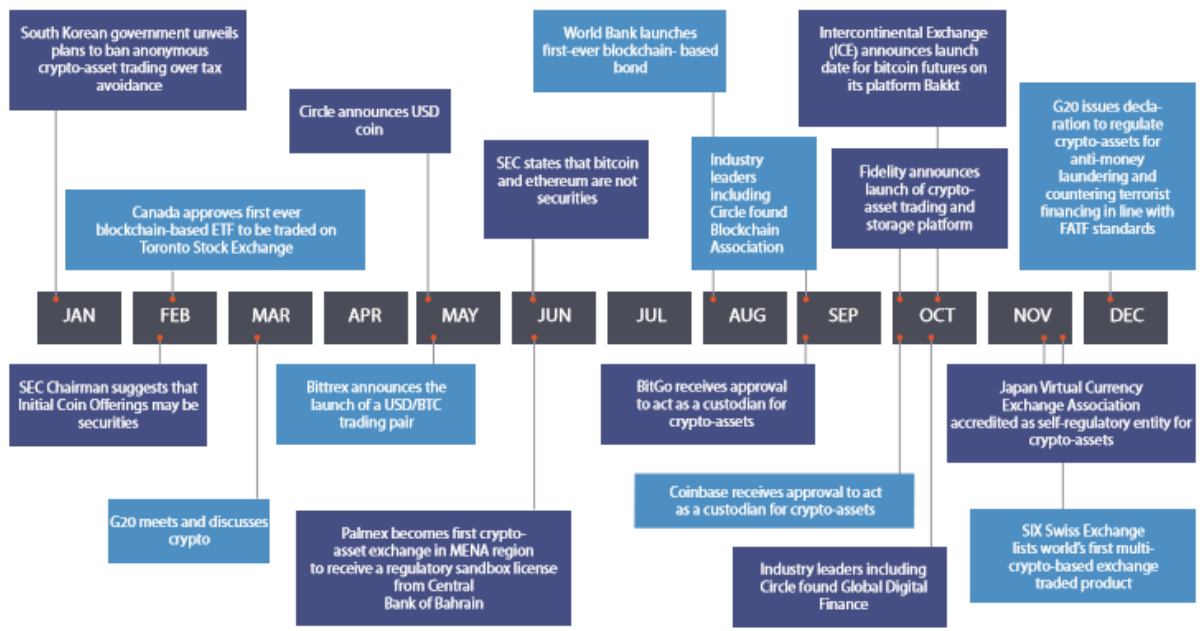


Figure 1. 3: Cryptocurrency industry developments over 2018

Source: Marsh&McLennan Companies, FireEye, Circle, 2019

There is still also a clear scalability and accessibility linked to the cryptocurrency. It is still not available for use by most of the population, mostly by ignorance, lack of belief or incentive for use. Radziwill (2018) also notes that the amount of energy necessary to “mine” a year’ worth of new Bitcoin is equal to the energy consumed by the country of Ecuador on the same period. However, the same author defends the importance of early adoption by managers and executives since it has far reaching implication of future issues of compliance and transactions management (Radziwilli, 2018).

However those issues seem to be well on the way of being tackled already, with the introduction of Bitcoin-linked funds by global investment banks, which worked to increase the accessibility of the crypto-currency to investors all over the world. Furthermore, this legitimacy increased even further in late 2017 with the launch of future contracts based on bitcoin.

Shahzad (2018) and his peers defend that those events should suggest that such events and the entire crypto market should not be ignored by the investing community, with possible dire consequences if such would happen (Shahzad, 2018).

### 2.1.3.2 *Creation of a government framework that regulates it and the trustworthiness of Bitcoin*

While this seems quite in contrast with the idea of having a decentralized currency, it is of high importance to have regulations on it, in order to bring about more transparency and clear out some of the misconceptions associated with bitcoin and crypto assets in general.

On the regulatory side, the Commodity Futures Trading Commission (CFTC) has already declared virtual money a commodity, just like crude oil or Gold and there has been a lot of improvement on this field by governments as well. While China banned the purchase and sale of cryptocurrencies in their land, they still allow for mining to be done; with china being in fact the largest miner in the planet with half of all bitcoin mining. (Marsh&McLennan Companies, FireEye, Circle, 2019)

We also have countries such as Australia which has recognized cryptos as a means of payment and an asset class, Japan where the government authorized the creation of a self-regulatory body for cryptos in the Japanese Virtual Currency Exchange Association, and France and Switzerland, whose governments have been keen to keeping up a leading role in the fintech and blockchain technology market. (Marsh&McLennan Companies, FireEye, Circle, 2019)

Nakamoto expands on how the blockchain system would increase the privacy and security of all transactions by breaking how the usual banking system is set up. Avoiding the necessity to share information with a third party, with his method the public can still see that someone is sending X amount to someone else. However, they would lack the possibility to link this transaction to specific parties (Nakamoto, 2008).

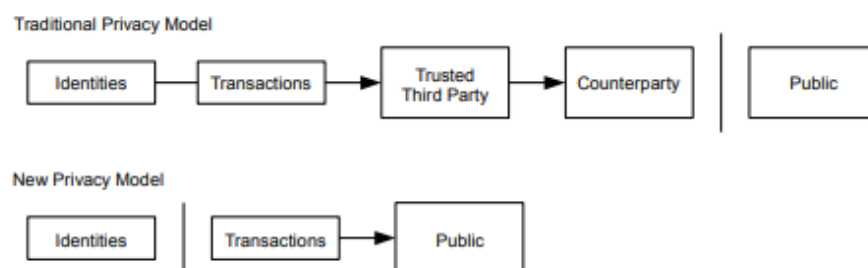


Figure 1. 4: Privacy model suggest by Satoshi Nakamoto

Source: Satoshi Nakamoto, 2008



There were still problems and lack of trust associated with bitcoin mainly because of hackers being able to steal billions worth of value from the coffers of bitcoin exchanges.

The most notable of those is probably the attacks on the Japanese Mt. Gox that occurred on late 2014. They could happen thanks to problems with Mt. Gox itself, that owned the cryptocurrency, and not thanks to the hacking of transactions themselves. However, some stigma has remained associated with bitcoin since thanks to that.

### ***2.1.3.3 Role as an instrument of diversification and safe-haven investment***

We define a safe haven investment as an asset that is not correlated or is negatively correlated with the portfolio in times of market downturns or turmoil. This does not mean that correlation cannot be positive in other times, but that it follows this behavior on the times mentioned above.

One of the crypto-currency's most important and talked about role is its potential not only as a mechanism of defense, not unlike gold and other similar commodities, to fight economic downturns and periods of general uncertainty on the equity markets. Capie et al. (2005) found in 2005 that gold can be used as a hedge against the dollar. The reasons for that being that it is not controlled by the same institution that controls the currency. Bitcoin clearly shares the same characteristics as no institution or even government in the entire world can control its production (Capie, Mills, & Wood, 2005). This leads us to think that the potential of bitcoin as a hedge measure can behave on the same way. However, one important note is that that Capie et al (2005) used data that ranged from 1971 to 2004 in their study. This data period encapsulated large fluctuations in the market, economic turmoil, downturns and rallies. This is still not possible to see with bitcoin, thanks to its recent inception.

Sherman (1982) and his research add value to this conclusion. He found that gold provides a good escape from the US financial environment, as it proves to have a less volatile price than those of stocks and bonds, while also giving greater rewards to market timing. This can be explained by Gold's low beta and extremely low correlation with the stock market. With bonds it also showed a good value for beta and an almost null correlation. Its alpha was also positive indicating good potential returns (Sherman, 1982).

Other, more recent, authors however such as Baur and Lucey (2010), defend that gold does not work as well as a safe haven anymore. It still shows this characteristics for the equity markets, but on the bond market it only functions as a safe haven for a limited time, with investors holding gold for more than 15 trading days after an extreme negative shock end up losing value with their gold investment (Baur & Lucey, 2010).

Dyhrberg (2015) seems to defend that the notions concluded by Capie et al (2005) apply to bitcoin as well, adding an interesting caveat. The inherent characteristics of bitcoin can give it an edge. The high and continuous frequency of trading alongside the lack of closed days in trading can give it an advantage over other hedging instruments. The author concludes that those rich advantages should be noticed and added to the hedging instruments of analysts (Dyhrberg, 2015).

The value of gold usually soars during contractions, economic downturns, or macro uncertainty. During these stress periods the value of the precious metal cannot be overstated. Commodities in general act as an instrument of diversification towards the stock markets. Bitcoin serves a similar purpose to fulfill those investor needs. According to Luther and Salter (2017) it has even been sought as by investors during stress periods such as the European debt crisis of 2010-13 (Luther & Salter, 2017). These ideas are built upon further by Corbet et al (2018), that defend the isolation of bitcoin from economic and financial variables (Corbet S. , Meegan, Larkin, Lucey, & Yarovaya, 2018).

Authors such as Shahzad (2018), further these notions by expanding this potential of bitcoin to other markets and stating that its potential to provide a diversified safe-haven for an investor does not only apply to the developed markets but in truth has a similar effect in emerging markets and the Chinese market, interesting considering the regulations imposed by the Chinese government that work against it (Shahzad, 2018).

Some authors such as Klein et al (2018) however defend that bitcoin works as a diversifying tool but not as a safe haven investment, with it showing a similar behavior to markets and declining when they do, which suggests that bitcoin is no hedge against equity investments (Klein, Pham Thu, & Walther, 2018).

## 2.2. Investment and Portfolio Framework

### 2.2.1 Markowitz Model

“Don’t put all of your eggs in one basket”

One of the first problems when constructing a portfolio, and perhaps also one of the most important ones, resides in the question of how much of it we want to assign to the risk-free asset, and how much do we want to then split between our other assets. Following on the ideas of (Markowitz, 1952) we can follow the methodology of:

- 1- Identify the risk-return combinations available between the assets we can choose from.
- 2- Identify the optimal portfolio composed by those risky assets available.
- 3- Decide how much we want to throw of riskless assets into the optimal risky assets to form the portfolio.

At first in order to solve this problem, we identify the risk-return opportunities that the investor can invest in by looking at the minimum-variance frontier of risky assets. What is this exactly? It can be summed up as the graph with the minimum variance that can be coupled with a portfolio’s expected return (Marcus J; Kane A, 2012).

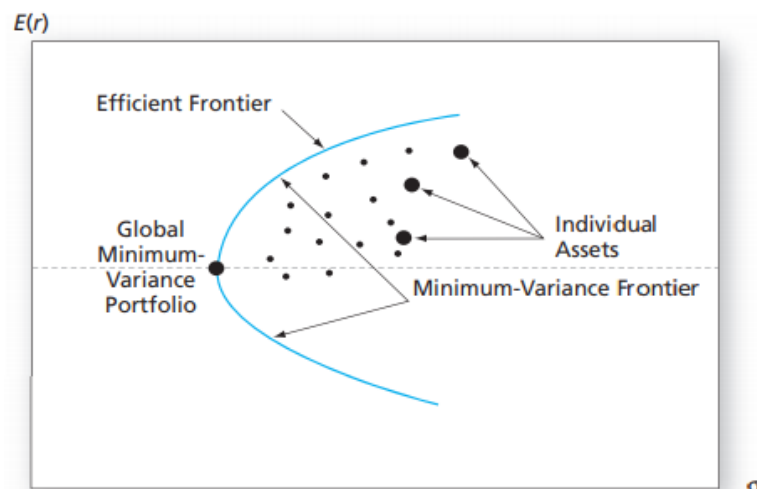


Figure 2. 1: Minimum Variance Frontier

Source: UCONN - University of Connecticut

By seeing that individual assets are on the inside of the efficient frontier we can also draw on an important conclusion, that portfolios composed by single risky assets are inefficient. It is also intuitive by looking at the figure that we want the portfolio

combination to be above the Global Minimum Variance Portfolio, which is the combination of assets that minimize risk on the market. As such the part of the minimum variance frontier that lies above this minimum variance portfolio is to be called the Efficient frontier, in which any combination of assets that make it up are potential candidates to be the optimal portfolio for investor X. Any combination of risky assets below the Efficient frontier are to be ruled out, as for the same risk we can very well find a portfolio with a higher positive expected return.

The next step lies on finding the capital allocation line that has the best return to its risk

Marcus and Kane (Marcus J; Kane A, 2012), identify this as the CAL that is tangent to the efficient frontier as it beats any of the others that “pass through” the curve of the efficient frontier (and evidently cross it over in two points thanks to that fact).

We finally arrive at two important formulas for this project. Those that allow us to reach the values of the expected return, variance and weights in each security, given by,  $E(r_p)$ ,  $w_i$  and  $\sigma_p^2$  respectively:

$$E(r_p) = \sum_{i=1}^n w_i E(r_i) \tag{1.1}$$

$$\sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j Cov(r_i, r_j) \tag{1.2}$$

The Markowitz model can be seen as the “first step of portfolio management” (Marcus J; Kane A, 2012), and his model is just that. The first step in identifying the efficient set of portfolios. The main idea behind it is simple, for any risk level of the investor we are only concerned in finding the one that fits that risk level and can achieve the highest expected return. We can also see this in the equivalent but opposite way. In order of variance it is the one that minimizes the variance for a set value of expected return.

Now is the moment in which we include the capital allocation line, which is the same as saying that we include the risk-free asset.

Let us understand why the portfolio that is “stuck” on the tangency point between the CAL and the efficient frontier is always the best choice. (unless that is there are outside constraints being imposed such as a dividend yield... any constraint will naturally lead to a worse portfolio when compared to the optimal one). This portfolio that stands on the tangency is the one that maximizes the reward to volatility ratio (Markowitz, 1952).

Why are not all portfolios equal then? Well that can be explained by the fact that not all investors have the same risk aversion, and will add different mixes of riskless assets to their portfolios depending on their aversion to risk (Marcus J; Kane A, 2012).

### **2.2.2 Index Modelling**

The Markowitz model while being incredible in opening the borders of portfolio management and portfolio construction however lacks in some critical areas. Those can be summarized as the need for a big number of estimates for the covariance matrix and the fact that it does not provide any reliable way to forecast future returns, as past returns are not reliable (Marcus J; Kane A, 2012).

Enter index models, which try to address these drawbacks but it is important for us to remember that while these models do simplify the amount of work put into the Markowitz model, they still rely on it as its basis and on its concepts of efficient frontier and optimization of the securities.

We rely upon a great example by the authors (Marcus J; Kane A, 2012) to show the impracticality of the Markowitz model and as such showcase why we feel that this model is the most indicated to perform our work.

Let us take 50 portfolio components... that gives us:

$n = 50$  estimates of expected returns

$n = 50$  estimates of variances

$\frac{(n^2-n)}{2} = 1225$  estimates of covariances

We can see how this can quickly prove itself troublesome...

### 2.2.3 Single Factor Model

By decomposing the risk into an expected and unexpected component we get that:

$$r_i = E(r_i) + e_i \quad 2.1$$

Now let us take this rational further and assume that the unexpected return can be made up by two components. One that is company dependent and is linked to the asset, which shall be our  $e_i$  and also another that is independent of the security but linked to the macroeconomic situation, which shall be our  $m$ . As such we can rewrite our previous expression as:

$$r_i = E(r_i) + e_i + m \quad 2.2$$

It is important to notice that both of them are not correlated with each other, as mentioned by the authors (Marcus J; Kane A, 2012), shocks that affect the firm are not those that affect the common factor that affects the entire macroeconomic scenario. We can then derive that our variance is given by:

$$\sigma_i^2 = \sigma_m^2 + \sigma^2(e_i) \quad 2.3$$

In where the  $\sigma_m^2$  represents our variance associated with the macroeconomic scenario and our  $\sigma^2(e_i)$  represents the specific one.

Analyzing the covariance, we can reach the conclusion that  $m$  is common between different securities as all of them will be affected by the same macroeconomic pressures. But as we saw above  $m$  is not correlated with the firm specific shocks, and as such we can resume the covariance between security  $i$  and  $j$  as:

$$Cov(r_i r_j) = Cov(m + e_i, m + e_j) = \sigma_m^2 \quad 2.4$$

We conclude with a refinement to the first return formula we presented above, with the fact that some firms are more sensitive to changes in their environment than others, as such we have to take this into account when applying the model, and we do just this by adding a sensitivity coefficient,  $\beta$  to our equation and we get the single-factor model.

$$r_i = E(r_i) + e_i + \beta_i m \quad 2.5$$

The same logic can be applied to the variance:

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma^2(e_i) \quad 2.6$$

And finally, to the covariance of any pair of securities:

$$\text{Cov}(r_i r_j) = \text{Cov}(B_i m + e_i, B_j m + e_j) = B_i B_j \sigma_m^2 \quad 2.7$$

It is important to put emphasis in the fact that we are assuming the normality of security returns, that in turn imply the normality of the portfolio returns (Marcus J; Kane A, 2012).

#### 2.2.4 Sharpe Ratio

We plan to use the Sharpe ratio to assert the characteristics of our wallet made of a combination of an index and bitcoin and how it fares when compared to other portfolios.

Beforehand an explanation of what this measure so commonly used in the world of finance is, is in order.

Sharpe defends in (Sharpe, 1994) his introduction of the metric. As he called it then with the term “*reward-to-variability ratio*”. This name however, opposite to the measure itself has not gained much popularity. And is commonly known now by the name described above: **Sharpe Ratio**. It has been known over the years also has Sharpe Index, Sharpe Measure amongst other names. In its core it is basically this. A measure that combines and summarizes the information given by two others (the mean and the variance).

Most of the performance measures are computed using historic data. They are however justified based on predictions. Sharpe (Sharpe, 1994) defends that historic results have at least some predictive ability of what is to happen in the future and how some commodities and assets will behave.

There is a separation of the ratio in two different applications. The so-called *ex ante* and the *ex post*. As the name implies, the first one is most commonly used in theoretical discussions while the second is more prevalent on the practical implementations.

In this thesis and as Sharpe takes the time to mention, we want to focus on the *ex-ante* version of the ratio, as the focus is to use it to make a decision (the one of the practicality and possible gains in the implementation of bitcoin on a wallet).

Initially the variables  $Rf$  are used to represent the return of a fund F and  $Rb$  to represent the return of a benchmark portfolio or security associated with the fund. Sharpe (1994) (Sharpe, 1994) then defines  $d$  as the *differential return* between the two:

$$\tilde{d} = \tilde{R}f - \tilde{R}b \quad 3.1$$

With then  $\bar{d}$  being the expected value of  $d$  and with  $\sigma_d$  being the predicted standard deviation of  $d$ , we have the Sharpe Ratio (S) given by:

$$S = \frac{\bar{d}}{\sigma_d} \quad 3.2$$

Or, in another form:

$$S = \frac{\tilde{R}f - \tilde{R}b}{\sigma_d} \quad 3.3$$

In this version of the ratio; **the ex-ante ratio**, it lets us know the expected differential return per unit of risk that is underlying to that return.

The author continues with the **ex post Sharpe ratio** that is given by a small change in the initial variables.

As said before the ex post ratio is prevalent in practical applications. Here it is used to evaluate past performance of a portfolio. As such we have access to its performance on a time  $t$ . Its actual returns are used in the calculations.

Having that slight change in the initial variables we have  $R_{Ft}$  that gives us the return of a fund F in the period  $t$ ,  $R_{Bt}$  which following the same logic gives us the return of the benchmark associated with the portfolio for the same period. Finally, we also have  $D_t$  being the differential of the two for the period in question, which leads us to:

$$D_t \equiv R_{Ft} - R_{Bt} \quad 3.4$$

Continuing as before, we have:



$$\bar{D} \equiv \frac{1}{T} \sum_{t=1}^T D_t \quad 3.5$$

Where  $\bar{D}$  represents the average value of  $D_t$  over the historic period that goes from  $t = 1$  to  $T$ .

The  $\sigma_D$  is therein given by:

$$\sigma_D \equiv \sqrt{\frac{1}{T-1} \sum_{t=1}^T (D_t - \bar{D})^2} \quad 3.6$$

From where we can get the ex post Sharpe ratio, that is also known as historic Sharpe Ratio:

$$S_h \equiv \frac{\bar{D}}{\sigma_D} \quad 3.7$$

An intuitive and simple way to understand the Sharpe Ratio is to imagine four investors:

- Investor 1 makes a lot of risky investments from which he manages a volatile income. In some years he makes a lot of money but in other years he also loses a lot of money.
- Investor 2 likes to play it safer therefore he only makes safe investments and as such he does not lose money. However, he also gains money at a very low rate.
- Investor 3 makes investments with low risk and still loses money.
- Investor 4 makes a lot of money in high growth investments that are still low risk.

The Sharpe ratio tells us which of these investors was the best. By looking at the upper part of the equation we have how much money we could have made when confronting our portfolio with a riskless asset. On the bottom part we have the risk associated with it. With the Sharpe Ratio we have the indication of how much that risk is worth when faced with the possible return. A high ratio will show us that the investor made a lot of money and faced a small amount of risk and a small ratio shows us exactly the opposite.

To add clarity, we think it would be interesting to show a simple example provided by Sharpe himself (Sharpe, 1994). In his example there is a strategy that can be made by a combination of 3 assets:

- X, which has an expected return of 5% and a standard deviation of 10%.
- Y, which has an expected return of 8% and a standard deviation of 20%.
- Riskless rate of interest of 3%

Applying the Sharpe Ratio here we have for X:  $S_X = \frac{(5\% - 3\%)}{10\%} = 0.2$

And for Y:  $S_Y = \frac{(8\% - 3\%)}{20\%} = 0.25$

So, we can see that fund Y is superior to fund X.

Imagine an investor that aims to have a standard deviation of 10%. He can either do that by exposing himself to 100% of fund X or by making an investment 50/50 between fund Y and the riskless asset. The latter provides a bigger expected return of 5.5%, which goes in line with the decision we get by implementing the ratio.

#### 2.2.4.1 Negative ratios

One interesting complication is that of a potential negative Sharpe ratio. This occurs when the expected return of an asset is negative, as that, coupled with a positive standard deviation will result in a negative Sharpe ratio. This can also occur with a positive expected return, in cases in which that expected return proves itself to be smaller than the riskless asset. When stacked up against one another this will result in a net negative on our numerator and as before result in a negative Sharpe ratio.

So, are negative Sharpe ratios useless for investors? And does this compromise the entire ratio as a whole?

McLeod and Vuuren, (McLeod & Vuuren, 2004) do not believe it to be so. In their paper they take use of the following two funds to make their case:

Risk free rate = 10%	Fund E	Fund G
Expected return $[E(R_p)]$	-12%	-8%
Risk $(\sigma_p)$	15%	10%
Sharpe Ratio	-1,467	-1,800

Figure 2. 2: McLeod's fund examples

Source: McLeod & Vuuren, 2004

As we can see, we have two funds with a negative expected return. And Fund E shows a higher Sharpe ratio while having a worse expected return and risk associated. How can this then be explained? Fund E should be more desirable than Fund G but it shows a lower expected return and a higher risk degree.

McLeod and Vuuren (2004) use a probabilistic interpretation to help bring light into this issue.

$$\begin{aligned}
 \text{Max Pr}(R_P \geq R_F) &\leftrightarrow \text{Max Pr}\left(\frac{R_P - E(R_P)}{\sigma_P} \geq \frac{R_F - E(R_F)}{\sigma_F}\right) \\
 &\leftrightarrow \text{Max Pr}\left(z \geq \frac{R_F - E(R_P)}{\sigma_P}\right), \text{ where } z \sim N(0,1) \\
 &\leftrightarrow \text{Min}\left(\frac{R_F - E(R_P)}{\sigma_P}\right), \text{ or, alternatively} \\
 &\leftrightarrow \text{Max}\left(\frac{E(R_P - (R_F))}{\sigma_P}\right) \\
 &\leftrightarrow \text{Max}(\text{Sharpe Ratio})
 \end{aligned}$$

This according to the authors would then represent an alternative interpretation of the goal of selecting funds with the highest Sharpe ratio. It is shown that the basis of selecting the highest Sharpe ratio is in truth selecting the fund that shows the highest probability of outperforming the risk-free rate. It is exemplified further by applying this notion to the funds represented on *Figure 2.2* above coupled with a risk-free asset of 10%:

$$\begin{aligned}
 \text{Probability}_{R_E > R_F} &= \text{Pr}\left(z \geq \frac{10\% - (-12\%)}{15\%}\right) \\
 &= \text{Pr}(z \geq 1.467) \\
 &= 0.071
 \end{aligned}$$

Applying the same notions to the other fund:

$$\begin{aligned}
 \text{Probability}_{R_G > R_F} &= \text{Pr}\left(z \geq \frac{10\% - (-8\%)}{10\%}\right) \\
 &= \text{Pr}(z \geq 1.800) \\
 &= 0.004
 \end{aligned}$$

With this we can now state that fund E has a slightly higher chance of outperforming the risk-free asset than that of fund G. And as such we can conclude that the fund with the highest Sharpe ratio, even when negative, remains the more attractive for investors.

#### **2.2.4.2 Time Dependence**

Sharpe in his work also studies the importance and impact of time in the sample and results of the ratio.

He defends that the ratio is dependent on the time period that it is subject to, either on the ex-ante or ex post version. To solve this issue and to still have a meaningful way to compare strategies and portfolios it is common practice to annualize the data that is correspondent to periods different from one year.

It is also good practice to get good estimates of the variables, it is a good idea to go over the returns of a given portfolio over a few decades (Rivin, 2018) . As this is most of the times impractical, mostly since markets change a lot over this kind of periods, it is in practice computed using the daily or monthly returns of the portfolio. For this to work we must assume that the returns are independent and identically distributed random variables.

If we look at more recent times, the Sharpe Ratio is mostly seen as it was intended by Sharpe, to be a measure of risk-adjusted returns that was made with the purpose of separating the strategies that were truly good from the ones where the portfolio manager simply ended up being “lucky”. As in, he simply managed to outperform the market in good times but did not do so well when the opposite happened, and the markets ended up down falling.

It is seen nowadays by the expression below:

$$S = \frac{E(R_p - R_f)}{\sigma_p} \quad 4.1$$

Here we have the difference between the expected return of the portfolio given by  $R_p$  and the expected return of the risk-free rate given by  $R_f$ . The ratio here is basically shown as the average return earned in excess to the risk-free rate per unit of volatility.

#### **2.2.4.3 Limitations of the Sharpe Ratio**

We feel that while a great and powerful metric that can greatly improve the investment decision process, it is also greatly important to look at the drawbacks

associated with the use of this metric in order to attempt to complement them further with other methods.

The first limitation that comes to mind is the fact that the method relies on the assumption that returns are normally distributed. This might not be and usually is not true as returns on the financial markets are skewed away from the average thanks to the existence of drops or the opposite in prices. The standard deviation assumes that price movements are equally risky either as they go up or down.

The second lies on the fact that the ratio can be manipulated by managers to boost how their results look on paper. This can be accomplished by incrementing the measurement interval and as such reduce the accuracy of the estimation. For example, by using monthly returns instead of daily returns.

Another weakness of the model rests on managers choosing a period of returns that is appropriate for the point they want to proof and as such distorting the results.

While the last two limitations do not really come up as a problem for the elaboration of this project, as our intention is merely to study the possibility of the bitcoin implementation and not the defense of that implementation, the first point is a valid consideration. We feel that the abundance of data to work with mitigates this issue and we look to further expand the potential of accurate results by complementing the Sharpe Ratio with other commonly used ratio that we will look at below.

### **2.2.5 Sortino Ratio**

While the Sharpe ratio is the most widely used ratio when it comes to measuring the risk-adjusted performance it is not without limitations. To tackle these, we find that the Sortino ratio improves on the Sharpe ratio in certain areas.

The Sharpe Ratio falls short as it cannot distinguish between upside and downside volatility. In fact, it is suggested that higher outliers have the effect of decreasing the value of the denominator, and as we are going to see below, subsequently decrease the value of the ratio (Rollinger & Hoffman, 2014). This does not make inherently sense as investors tend to welcome large positive returns.

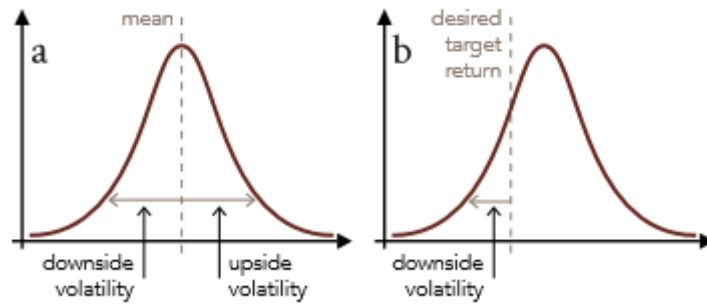


Figure 2. 3: Sharpe Ratio and upside/downside volatility

Source: Rollinger & Hoffman, 2014

Building further, for a positively skewed return distribution such as a trend following strategy, the Sharpe Ratio can actually show a higher value if we remove positive returns, which is obviously not something that a rational investor looks fondly upon. It is particularly poor as a performance metric when compared with positively skewed strategies such as option selling (Rollinger & Hoffman, 2014).

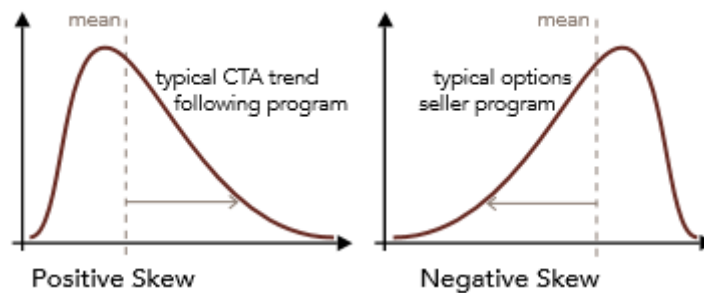


Figure 2. 4: Sharpe Ratio on skewed strategies

Source: Rollinger & Hoffman, 2014

So, what is Sortino Ratio?

It is a variation of the Sharpe Ratio that manages to have a bigger emphasis on differentiating the harmful volatility from the total overall volatility by using the asset's standard deviation of negative portfolio returns, called the downside deviation, instead of the total standard deviation. This is important because, as mentioned above, upside volatility is beneficial to investors and is not something that most investors worry about.

It manages to achieve this by means of using the portfolio's return and subtracting to it the risk-free rate (or other benchmark) and then using the asset's downside deviation as the denominator of the ratio. This being the difference it shows when compared with

the Sharpe Ratio and a main reason on why it is thought to give a better view of the portfolio's risk-adjusted performance, since positive volatility is a benefit.

As written by the authors, the Sortino ratio has for the numerator the difference between the return of the portfolio and the minimum acceptable return, which is often taken as the riskless asset (Sortino & Price, 1994). The denominator for it is different from the Sharpe Ratio, as said above, being the downside deviation.

As such, and assuming the same considerations as given to the Sharpe Ratio, we have the Sortino Ratio given has:

$$S_o = \frac{Rp - rf}{\sigma_d} \quad 5.1$$

Where  $Rp$  stands for the return of the portfolio,  $rf$  for the risk-free rate and  $\sigma_d$  for the standard deviation of the downside.

Sortino complements that if we look further into the theoretical differences of both ratios we see that while the Sharpe Ratio indicates how much excess return above the risk-free rate is received for the risk associated, then the Sortino ratio indicates how much excess return above the minimum acceptable return is received for the risk of not achieving that minimum acceptable return (Sortino & Price, 1994).

One interesting piece of trivia regarding this ratio lies in the fact that the Nobel prize winner Harry Markowitz recognized that since only downside deviation is relevant for investors, using it would be the most appropriate measure of risk. However, he did not consider it in his work as using it was impractical at the time (Rollinger & Hoffman, 2014).

Like the Sharpe Ratio, from which it obviously draws many similarities, a higher Sortino ratio result is naturally better. When comparing two different investment strategies the investor would obviously be more impressed by the one that earns more return per unit of bad risk that it takes on itself.

### 3. Data Analysis - Bitcoin

We feel it is important to analyze both Bitcoin's performance over its inception as well as that of the markets that we will pit Bitcoin against in attempts to try to achieve a diversified wallet.

It is important to consider the scenario that this data is set on. Bitcoin's inception data used on this work begins at the 29<sup>th</sup> of April of 2013 and goes forth to end in 27<sup>th</sup> of November of 2019. Coincidentally this is right in the middle of one of the greatest and longest bull runs the market has ever seen, before toppling down quite a bit in a correction that happened in December of 2019.

We feel it is important to understand the context the overall markets lived in at the time. The general bull euphoria was at all-time highs and unprecedented returns were seen across several markets. Bitcoin does not exclude itself from this general panorama. This macroeconomic feeling of confidence in the markets was indeed quite pivotal in making bitcoin what it is today.

Let us look at the numbers, starting by looking at bitcoin's cumulative returns since its inception, as mentioned before being April of 2013.



*Figure 3.1. 1: Bitcoin Cummulative return from 2013 to 2019*

*Source: Processed from CoinMarketCap data (2020)*

It is obvious by just looking at the data that the greatest spike in trading and returns occurred in the years 2017 and 2018. Trading euphoria was high on that period, seeing



bitcoin going from an already respectable value of over 1000 dollars per bitcoin, all the way to a staggering 19,497 dollars at its all-time high on the 16<sup>th</sup> of December of 2017.

When we put this into relative terms. The numbers are even more shocking. In the period where it saw its steepest bull run, bitcoin went up some shocking 1,704.5% in a bit more than 7 months. Really lucky investors could have made in that period just a bit over half a year a fortune that would last them several lifetimes over.

However, not only those were fortunate. When we join luck with patience the results are much higher. If we look at the astronomically unlikely scenario of an investor that bought in at the start and sold at the all-time high, that investor would have made a 13,389.3% on his initial return. Those numbers are unseen in any other commodity and speak volumes to the potential that bitcoin has.

The event described above is quite unlikely. Interest remains in seeing what would have happened to those so called “diamond-hand” investors. Those that bought and held on through the entirety of the asset’s life for believing in its potential. If we look at those cases of investors that bought in at start and still had it in their portfolios at the end of the data collected, they would have also achieved not a shabby return at all. They could consider themselves 5,110.8% richer on that investment.

As commonly is in life, it is not just upsides. There are some big and steep downsides that turn investors away from the cryptocurrency such as a higher aversion to risk and potential to downswings. Chiefly among those stand the volatile characteristics of the asset.

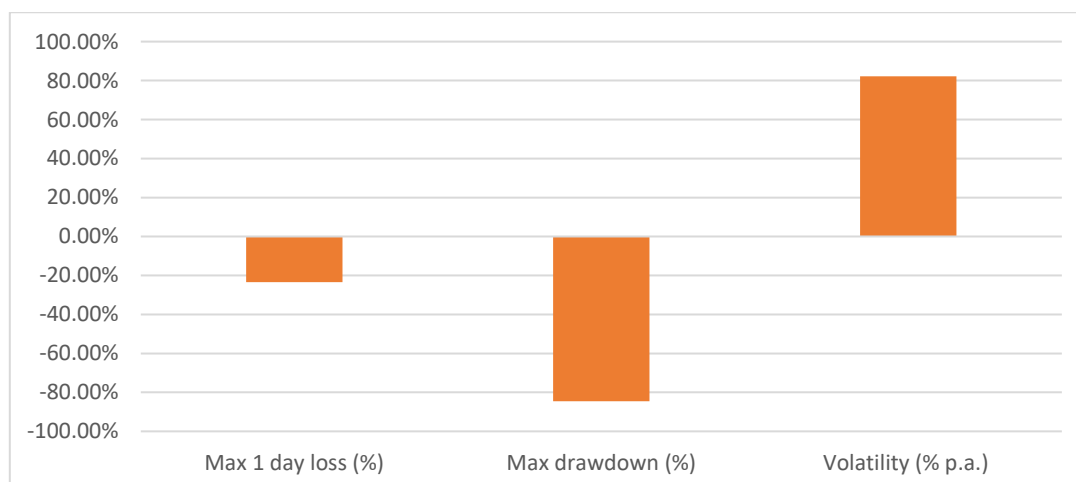


Figure 3.I. 2: Bitcoin drawdown and volatility characteristics

Source: Processed from CoinMarketCap data (2020)

Looking at the data above it is quite clear where the weakness lies. We can see that the maximum loss that occurred in one trading day (specifically on the 18<sup>th</sup> of December of 2013) was quite high standing at almost ¼ of the entire value of the asset at the time, losing 23.37% on a single day, while on average, whenever the market closed red the loss was around 2.69% percent of the portfolio.

When we focus our attention on how much the portfolio receded over time before starting to recover from its losses and having its first positive day, that is the drawdown, we can see that it stands at a high -84.5%.

With the information reflected above it is not shocking that the volatility inherent to bitcoin is high. Over the period and when adjusted with the square root of time it stood at a high 82.3% *per annum*.

It stands as important to notice as an afterthought that the number of upswings exceeded that of the downswings over the period, with values of 1299 and 1104, respectively. This is interesting to conclude as it shows that the trend has been of constant growth. It is not only that the swings in price are higher when it goes up than when it recedes, but also that those upswings occur more often.

It is important to also study the effect of volatility more in depth and try to analyze its characteristics and trends.

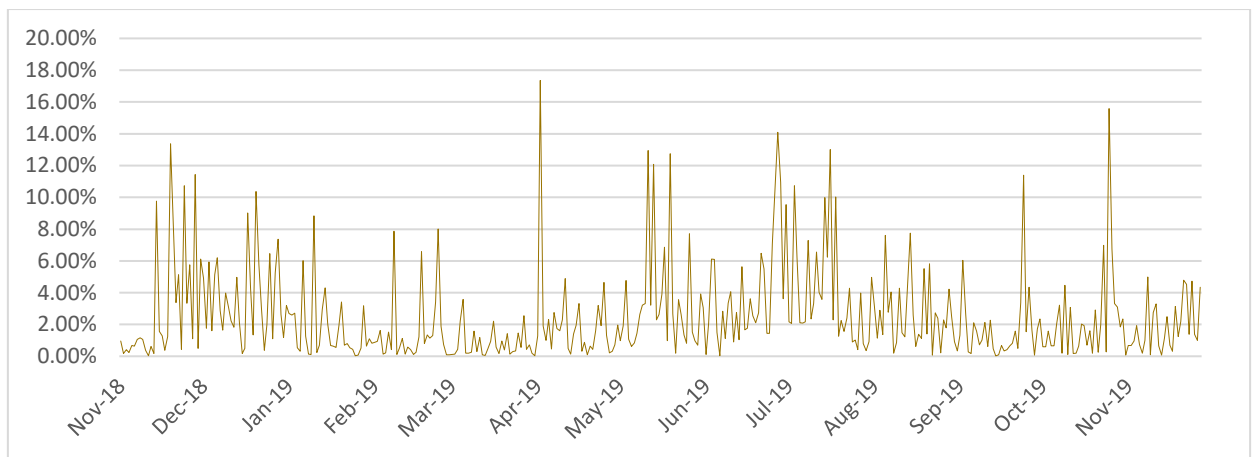


Figure 3.1. 3: Bitcoin rolling 1 year volatility

Source: Processed from CoinMarketCap data (2020)

On the above figure we can see how the volatility has behaved over time, with the necessary adjustment to consider the square root of time. As expected, the rolling volatility over one year shows a consistently high value, starting by almost reaching 100%. These values tended to average out over time and hovered around a “milder” 60% over the period. As expected, we can see that volatility trends well with the returns of the cryptocurrency, finding its highest ridge happening around the same period when the asset spiked in price, i.e. in the period between April 2018 and 2019.

When we look at the past year, volatility seems to have stabilized being quite often below the 10% range. However, as shown on Figure 3.1.4, the number of peaks surpassing this value still happen quite often, reaching almost 18% at one time. The cryptocurrency inherent volatility is still high by any measure.



*Figure 3.1. 4: Bitcoin absolute daily 1-year volatility*

*Source: Processed from CoinMarketCap data (2020)*

The same analysis has been made for the other assets covered in this project (S&P 500, MSCI, NASDAQ, and Gold) and can be found in Appendix A.

### 3.1.Comparison between Assets

#### 3.1.1 Performance

With all the above said we can now look at the numbers together and see how they all add up.

One subject not yet looked at, at least directly for it has been talked about repeatedly indirectly by basis of the cumulative returns of the portfolios, is the performance that each asset showed over the years. This performance has been annualized for periods above one year so that the ease of comparison between them is higher.

<b>Performance</b>	<b>Since Inception</b>	<b>5 Yrs (p.a.)</b>	<b>3 Yrs (p.a.)</b>	<b>1 Yr</b>	<b>6 Months</b>
<b>Bitcoin</b>	82.4%	82.9%	117.2%	99.3%	-13.2%
<b>S&amp;P 500</b>	10.9%	8.8%	12.7%	18.0%	11.6%
<b>NASDAQ</b>	15.9%	12.8%	17.4%	22.9%	14.0%
<b>MSCI World</b>	9.3%	7.8%	12.4%	17.6%	10.8%
<b>Gold</b>	-0.6%	3.5%	6.6%	18.5%	13.0%

*Table 1. 1: Performance comparison for all assets over time*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

The above table highlights those same performances mentioned above.

Considering all seen before, it is no surprise that bitcoin shows the greater numbers of the bunch. It shows incredible growth over pretty much the entirety of the investment period. An investor that put his trust and money into the cryptocurrency could have expected to get an increment of 82.4% on his portfolio per year over the period it has existed.

Those numbers peak over the three-year period, in which the cryptocurrency saw its higher returns. This fact one could argue infer from figure 3.6.2 below where it is evident that bitcoin saw its highest upswing in returns over the period between 2016 and 2019, even when taking in consideration the also colossal downswing that followed the peak at the end of 2017.

Overall, this allows us to see that bitcoin greatly outpaced the entirety of the market on returns and an investor with capacity for holding the asset over a middle to longer period would make good bank on his investment. Day traders, our fast frequency ones, would however find their lives much trickier, thanks to the much higher volatility

and maximum one day losses incurred by this asset over its reported lifetime. These facts can be seen with more detail below on table 1.2.

But how have the other portfolios fared? As mentioned before, this data encompasses one of, if not the, biggest bull runs the financial markets have seen. So, it is natural to be expectant of good results from the markets. All of them can be said to have performed just so. While S&P is seen as a flagship of the American economy, it was NASDAQ which outperformed all the others, being some solid 5% above the other American index since the inception period. This advantage reflects what was seen over the other time periods considered, however it is seen narrowing over the shorter 6 months period.

MSCI composite seems to follow the same trajectory as the indexes above, albeit with the constraint of being lagged by the more fringe and weak economies that compose it along with the American one.

We turn to Gold and we see some entrancing characteristics. It shows a solid constant upward trend. While its performance since inception is the only one of the bunches that is negative, it also is the only one that we can see constantly growing as the periods of time shorten. Paramount to that is the fact that it went to outpace both the S&P and MSCI World composite over the 1 year and 6 months more recent periods. Along with worst performances seen overall by all the other markets, this could be interpreted as a rise of fear in investors, that seek the more secure alternative. It is true that stocks and gold are often inversely correlated. Also, true appears to be the fact that the same pattern occurs with bitcoin, that is seen in investors eyes more as a common stock, than as a commodity rivalling gold.

One last fact that sticks to mind is that, disregarding gold's since inception return, only bitcoin shows a negative return on the periods studied. This occurs over the recent 6 months in which it presents us with a negative return of 13.2%.

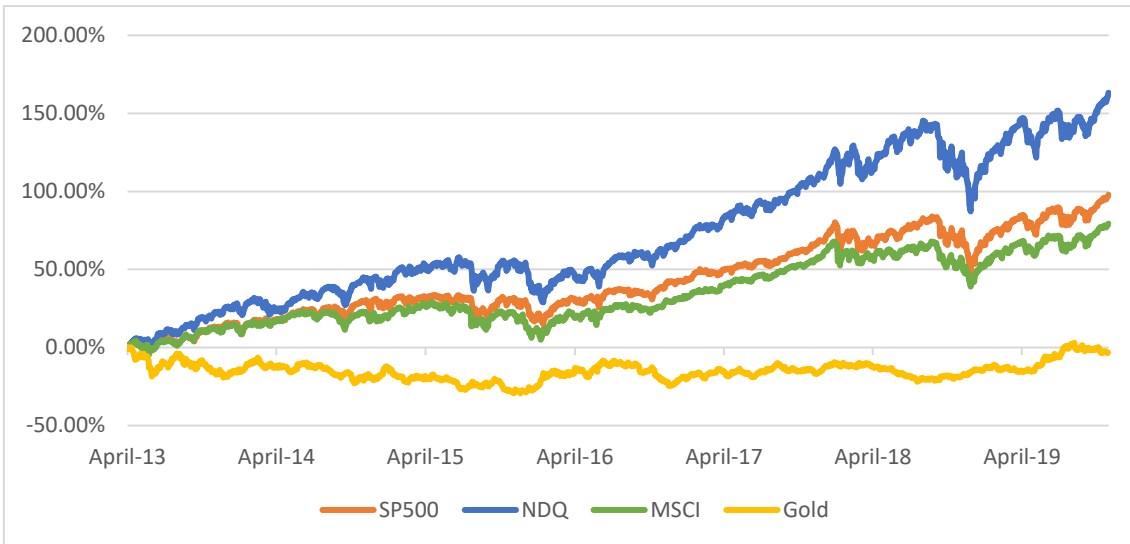


Figure 3.2. 1: All Assets excluding Bitcoin Cumulative return from 2013 to 2019

Source: Processed from Yahoo Finance data (2020)

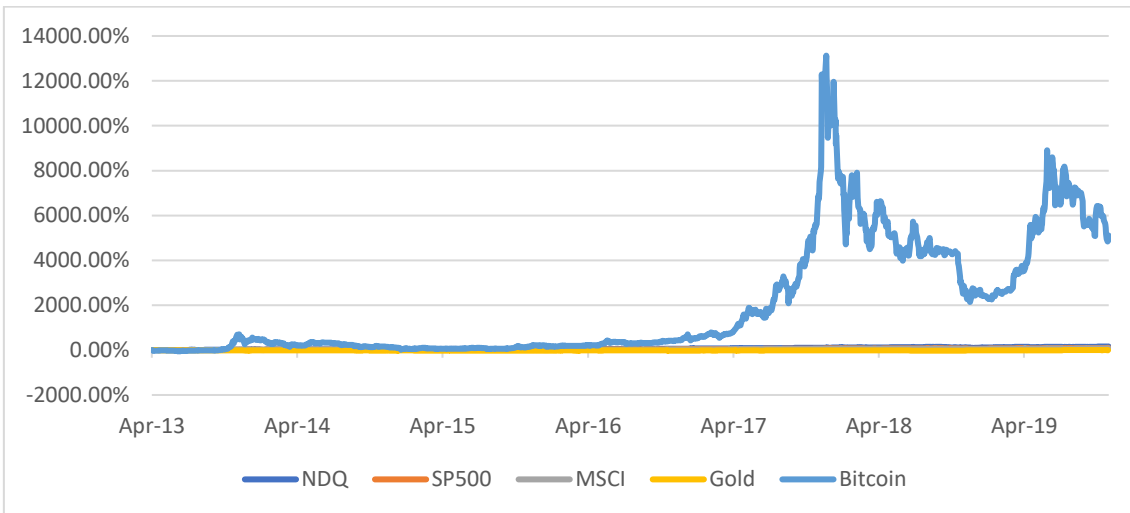


Figure 3.2. 2: All Assets Cumulative return from 2013 to 2019

Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)

In the figures above we can see two similar but very distinct cases. It barely looks like the same graphic at all. On the first, that being figure 3.6.1 we have the cumulative returns of all the portfolios in study and mentioned above. In them we see all the characteristics already mentioned before when we went through them individually. The most striking appears to be how, apart from gold, the 3 indexes perform relative to one another. Their trends are extremely similar over time, performing each in line with the other.

When we had bitcoin to the comparison we can see truly how gargantuan its performance was when compared with the others, being so overwhelming that it dwarfs

all other returns (that as we previously stated are pretty respectable) into a straight line that barely appears to leave the X axis.

### 3.1.2 Volatility

Now let us shift focus for a minute and follow with the volatility characteristics comparison.

<b>Volatility</b>	<b>Max 1 day loss (%)</b>	<b>Max drawdown (%)</b>	<b>Volatility (% p.a.)</b>	<b>Average daily volatility</b>
<b>Bitcoin</b>	-23.4%	-84.5%	82.3%	2.7%
<b>S&amp;P 500</b>	-4.1%	-19.8%	13.2%	0.6%
<b>NASDAQ</b>	-4.4%	-23.6%	15.8%	0.7%
<b>MSCI World</b>	-4.5%	-18.6%	12.7%	0.6%
<b>Gold</b>	-5.4%	-29.6%	14.4%	0.6%

*Table 1. 2: Volatility comparison for all assets over time*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

As before, immediately it is evident just how much bitcoin outpaces all the other portfolios. Where annualized volatility is concerned, it stands almost 60% above the second most volatile portfolio, while all the others stand pretty much all close to each other. There is no surprise here, if only there could be in just how much more the volatility is. This is probably the most dangerous characteristic of the cryptocurrency and what keeps investors away the most. If one could pinpoint its biggest weakness, volatility would for sure be it. All the other columns appear to be in line with this much higher volatility, with bitcoin presenting much higher values for all the other fields in analysis.

All the stock dominated portfolios, that being S&P, NASDAQ and MSCI composite show values that are much in line with one another. The biggest discrepancy to note is on the Max drawdown suffered by NASDAQ that deviates, albeit not by much, from that trend. Considering only these characteristics, gold appears to be a weaker investment, with overall weaker values than the stock portfolios.

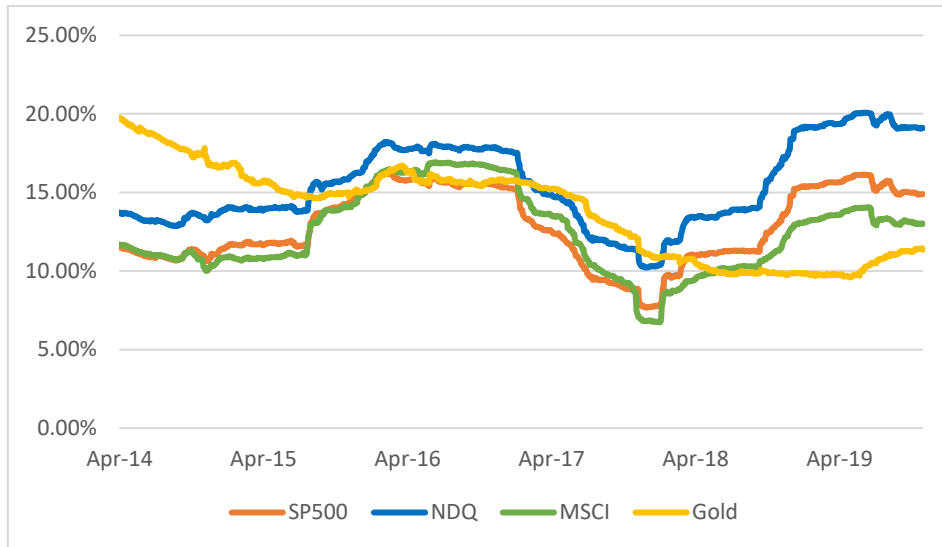


Figure 3.2. 3: Rolling 1 year volatility for all assets except Bitcoin

Source: Processed from Yahoo Finance data (2020)

The rolling over one-year volatility, as displayed on the figure above, appears to be well in line with what was stated about the stock portfolios and gold. As is plain on the graph they all appear to follow the same trend almost perfectly. Of course, gold is the exception to that rule, showing an independent trend.

We can easily see nonetheless that they all are generally on the same range, not differing by great magnitudes from one to the other. This *status quo* changes when we add bitcoin to the graphic, as seen below:

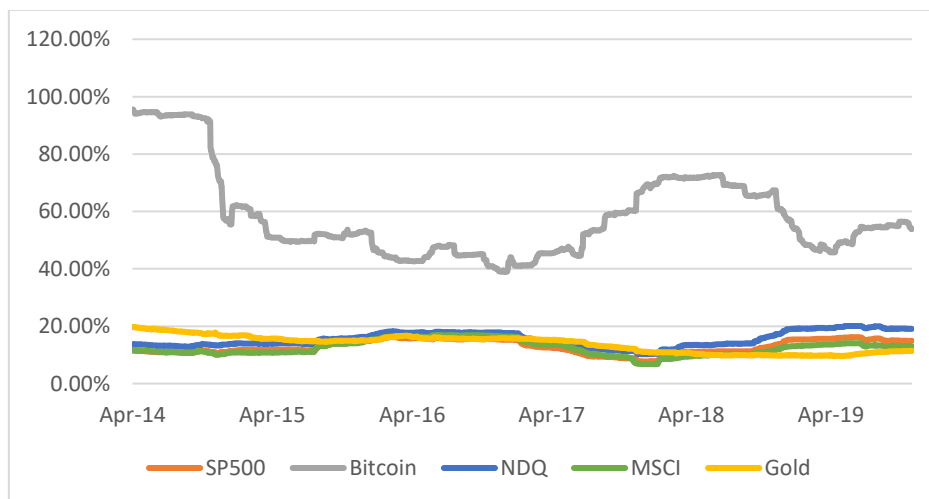


Figure 3.2. 4: Rolling over 1 year volatility for all assets

Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)



Here it is even more evident how bitcoin differs from these volatility characteristics. It is extreme to the point of dwarfing all the other assets so that they are imperceptible from one another.

As we can see when compared with some of the most known market replication portfolios in the world such as those ETF's, it is quite obviously clear that bitcoin stands on a league of its own where returns are concerned. However, it would be unfair to them not to consider the recent bias associated with the commodity and comparing it with established market replication strategies with much less potential for up and downswings. The inherent volatility linked to bitcoin, its status as a new commodity and the potential associated with it is what potentiated this climb.

### 3.1.3 Correlations

One of the most important rules to be a successful investor, as evidenced by Markowitz and mentioned earlier, is diversifying risk as much as possible. Both the systematic risk. That which for example two companies of the same sector share, as the specific one, being the one linked to a specific company.

Taking that into account and bringing it to our study, with bitcoin being a new investment, it is interesting to see how it fares when correlated with the other mature assets. With an initial hunch one could see it being similar with the stock market, seeing it being traded as such and with such an exponential growth that is in line (or even higher) than well-known companies of the tech sector such as that of Amazon and more recently Tesla, the comparison with the latter one being especially interesting taking into account the high price to earnings ratio of Tesla bringing into itself the “bubble” nickname, as it has happened with bitcoin recurrently. On the other hand, a weaker argument could be made for it being close to gold, taking into account its similarities as an alternative investment; weaker argument it remains as we have seen before that the behavior of bitcoin in nothing compared to that of the precious metal.

#### Correlations

	SP500	NDQ	MSCI	GOLD	BITCOIN
SP500	1,00	0,95	0,61	-0,13	-0,01
NDQ	x	1,00	0,56	-0,13	0,00
MSCI	x	x	1,00	-0,08	-0,01
GOLD	x	x	x	1,00	0,00
BITCOIN	x	x	x	x	1,00

Table 1. 3: Correlations between Assets

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

Some interesting conclusions can be taken from this data. Some go well in line with the inborne expectations, some other break the mold of what was initial thought to happen.

As we can see there is a proximity between the S&P 500 and the NASDAQ. This is well within the realms of the expected seeing at all similarities that the indexes show between them. With the S&P 500 being an agglomeration of the US's largest companies, a considerable amount of those are also present on the NASDAQ, that by itself more than explains the correlation of the two being so close to one.

If we explore further, we can easily infer that a similar, albeit less extreme situation occurs with the MSCI index. While not as blatantly obvious as the previous case, with it being composed by stocks we can expect those to follow an overall similar trend, as for the most part they are both similarly affected by the macro environment.

With gold we are exposed to an inverse correlation. This too is expected, as gold has historically (and continues to be...) seen as a haven for investors when the waves of the stock market start crashing down. On bear markets it is common to see a surge of demand for gold and as such its price naturally rises. This is not a perfect inverse correlation however, as gold has more or less kept a stable value over the studied periods. As such the small inverse correlation is correspondent to expectations.

Now comes the most interesting part, being bitcoin. It does not show neither a correlation nor inverse correlation with any asset. Bitcoin appears to be on a dimension of its own, with its value not being impacted by either upturns or downturns in the stock market. This is unexpected and interesting, as one could most directly identify it to a stock, and as such see at least some semblance of correlation with one of the indexes.

One interesting idea for future research would be to compare bitcoin with the behavior of some high growth stocks, such as some periods of the ones mentioned before. As we stand however, bitcoin remains as an almost independent event. This occurrence can be highly valued by investors that seek to distinguish their investment from the norm. They can possibly see in bitcoin just that.

We end this segment as we started. Bringing back the notion stated by Markowitz of the importance of diversifying. Bitcoin stands out for the investor that seeks such paths.

## 4. Methodology

As presented before, methodology gives an idea of the approach to take now that we analyzed the data and the nuances involved.

Our goal now passes by creating portfolios that incorporate bitcoin into their framework as a possible investment choice for investors. We aim to explore the widely accepted ideas divulged by Markowitz and display a portfolio that is composed by a combination of assets that stands on the Markowitz frontier of efficiency, being the minimum variance portfolio explored on the literature. To do this we will apply the method of finding the frontier equation, from where we will derive the correct allocation to be allocated to each of the two assets.

For comparison of results not only between different assets, but also between portfolios including them, we will also take the approach of building a portfolio composed by 50% of bitcoin and 50% of the complementary asset that will be used to build the wallet in each particular iteration.

Taking into account that the adjunct assets used with bitcoin are, in the exception of gold, some already extremely diversified assets, as such is the nature of market replication indexes, we feel that the combination of those 2 assets more than justifies the need for diversification. With that said, we think that the exercise of combining bitcoin and gold also proves itself interesting and as such the same methodology will be applied to it

Looking forward, we will focus on calculating the ratios that we also explored on the literature review of this project, and with those take conclusions on the usefulness and adequacy of adding bitcoin to our investments. We will search for those metrics on both the minimum variance portfolio of each pair of assets, and the more naïf 50/50 approach taken, with comparisons between the two hopefully adding value to the overall goal. For the application of those ratios, a risk free active needs to be taken. We will opt by using the US one-year treasury yield, as that for such calculations, that is the one of the most widely regarded financial instrument that guarantees a riskless return for the investment. At the time where our data ends that value stood at 1.6%.

Henceforth measures will be taken to assure the most accurate interpretation possible of the data, with different time frames being chosen from bitcoin's run. We will

try to look at the entire run, and at specific intervals such as more bullish and bearish periods and try to understand how and how much those different environments will end up affecting our results and considerations.

When interpreting the results from the ratios we will make use of the consensus in what regards a good and bad result on the ratio, knowing that a good one would give a good amount of expected return exposed to the least amount of risk possible we get that:

- A positive Sharpe ratio is considered acceptable
- When higher than 1 it is considered good.
- Higher than 2 is considered very good
- Above 3 is considered excellent
- Below 1 is considered not good and worthy of investing in (of course, not withstanding specific situations and investor needs), albeit acceptable if positive (as stated above)

With this approach it is believed that we will be able to capture a vast enough array of situations and time frames and as such be confronted with more nuances and details that might help with our final decision (if a clear one is possible to be taken).

In a more detailed description, for the calculus of the Sharpe Ratio we will first calculate the annualized returns and standard deviations of each of our combinatory assets. We will also see how each of the different assets correlate between themselves. To reach these results we are armed with the daily prices of every active. Once treated we will make use of the daily change in value to make our calculations.

Next, use will be made of the Markowitz equation to reach the optimal MVP portfolio weights. Those will be used to answer the question of the value of the expected return for the portfolio and its respective standard deviation. The same exercise will be ensured for the 50/50 portfolio. This will be applied for 3 different time scenarios. Scenario 1 will encompass our entire data frame. Scenario 2 will emphasize the bearish market conditions that occurred between December 2017 and December 2018. And finally Scenario 3 will take into account the more bullish market that occurred from December 2018 until the end of our data on November. Those scenarios naturally will have different characteristics between themselves. We hope that this will help in achieving more robust and solid conclusions.

With all of this we are ready to reach our Sharpe Ratio which will be then calculated, compared between the different scenarios of the same combination of assets and between different combinations and conclusions will then be taken.

For the Sortino ratio the story is slightly more complex. We can easily calculate the Sortino ratio of the individual assets. For that calculation we will for every scenario assume a risk-free asset to which we will compare the daily returns to. Our goal of course is to outperform those risk-free daily returns. The days where that does not happen will be the ones used in the calculation of downside volatility, our semi variance and semi standard deviation and ultimately our Sortino Ratio.

With those calculated we will count the number of observations of our data and divide the squared sum of occurrences by that same number to reach our semivariance that after will be squared and annualized to reach our goal of the annualized semi standard deviation.

Now armed with the denominator of our Sortino ratio equation we will easily reach the numerator by means of the difference between our expected returns and our risk-free rate and then achieve the goal of calculating the Sortino Ratio.

This is considerably more detailed for the Sortino ratio of the MVP and 50/50 portfolios that we used in the Sharpe ratio. For those we will have to recalculate our daily returns using respectively, the weights that we calculated before for the frontier portfolio and a mix of half of each asset for the 50/50 portfolio. We will then consider those returns when applying the method described above for the individual assets.

When we finally reach our Sortino ratio for all portfolios we will compare it to the same result achieved by the Sharpe ratio in each situation and see how they compare and what conclusions can be taken about the relative and absolute performance of every combination of assets.

## 5. Results

### 5.1. Sharpe Ratio

#### 5.1.1 Entire data

Let us bring back the Sharpe Ratio formula that we explored beforehand in which we needed the Return of the portfolio, the return of the risk free asset (which we declared beforehand to be the US one year treasury yield rate of **1.6%**) and the standard deviation of that same portfolio of which we are fetching the return of.

These returns need of course to be annualized to give any semblance of sense to the results. As such we proceed to do that step and we get the following expected returns:

Asset	Return	Standard Deviation	Sharpe Ratio
Bitcoin	157.6%	83.1%	1.88
S&P 500	12.0%	13.1%	0.79
NASDAQ	17.4%	15.7%	1.00
MSCI World	10.2%	12.6%	0.68
Gold	0.4%	14.2%	-0.09

*Table 2. 1: Annualized expected returns, annualized standard deviations and Sharpe ratios*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

These results are highly in line with what was expected with bitcoin clearly showing a much higher return than those of the indexes and gold.

When we look at the behavior of the standard deviations after being annualized below, we see the exact same behavior. Bitcoin still pulls ahead by a landslide when compared to the other assets, however with the slight nuance that the difference between them is not as accentuated as it was regarding expected returns.

With all this data in hand and after appropriate treatment of it to get the accurate annualized percentages for the portfolio that can be found on Appendix B it becomes quite a simple exercise to build our generic 50/50 portfolio composed of bitcoin and one of the other 4 assets.

50/50 Portfolio	Return	Standard Deviation	Sharpe Ratio
Bitcoin + S&P 500	69.9%	48.1%	1.42
Bitcoin + NASDAQ	74.0%	49.4%	1.46
Bitcoin + MSCI World	68.6%	47.9%	1.40
Bitcoin + Gold	60.9%	48.7%	1.22

*Table 2. 2: Expected Return, Standard Deviation and Sharpe Ratio of the 50/50 portfolios*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

All, except for gold, are in line with each other in what Sharpe Ratio is concerned. This is explained by the presence of bitcoin having such an overwhelming effect on the portfolio with its insanely high returns and volatility that it gobbles up much of the effect that the other asset would have in bringing it down. This also explains the fact that gold has such a high Sharpe Ratio, relatively close to the markets all things considered, when it has such a lower expected return compared to them (0.4% vs 13.1% when compared to the S&P 500) and a similar standard deviation (14.2% vs 13.1% of the S&P 500).

If we look only at this 50/50 wallet, we can see that the investment is clearly quite profitable, showing some good to very good Sharpe ratios. If we take into consideration the previously mentioned criteria for analyzing the ratios, we can see that they are between 1 and 2 in absolute value, which represents a good to very good investment.

It is interesting to compare these numbers with the absolute Sharpe ratio we would show for our efforts if we only owned one of those investment options, which serves as reinforcement to the previously mentioned notion that bitcoin is the one pulling most of the weight in our 50/50 portfolio. Over the time frame that captures our entirety of data we can see that adding any other asset to bitcoin would only serve to dampen its Sharpe ratio, and as such attractiveness for investors. Obviously, this while being an interesting conclusion to take on an ad hoc approach comes with a severe risk involved. That of having a portfolio composed in its entirety by one asset, and one so recent and volatile as bitcoin. With that said, there is no surprise in these results... if only the most surprising fact being that gold shows such a low ratio, being the only negative one. We were already aware of the meteoric rise in price and spotlight that the cryptocurrency had over its inception.

Which brings us already to one further conclusion. If we are to look only for the highest possible Sharpe ratio, we found it. It is linked with the portfolio composed in its entirety by bitcoin. Diluting the % of bitcoin on the wallet will only have the effect of also diluting the Sharpe ratio, as we see no other asset with a low enough risk that is able to raise a challenge to the overwhelming effect that bitcoin's returns presented.

This is not feasible as an investment for many investors. The majority of those are not likely to decide turning all their portfolios into bitcoin. A rational investor could very likely seek the minimum variance portfolio, and that is exactly the rational that we are to follow.

And so, over the entire timeline of our data, we can follow the Markowitz method mentioned on the literature and we would reach the following results (the calculations can be found on the appendixes of this project for the weights of the minimum variance portfolio:

- Bitcoin + S&P 500:

$$W_{Bitcoin} = 2.6\%$$

$$W_{S\&P\ 500} = 97.4\%$$

- Bitcoin + NASDAQ:

$$W_{Bitcoin} = 3.5\%$$

$$W_{NASDAQ} = 96.5\%$$

- Bitcoin + MSCI World

$$W_{Bitcoin} = 2.4\%$$

$$W_{MSCI} = 97.6\%$$

- Bitcoin + Gold

$$W_{Bitcoin} = 2.8\%$$

$$W_{GOLD} = 97.2\%$$

Now with the portfolio weights found we proceed with the same method as done before with the 50/50 portfolio, using the expected returns and standard deviations to reach our Sharpe ratios, we get the following:

<b>MVP Portfolio</b>	<b>Return</b>	<b>Standard Deviation</b>	<b>Sharpe Ratio</b>
Bitcoin + S&P 500	14.4%	14.9%	0.86
Bitcoin + NASDAQ	20.6%	18.1%	1.05
Bitcoin + MSCI World	12.5%	14.3%	0.76
Bitcoin + Gold	3.1%	16.1%	0.09

*Table 2. 3: Expected Return, Standard Deviation and Sharpe ratio of the MVP portfolios*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

We are faced with considerably lower Sharpe Ratios with most of them being below the 1 threshold. These numbers are however still highly interesting for investors if we consider some reflections.



Let us proceed to the exercise of comparing the results for our portfolio alternatives side by side:

	Sharpe Ratio	
	50/50	MVP
Bitcoin + S&P 500	1.42	0.86
Bitcoin + NASDAQ	1.46	1.05
Bitcoin + MSCI World	1.40	0.76
Bitcoin + Gold	1.22	0.09

*Table 2. 4: Side by side comparison of the 50/50 and MVP portfolios Sharpe ratios*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

Chiefly and most importantly, we can conclude that while the weight allocated to bitcoin has been drastically reduced in the MVP portfolio, the Sharpe ratio of the portfolios that were bundled with indexes did not fall in the same proportion. The drop was relatively lesser. This serves as proof that bitcoin would have brought a positive addition to investors that decided to allocate as little as a range of 2.5% to 3.5% to their market replication indexes.

The second biggest argument to be made for the implementation of bitcoin in an investor’s portfolio is seen when we analyze how much it increased the Sharpe Ratio when compared to a portfolio composed 100% by the second asset.

Let us take into example the NASDAQ. Its Sharpe ratio as a standalone investment was of 1.0. However just by introducing into our wallet 3.5% of bitcoin, we can see our overall investment’s Sharpe raise to 1.05. That is an increase of 5% on our risk-adjusted performance, while only increasing our exposure to bitcoin by 3,5% of the portfolio’s value. This is true for all portfolios, with gold showing a negative result of 0.09 as a standalone portfolio, saw that value raise to a positive value of the same magnitude with the introduction of 2.8% of bitcoin.

The presence of the risk-free asset into our considerations also has a bigger impact on the MVP portfolio. As our returns show a big decrease in absolute value, (taking the S&P 500 case, from some 69.9% to 14.4%) the subtraction of the 1.6% of the US one year treasury yield takes a much higher relative relevance, which naturally impacts negatively the Sharpe ratio.

### 5.1.2 Bearish scenario from December 2017 to December 2018

Our next step lies in order to expand the range and accuracy of our conclusions lies in taking a look at one of bitcoin's largest bear run, that occurred between the all-time high of December of 2017 and one of its minimums that occurred at December of the following year.

Taking into account the short timeframe involved on this piece of data, naturally this is to be seen as an extra piece of information for decision making, attempting to describe how the value of our investment portfolio could be in a repetition of the worst period experienced.

For this period, we also retrieved a different risk-free rate that is set at the end of the period, so that the results are more accurate and in line with reality. At the 15<sup>th</sup> of December of 2018, this rate was of **2.68%** and it is what shall be used in all calculations henceforth.

As before let us take a look at the annualized expected returns, standard deviations and Sharpe ratio of the individual assets:

Asset	Return	Standard Deviation	Sharpe Ratio
Bitcoin	-75.5%	78.7%	-0.99
S&P 500	-1.7%	15.5%	-0.28
NASDAQ	1.5%	19.4%	-0.06
MSCI World	-3.6%	14.4%	-0.43
Gold	-1.4%	9.5%	-0.42

*Table 2. 5: Annualized expected returns, annualized standard deviations and Sharpe ratios on the bearish scenario  
Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

As expected, when compared to our previous results, our expected returns are considerably worse. The biggest absolute change in value appears to be linked to bitcoin. This too is too be expected as it is the asset with the most volatility associated. On the other hand, gold shows an inversed behavior, displaying the lowest change in expected returns. All the market indexes appear to behave quite similarly to one another. This mirrors the behaviors and correlations that we found above.

On what the deviations are concerned, those too show some worse results. With the notable exception of Gold, which shows an increased degree of stability over the run.

### Correlations

	SP500	NDQ	MSCI	GOLD	BITCOIN
SP500	1.00	0.95	0.46	0.03	0.13
NDQ	x	1.00	0.39	0.03	0.12
MSCI	x	x	1.00	0.13	0.06
GOLD	x	x	x	1.00	-0.07
BITCOIN	x	x	x	x	1.00

*Table 2. 6: Correlations of the bearish scenario*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

Exploring the correlations, we notice that there are some big differences on gold's and bitcoin's when compared to those found on *Table 1.3*. While bitcoin beforehand appeared to be quite uncorrelated with most assets, it now shows a higher degree of correlation with the index funds, especially with S&P 500 and NASDAQ. This is noteworthy as it shows a possible change in paradigm and on the mindset of bitcoin investors, that tend to start seeing it more easily compared to stocks as time goes by. Gold on the other hand stopped showing an inverse correlation with the indexes and now even shows a minor correlation with them. With bitcoin it now also shows a negative correlation.

Armed with this new information we proceed to, as before, calculate our 50/50 and MVP portfolios, noting that all details beforehand mentioned regarding portfolio composition still apply and naturally, starting with the former:

50/50 Portfolio	Return	Standard Deviation	Sharpe Ratio
Bitcoin + S&P 500	-50.9%	47.1%	-1.14
Bitcoin + NASDAQ	-50.1%	49.1%	-1.08
Bitcoin + MSCI World	-51.4%	46.6%	-1.16
Bitcoin + Gold	-50.8%	44.1%	-1.21

*Table 2. 7: Expected Return, Standard Deviation and Sharpe Ratio of the 50/50 portfolios on the bearish scenario*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

As expected, all Sharpe ratios are now negative. As such we revert to the literature and aim to compare them between themselves, knowing however that none has shown signs of being a profitable investment over the timeframe. As before the order of investor preference seems to be the same. With NASDAQ still being the most attractive portfolio and Gold the worse.

As before the greatest driver for this negative Sharpe ratio is still Bitcoin, whereas before it showed the highest standalone Sharpe ratio, it now also shows the worst, as we can see above on *Table 2.5*.

One interesting conclusion to take here is that even during the worst bear run showed by the bitcoin market, it still shows a Sharpe ratio of -0.99. In relative values the Sharpe ratio shown over the total timeframe studied before of 1.88 shows a bigger difference from 0. This trend might be a clue as to the overall future profitability of the asset.

Moving forward with the application of the Markowitz method to achieve our MVP portfolio we reach the following weights:

- Bitcoin + S&P 500:

$$W_{Bitcoin} = 1.3\%$$

$$W_{S\&P\ 500} = 98.7\%$$

- Bitcoin + NASDAQ:

$$W_{Bitcoin} = 3.1\%$$

$$W_{NASDAQ} = 96.9\%$$

- Bitcoin + MSCI World

$$W_{Bitcoin} = 2.3\%$$

$$W_{MSCI} = 97.7\%$$

- Bitcoin + Gold

$$W_{Bitcoin} = 2.2\%$$

$$W_{GOLD} = 97.8\%$$

With those in mind we reach the Sharpe ratios for our MVP portfolios:

<b>MVP Portfolio</b>	<b>Return</b>	<b>Standard Deviation</b>	<b>Sharpe Ratio</b>
Bitcoin + S&P 500	-3.4%	16.3%	-0.37
Bitcoin + NASDAQ	-2.9%	21.3%	-0.26
Bitcoin + MSCI World	-5.2%	15.9%	-0.50
Bitcoin + Gold	-3.1%	11.1%	-0.52

*Table 2. 8: Expected Return, Standard Deviation and Sharpe ratio of the MVP portfolios on the bearish scenario*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

If on our previous timeframe the MVP results were mellowed out by the lower exposition to bitcoin, now the same follows suit. However, none shows a positive Sharpe ratio, that is expected as we just saw that even the individual assets all showed negative ratios.

	<b>Sharpe Ratio</b>	
	<b>50/50</b>	<b>MVP</b>
Bitcoin + S&P 500	-1.14	-0.37
Bitcoin + NASDAQ	-1.08	-0.26
Bitcoin + MSCI World	-1.16	-0.50
Bitcoin + Gold	-1.21	-0.52

*Table 2. 9: Side by side comparison of the 50/50 and MVP portfolios Sharpe ratios on the bearish scenario*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

The side by side comparison helps us recap a conclusion already stated. On the most bearish scenario that we analyze here we conclude that the implementation of bitcoin has an opposite effect on the attractiveness of a portfolio for investors as it had before. While formerly its inclusion was the propelling force of a larger attractiveness now it is optimal to be exposed to smaller amounts of bitcoin in our most efficient wallet to limit our downside potential.

One caveat to add is the inclusion of a stronger risk-free asset in this scenario that weakens the Sharpe ratio. It proves itself more attractive for investors relatively (when compared to the lower yield used before) and absolutely, as it is now a better investment alternative for investors.

### **5.1.3 Bullish scenario from December 2018 to November 2019**

Finally, we will analyze the opposite direction. What would happen to our conclusions on a more bullish bitcoin run.

For this we will take data from the end of our above analyzed bear run, with start at December 2018 and go past the second most glaring peak evidenced by bitcoin all the

way into our data end, which occurs on November of 2019. We feel that there is more value to be added by including data past the peak as that would give us a more interesting look at how bitcoin has behaved over a longer recent period, while still being able to take conclusions of it as a bull run since it shows a considerable price increase since the low in December.

For this period, we will return to our first risk-less asset of **1.6%** as that is the value of the US one-year treasury yield as of November of 2019, otherwise our method will follow suit.

<b>Asset</b>	<b>Return</b>	<b>Standard Deviation</b>	<b>Sharpe Ratio</b>
Bitcoin	208.5%	83.1%	2.82
S&P 500	21.5%	13.1%	1.33
NASDAQ	26.6%	15.7%	1.22
MSCI World	20.8%	12.6%	1.37
Gold	18.4%	14.2%	1.45

*Table 2. 10: Annualized expected returns, annualized standard deviations and Sharpe ratios on the bullish scenario*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

Over this period, we see a curious development on our returns. Relatively, when compared to our initial data we see that now it was all other assets that showed an increase. Bitcoin of course also increased in its absolute expected return, but it was not as evident. Our most noticeable increase lied in fact in the gold asset, that went from a 0.4% annualized return to a new 18.4%.

Regarding the standard deviations, we see below that apart from bitcoin and gold they also increase slightly over the period. This could be of course linked with the market correction that occurred around December of 2018 in the equity markets. All of these make for some interesting market conditions.

Below we analyze the correlations between the portfolio components during this time-period.

<b>Correlations</b>					
	<b>SP500</b>	<b>NDQ</b>	<b>MSCI</b>	<b>GOLD</b>	<b>BITCOIN</b>
<b>SP500</b>	1.00	0.97	0.73	-0.22	-0.19
<b>NDQ</b>	x	1.00	0.71	-0.22	-0.17
<b>MSCI</b>	x	x	1.00	-0.22	-0.09
<b>GOLD</b>	x	x	x	1.00	0.25
<b>BITCOIN</b>	x	x	x	x	1.00

*Table 2. 11: Correlations of the bullish scenario*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

As we can see all of them seem to be in absolute terms higher than they were during our initial data set. This seems to be the opposite of what occurred during our bearish scenario where the correlations between assets were lower.

On the other hand, we see that bitcoin does not appear to follow this trend. Showing correlation absolute values that appear to be much more in line with those of the previous scenario, albeit in the opposite direction, being more inversely correlated in this case, when compared to the positive correlation that occurred before. During the correction of December of 2018 of the stock market, bitcoin was rising and while the markets rallied afterwards bitcoin started slumping leading to this inverse correlation that we see here.

Interesting too to see how bitcoin showed his biggest correlation of our entire study during this period with gold, valued at 0.25. This could be seen has a recent paradigm shift in which investors start seeing bitcoin more and more alongside gold as an investment to invert the stock markets.

Lastly, during these two bearish and bullish last scenarios we see bitcoin correlations that are in absolute value much higher than those of our initial data. This of course can be explained by the fact that that data encompasses bitcoin's inception period in which naturally there was still little awareness to it as an asset class. Only after that growth phase and early adopters got in could we expect to see a more realistic degree of correlations that we are starting to see now. An important caveat that shadows this entire study is the fact that we are working with a still extremely young new asset class. And as such there is still much that can change in a very little timeframe.

<b>50/50 Portfolio</b>	<b>Return</b>	<b>Standard Deviation</b>	<b>Sharpe Ratio</b>
Bitcoin + S&P 500	93.7%	44.2%	2.08
Bitcoin + NASDAQ	97.7%	47.0%	2.05
Bitcoin + MSCI World	93.1%	43.7%	2.09
Bitcoin + Gold	91.2%	42.5%	2.11

*Table 2. 12: Expected Return, Standard Deviation and Sharpe Ratio of the 50/50 portfolios on the bullish scenario*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

Above we see our usual portfolio half composed by bitcoin and the other half by one of our other assets. We can see that the ratios are yet again highly positive, but we also note that the difference in absolute values between the former and those of our portfolio made by the entire dataset is lower than the one between the portfolio of our bearish scenario and the initial one. This occurs by the fact that the market downturn was bigger than the rally that it went through shortly after.

We see that all the portfolios have Sharpe ratios hovering near 2, which as we stated before is very good. The biggest surprise comes from the Bitcoin + Gold portfolio that presents the highest of the bunch. While gold always lagged the data it now enjoys the best ratio because of the exceptional expected returns and low standard deviation that occurred during this data period.

Below, we see the individual Sharpe ratios that we would incur if we had a portfolio composed entirely of each of the assets in study.

In line with the expectations set before we see that gold is (after bitcoin naturally) the highest performer over the period. Interestingly, NASDAQ is behind all others even though it led them over the past two periods in study (again, after bitcoin).

We proceed then with the usual determination of our MVP portfolios:

- Bitcoin + S&P 500:

$$W_{Bitcoin} = 7.2\%$$

$$W_{S\&P\ 500} = 92.8\%$$

- Bitcoin + NASDAQ:

$$W_{Bitcoin} = 10.6\%$$

$$W_{NASDAQ} = 89.4\%$$



- Bitcoin + MSCI World

$$W_{Bitcoin} = 5.0\%$$

$$W_{MSCI} = 95.0\%$$

- Bitcoin + Gold

$$W_{Bitcoin} = -1.5\%$$

$$W_{GOLD} = 101.5\%$$

With those we proceed to the Sharpe ratio result of the portfolios:

<b>MVP Portfolio</b>	<b>Return</b>	<b>Standard Deviation</b>	<b>Sharpe Ratio</b>
Bitcoin + S&P 500	30.0%	19.2%	1.48
Bitcoin + NASDAQ	39.2%	26.2%	1.44
Bitcoin + MSCI World	26.6%	17.0%	1.47
Bitcoin + Gold	16.7%	10.7%	1.41

*Table 2. 13: Expected Return, Standard Deviation and Sharpe ratio of the MVP portfolios on the bullish scenario*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

One of the most notable takeaways lies with the Bitcoin + Gold portfolio. While it led the competition with the other portfolios on the 50/50 combinations it now lags by a considerable amount, with all the other being much in line with each other.

We see in our Bitcoin and Gold wallet that the most efficient portfolio is composed by a negative percentage of bitcoin. This shows that the combination of returns and standard deviations that gold has enjoyed over the period are too attractive to pass by on. We know that bitcoin has shown great returns over the data period and as such it is not its fault for the negative percentage on the portfolio. This would result in the act of shorting bitcoin.

Albeit not as easy as in the more regulated markets such as the equity market, this is still possible by using platforms that allow the trading of crypto currencies with margin calls or by means of other derivatives (even though with considerable more risk) such as options and futures.

	Sharpe Ratio	
	50/50	MVP
Bitcoin + S&P 500	2.08	1.48
Bitcoin + NASDAQ	2.05	1.44
Bitcoin + MSCI World	2.09	1.47
Bitcoin + Gold	2.11	1.41

*Table 2. 14: Side by side comparison of the 50/50 and MVP portfolios Sharpe ratios on the bullish scenario*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

When comparing both side by side we notice that while in the 50/50 composition gold pulled ahead of all other portfolios, it showed the opposite behavior in the MVP one. This occurs since the presence of the risk free active is more attractive in that composition has it has a bigger impact on the calculus thanks to the returns being smaller in absolute value. One interesting detail lies in the fact that this is the closest the MVP portfolio has been on relative value to the 50/50 one for most of the portfolios.

#### 5.1.4 All scenarios

50/50 Portfolio	Scenario		
	1	2	3
Bitcoin + S&P 500	1.42	-1.14	2.08
Bitcoin + NASDAQ	1.46	-1.08	2.05
Bitcoin + MSCI World	1.40	-1.16	2.09
Bitcoin + Gold	1.22	-1.21	2.11

*Table 2. 15: 50/50 Portfolio Sharpe Ratio of All Scenarios*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

MVP Portfolio	Scenario		
	1	2	3
Bitcoin + S&P 500	0.86	-0.37	1.48
Bitcoin + NASDAQ	1.05	-0.26	1.44
Bitcoin + MSCI World	0.76	-0.50	1.47
Bitcoin + Gold	0.09	-0.52	1.41

*Table 2. 16: MVP Portfolio Sharpe Ratio of All Scenarios*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

At last we have the results of our three scenarios side by side, with of course each scenario being numbered by order of appearance.

We see that naturally the most profitable one is Scenario 3, which is our most recent scenario. This is good news for the cryptocurrency as it shows a bullish sign for the future and its potential to serve as a useful tool for investors looking to diversify their portfolios. Of course as a still growing investment alternative it is not for the faint of

heart, which is shown by the fact that during Scenario 2, its worse period since its inception it was responsible for bringing down the entirety of the portfolio.

In all situations and scenarios, we see how bitcoin exacerbates the effect of the portfolio by a large amount, this happening even to an investor exposed to the lower percentages of the MVP portfolio. As such investors that have a lower risk tolerance should steer well away from this asset.

We also see that it can be coupled with different investment options and be coupled well with them. There was not one combination of bitcoin with another asset that pulled ahead of the others in all scenarios. The combination with NASDAQ seems however to be the one that better performed more often. Even so there are periods where it was the worse such as the 3<sup>rd</sup> period of our 50/50 mix.

Even gold, which has often shown the lowest Sharpe ratio had periods in which it outperformed all others. This too looks well for the cryptocurrency as it can be matched with different types of commodities to match the needs of investors in different market scenarios and not merely work as a complement of diversification to the equity markets.

If we evaluate the difference between the performance of the portfolios during their best and worst performance timeframe, we can get a clue to how much an investor can expect its Sharpe ratio to swing. This might clue us in on the volatility that those returns were linked to and give us yet another glimpse on how the investment developed.

While being similar in values, the biggest swing on the 50/50 portfolio occurred in the Bitcoin + Gold combination, where the portfolio went from a ratio of -1.21 on Scenario 2, to 2.11 on Scenario 3. On the MVP portfolio this occurred on the Bitcoin + MSCI World combination that went from -0.50 to 1.47. This serves to highlight the fact that the timeliness in which an investor entered his investment would have brought some massive differences to his portfolios Sharpe return, more so than one would reasonably expect to have with more traditional and risk averse investments.

Lastly, looking at our first scenario and over the entire timeframe of our data we state that apart from the worst pullback that the crypto asset faced, it would have been well used as a diversification alternative. Making up for its risk and assuring a very good Sharpe ratio for investors over most of its lifetime.

## 5.2.Sortino Ratio and how does it compare?

### 5.2.1 Entire Data

Now we will proceed with the same method for the Sortino Ratio, analyzing how the different portfolios behave in the different scenarios and see if by focusing on the semi-deviation, that being severity of the downside risk of an investment. For this we will set for our daily returns an objective of achieving a minimum return. This minimum return will change between the scenarios but will be the daily return achieved by the risk-free rate in the different scenarios.

For the calculation of our semi standard deviation we will be interested in two things:

The first being the sum of the squared differences between the portfolio daily return and our minimum acceptable return (given by the daily return of the risk-free asset). And the second being the number of observations that we have on our data set, either them being downsides or not.

With the division of the first by the total number of observations we arrive at our semi variance, its squared root is then the semi standard deviation which will immediately after be annualized to match the format of our previous data.

With all of this we are ready to calculate our Sortino ratios and apply them to our portfolios. We will be using the risk-free assets used before in the Sharpe ratio calculations for our Sortino ratio.

Again, the distinction between the Sharpe and Sortino is important, with the latter being linked with the possibility of loss. A higher Sortino ratio implies a lower chance of incurring in losses.

But before that let us use our previous expected returns and risk-free asset and with those look at the ratio of each of our individual assets:

	<b>Sortino Ratio</b>	<b>Sharpe Ratio</b>
Bitcoin	2.97	1.88
S&P 500	1.02	0.79
NASDAQ	1.31	1.00
MSCI World	0.87	0.68
Gold	-0.11	-0.09

*Table 3. 1: Sortino and Sharpe Ratios of the different assets*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

While hard and not much important to compare them in absolute values, it is interesting to see what trends they follow when both ratios are compared. That is, the behavior that each portfolio displays between ratios.

As expected, the values for the Sortino ratio show themselves as much higher than those of the Sharpe. As we use the risk-free asset as a target of profitability the numerator of our calculus will be shared between ratios. However, the denominator for the Sortino ratio will be the downside risk only and as such it is expected that the ratio shows these results exacerbating the ones shown by the Sharpe.

We can state that all investments apart from Gold were good over the period that encompasses our entire frame of data. And investors that give more importance to the Sharpe ratio would agree with those that focus on the Sortino.

Below we have data for both the 50/50 and MVP portfolios and the side by side comparison with their respective Sharpe Ratio.

<b>50/50 Portfolio</b>	<b>Sortino Ratio</b>	<b>Sharpe Ratio</b>
Bitcoin + S&P 500	2.53	1.42
Bitcoin + NASDAQ	2.66	1.46
Bitcoin + MSCI World	2.48	1.40
Bitcoin + Gold	2.20	1.22

*Table 3. 2: Sortino and Sharpe Ratios of the different assets for the 50/50 portfolio*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

We can see that the Sortino ratio appears to follow a similar trend to that of the Sharpe ratio, with increases on the former being seen over all the equity markets. While all of them showing increased Sortino ratios when compared to the Sharpe, Bitcoin is the

one with the highest increase, and as such one could expect these results as it would pull the 50/50 portfolio to a higher investment value.

Over all portfolios the conclusion appears to be the same for both the Sharpe ratio and the Sortino ratio, albeit with the results being much better for the latter which confirms the conclusions taken by the Sharpe and supports the observation that the 50/50 portfolios over the period were really good to excellent investment opportunities.

<b>MVP Portfolio</b>	<b>Sortino Ratio</b>	<b>Sharpe Ratio</b>
Bitcoin + S&P 500	1.29	0.86
Bitcoin + NASDAQ	1.63	1.05
Bitcoin + MSCI World	1.11	0.76
Bitcoin + Gold	0.14	0.09

*Table 3. 3: Sortino and Sharpe Ratios of the different assets for the MVP portfolio*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

With the MVP portfolio the results appear to be similar, with all showing a positive result, higher than Sharpe's. Compared to the 50/50 portfolio the values appear to follow the same trend, with all investments, even the combination of Bitcoin + Gold being positive investments.

The difference between the 50/50 and MVP portfolios as before can be explained by the lesser exposure to bitcoin which as we saw beforehand by the Sortino ratios of the individual assets showed a much higher value than the others.

### **5.2.1 Bearish scenario from December 2017 to December 2018**

Over our bearish stretch naturally and as expected the results appear much grimmer which can be seen below by the Sortino ratio achieved by the individual assets.

	<b>Sortino Ratio</b>	<b>Sharpe Ratio</b>
Bitcoin	-1.48	-0.99
S&P 500	-0.34	-0.28
NASDAQ	-0.07	-0.06
MSCI World	-0.51	-0.43
Gold	-0.49	-0.42

*Table 3. 4: Sortino and Sharpe Ratios of the different assets on the bearish scenario*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

As before the Sortino ratio seems to exacerbate the values we have seen by the Sharpe to a higher range. While the Sharpe remained positive, the Sortino was even higher, and now the opposite also holds true.

All assets show a negative value on the Sortino ratio and are to be seen as a bad investment choice, with Bitcoin being seen as the worst of them all. Curiously, the NASDAQ still displays a value that is closer to being positive than the other equity markets, albeit still negative.

<b>50/50 Portfolio</b>	<b>Sortino Ratio</b>	<b>Sharpe Ratio</b>
Bitcoin + S&P 500	-1.92	-1.14
Bitcoin + NASDAQ	-1.86	-1.08
Bitcoin + MSCI World	-1.89	-1.16
Bitcoin + Gold	-1.97	-1.21

*Table 3. 5: Sortino and Sharpe Ratios of the different assets for the 50/50 portfolio on the bearish scenario*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

On our 50/50 portfolio, as seen above, the Sortino ratio was much worse than the Sharpe ratio over the period with the investment much more clearly regarded as a bad one. In binary terms of good and bad the investments are seen the same way by an investor looking at the Sharpe or Sortino ratio, with the investment being bad and not worth it. However, with more range to the analysis we can easily see that the investor looking at the Sortino ratio would regard it much more poorly.

<b>MVP Portfolio</b>	<b>Sortino Ratio</b>	<b>Sharpe Ratio</b>
Bitcoin + S&P 500	-0.48	-0.37
Bitcoin + NASDAQ	-0.35	-0.26
Bitcoin + MSCI World	-0.64	-0.50
Bitcoin + Gold	-0.70	-0.52

*Table 3. 6: Sortino and Sharpe Ratios of the different assets for the MVP portfolio on the bearish scenario*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

On the MVP portfolio the trends that have been seen between both investments seem to also be present. Corroborating the previous findings and conclusions regarding investors points of view. The investments are still seen as bad for the MVP portfolios over our bearish scenario.

## 5.2.2 Bullish scenario from December 2018 to November 2019

Opposite to the previous scenario now the expectations are of an increase in the Sortino ratio, which seems to be the case when a glance is taken at the Sortino for the individual assets over this timeframe.

	Sortino Ratio	Sharpe Ratio
Bitcoin	4.63	2.82
S&P 500	1.82	1.33
NASDAQ	1.90	1.22
MSCI World	2.03	1.37
Gold	1.92	1.45

*Table 3. 7: Sortino and Sharpe Ratios of the different assets on the bullishh scenario*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

When compared with Table 3.1 above we see an increase in all Sortino Ratios, with Bitcoin and Gold showing the biggest increase. This seems to follow a similar trend shown by the increase in the Sharpe ratio, with their behavior mirroring each other.

We can state that all individual assets over this period have shown a good level of attractiveness for investors, with bitcoin being the clear winner with more than double the Sortino ratio of all the other individual assets. Interesting to note that the Sortino ratio over this period ended up following the behavior it has shown over our initial timeframe with the Sortino retaining its characteristic of displaying more extreme values than those of the Sharpe.

Most of these conclusions can also be seen on our 50/50 portfolio:

50/50 Portfolio	Sortino Ratio	Sharpe Ratio
Bitcoin + S&P 500	4.09	2.08
Bitcoin + NASDAQ	4.20	2.05
Bitcoin + MSCI World	4.01	2.09
Bitcoin + Gold	3.77	2.11

*Table 3. 8: Sortino and Sharpe Ratios of the different assets for the 50/50 portfolio on the bullish scenario*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

As before it shows a very similar behavior to that of the Sharpe ratio, with a similar increase and with all portfolios displaying a higher Sortino ratio. All values display a strong ratio, ensuring that over this period all investments were a really good option to investors and would result in great excess returns above the minimum



acceptable return set by investors. Investors looking at the Sortino ratio would consider the whole set of investments as excellent and would be hard pressed to find reasons stating otherwise.

Finally, we look at our last scenario, the MVP portfolio for the Sortino ratio:

<b>MVP Portfolio</b>	<b>Sortino Ratio</b>	<b>Sharpe Ratio</b>
Bitcoin + S&P 500	2.84	1.48
Bitcoin + NASDAQ	3.16	1.44
Bitcoin + MSCI World	2.81	1.47
Bitcoin + Gold	1.72	1.41

*Table 3. 9: Sortino and Sharpe Ratios of the different assets for the MVP portfolio on the bullish scenario*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

As earlier the behavior seems similar with the Sortino ratio showing higher values when compared with the initial scenario. However, the difference between the two ratios is much higher on this scenario. With the Sortino ratio in some cases almost doubling Sharpe's. We can conclude that while the gain that an investor can make by risk unit is already showing a considerably high value, the possibility of incurring in losses is showing in relative terms an even lower chance. All the equity markets and bitcoin portfolios hover around being excellent, with the combination of Bitcoin and Gold pulling ahead.

### **5.2.3 All scenarios**

Again, we have the data representative of all of the above scenarios displayed side by side for both the 50/50 and MVP portfolios:

<b>50/50 Portfolio</b>	<b>Scenario</b>		
	<b>1</b>	<b>2</b>	<b>3</b>
Bitcoin + S&P 500	2.53	-1.92	4.09
Bitcoin + NASDAQ	2.66	-1.86	4.20
Bitcoin + MSCI World	2.48	-1.89	4.01
Bitcoin + Gold	2.20	-1.97	3.77

*Table 3. 10: 50/50 Portfolio Sortino Ratios of All Scenarios*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

MVP Portfolio	Scenario		
	1	2	3
Bitcoin + S&P 500	1.15	-0.48	2.84
Bitcoin + NASDAQ	1.59	-0.35	3.16
Bitcoin + MSCI World	1.10	-0.64	2.81
Bitcoin + Gold	0.09	-0.70	1.72

*Table 3. 11: MVP Portfolio Sortino Ratios of All Scenarios*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

The values seem to mirror the behaviors displayed by the ones obtained earlier for the Sharpe ratio very closely, with the main takeaways being the same for both ratios.

All scenarios, with the exception of the bearish one, are profitable, with the period encompassing all data since the inception of bitcoin showing a positive Sortino ratio for all asset combinations for both the 50/50 and MVP portfolios.

As before our most profitable scenario is the 3<sup>rd</sup> one with the greatest Sortino ratio, values with Bitcoin remaining largely the greatest culprit of these results, still showing the effect of exacerbating all portfolios it is included in and bringing it to a more extreme outcome.

The Bitcoin + NASDAQ combination of assets remains the one that pulls ahead with the best Sortino ratio in all scenarios and portfolio weight combinations when compared with the other portfolios.

The Sortino ratio when compared to the Sharpe however seems to fluctuate more, displaying absolute values on Scenarios 2 and 3 that are further apart from Scenario 1 than what occurred with the Sharpe ratio.

These conclusions back up the notion of the huge impact that market timeliness asserted on the results, with the impact of a bad entrance or wrongly timed exit having the potential to really dent an investor portfolio, more so than with more traditional investments. The Sortino ratio had even bigger swings than the Sharpe ratio, highlighting even more this characteristic of the Bitcoin portfolios.

Still, the biggest takeaway lies in the fact that the Sortino ratio serves its use and mostly corroborates the results found by the Sharpe which naturally brings more robustness to the conclusions taken.

## 6. Conclusion

In this project we decided to study the implementation of Bitcoin as an alternative investment and diversification tool for investment portfolios. We are aligned with most of the literature (even though there are still a lot of conflicting ideas and discussions occurring with no clear consensus) in the opinion that the emergence of the new cryptocurrency asset class will have a big impact as a diversification method available for investors in the future. The entire market is, even after more than 10 years, still in its infancy and with a lot of room for growth.

It has been an incredible first decade for Satoshi Nakamoto's ambitious vision. The fast development that crypto-assets and the blockchain technology have been showing and their successful and every day more common use in financial services suggest that the building blocks for the next digital transformation of financial markets are already underway.

With the investment on a strong and credible market infrastructure, coupled with an appropriate regulatory framework and enhanced security the bold promise of these innovations may well be within reach.

It was decided the use of Bitcoin as it is the most established of all the crypto currencies, being the first. It also enjoys the greater spotlight and is seen as the "face" of the entire market at times, with most other crypto currencies displaying some sort of correlation to it. Bitcoin also went through an incredibly rich run since its inception made of vertiginous highs and amazing crashes and dips. There was some luck involved with the literature as most of it was incredibly recent and found it well centralized in some key papers and journals that we could make use of.

We combined Bitcoin with three of the biggest and better diversified equity markets indexes: S&P500, NASDAQ, and the MSCI World Index. Along with Gold to have some exposure to other asset classes. And then followed the literature set by Markowitz to achieve the minimum variance portfolio of the combination of the former assets with Bitcoin. The diversity of the used assets served to highlight Bitcoin's impact when majorly exposed to American equities, equities of all over the world or a standard commodity such as Gold and as such bring more reliability to the results.

The main results are taken from the employment of the Sharpe and Sortino ratios to understand the attractiveness of the investments over the studied period.

The results show that the addition of Bitcoin to diversified investments served to improve the latter. All assets seemed to improve on both the Sharpe and Sortino ratio outlook when compared to the portfolios in which Bitcoin is added to it. The cryptocurrency seems to exacerbate the effect of the portfolios it is added to while adding diversification to them. While over its entire inception it showed great ratios (adding emphasis to the fact that it was indeed a good investment tool), it showed even better results over the last year of our study and the most recent time period. However, it would have a disastrous effect if added during the period that occurred between December of 2017 and December of 2018, where it had its worse performance of all time and served to drag already negative ratios to even lower numbers.

Thus, this project brings confirmation and helps strengthening the growing notion of Bitcoin as a diversification tool. While more risk averse investors could still be unattracted by the higher volatility and risk that the cryptocurrency adds to their portfolio, we conclude that by finding the minimum variance portfolio those fears can be tranquilized by these results.

There were some limitations with this project. Namely in finding up to date and recent data. We feel that a good compromise could be had with the data set ending in November of 2019, however ideally, we would like that to be updated to at least December 2019. While efforts were made with that aim, the quality of that data left much to be desired and we felt it would be best to work with the more trustworthy data set.

It would be an understatement to say that bitcoin, and on a broad view, the entire cryptocurrency market is new and volatile. We have seen that firsthand during this project.

As such it makes it hard to capture all the new and relevant developments that such a young market is sure to display. Over the time that passed since the start of this work and its conclusion a great number of such events happened that completely could change our conclusions and findings. The most glaring example of this must be the crisis induced by the Covid-19 virus which has shown great implications for the financial and economic global panorama. Much of these consequences we have not yet started to see and will be sure to develop over the coming years.

While we stand by our results and findings, we find that it would be important to extend the data frame of this study further such has to try and consider these new developments.

It would also be interesting to explore deeper the connection with gold and how do they compare further, alongside with that development one could make a case for adding more specific indexes and ETF's to the study, such as those with a big focus on tech or industry. One last interesting bit would be comparing its development with the exchange rates and the strength of currencies over the world.

Some more interesting comparisons would lie in contrasting bitcoin with the Emerging Market Indexes and see how it correlates with some known and recent growth stocks such as Amazon and Tesla to investigate the similarities between the two assets.

Further research could also see how Bitcoin relates and correlates with other cryptocurrencies and how those would pin themselves against the comparisons done in this project and mentioned above.

We feel that the most immediate further research point of interest would be the development of more complex combinations of assets that better mimic those found in practice. It would be interesting to see how bitcoin would impact a more diversified portfolio composed of more asset classes such as REIT's, Emerging Markets, Government and Corporate bonds, etc. Nevertheless, we think that we already used in this project such well diversified indexes and markets, which somehow excuses this flaw.

Aiming at the data, the Sortino ratio results could be brought some robustness by using the daily risk-free rate that really occurred in each day. Thanks to data constraints we were made to work with a daily return obtained from the one-year risk-free rate. Even if the latter is still an appropriate method as it is a reasonable target for a minimum acceptable return.

*Bitcoin will continue its run, and, in the interim, fortunes and massive investments will be made and lost.*

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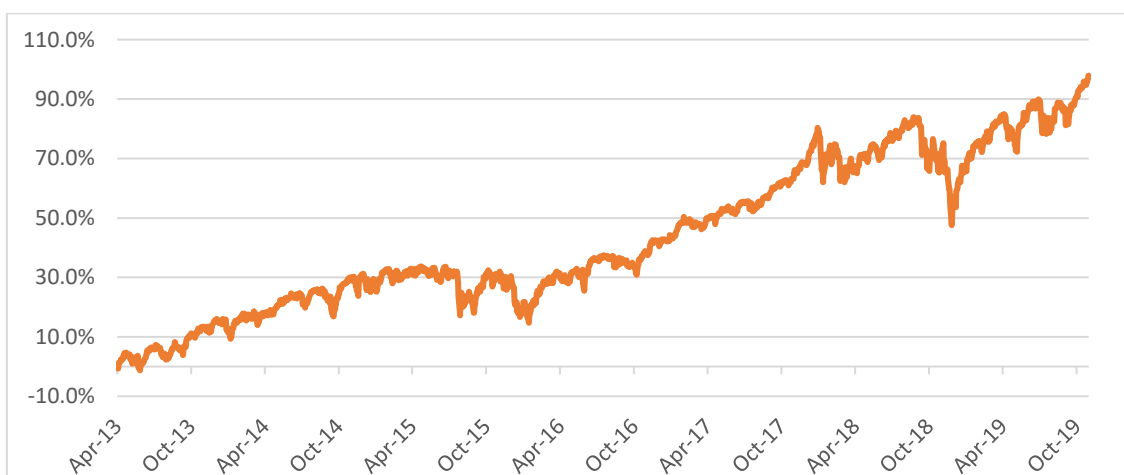
## Appendices

### Appendix A

#### S&P 500

These numbers by themselves are meaningless, however. It is important to compare them with the other possible constituents of the wallet we are trying to build in this study.

Our first example is the famous Standard & Poor's 500, commonly known by the shortened forms S&P 500 or S&P. It is a stock market index composed by 500 of the largest American companies listed in the New York Stock Exchange (NYSE). It is famous for being one of the most followed equity indexes and it is generally regarded as a representation of the entire U.S. stock market.



*Figure A. 1: S&P 500 Cumulative return from 2013 to 2019*

*Source: Processed from Yahoo Finance data (2020)*

When we look at returns it is immediately evident and to be expected that it would not reach the crazy high rate of return on investment that we saw previously with the bitcoin chart. Albeit more modest, the market performed quite well over the period, with investors that put their confidence in trackers of the most famous American index finding their money almost doubled at the end of the period with a cumulative return of 97.9%. This in absolute terms contrasts a price of around 1,593 dollars on close at the 29<sup>th</sup> of April 2013 with one of 3,153 at the end of the data collected, that being 27<sup>th</sup> of November of 2019. While still being an extremely attractive prospect of investment (as it has been for decades now for savvy investors that seek a low effort approach to investment) the



difference in performance between the market and bitcoin is immense. This is offset greatly by other facts. While bitcoin is still seen as an unproven commodity to many, S&P lies in the opposite side of the spectrum. It is seen as one of the safest avenues for stock market investing.

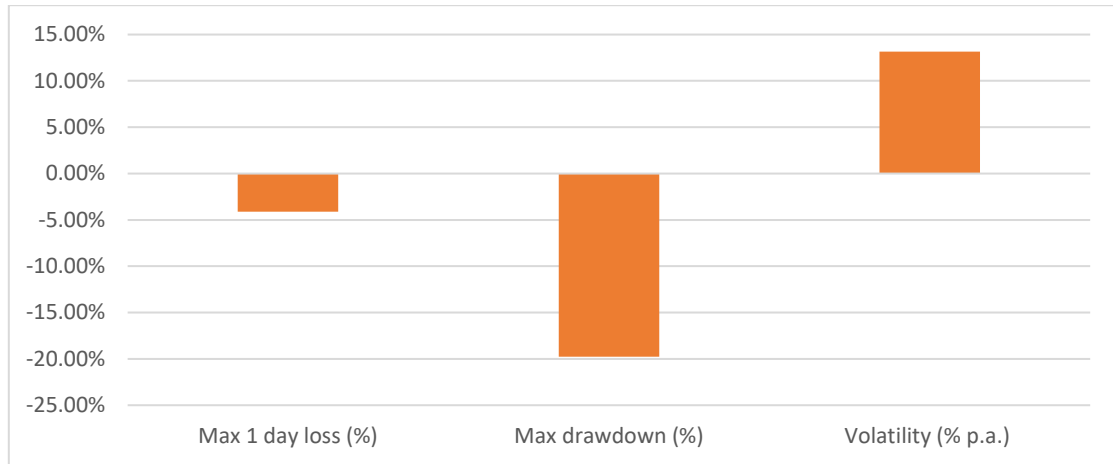


Figure A. 2: S&P 500 drawdown and volatility characteristics

Source: Processed from Yahoo Finance data (2020)

The figure above illustrates these facts. As we will see when more markets are exposed, with a volatility of 13.2% *per annum*, the S&P 500 stands as the least volatile of them all. The other measures seen above are merely a reflection of that, with the maximum loss that occurred on a trading day being of 4.1% and the maximum value that the index recessed before closing green again being also a modest 13.2%.

When we look at how volatility behaved over the last year, we can see that the trend it presents is also much different from that of the cryptocurrency.

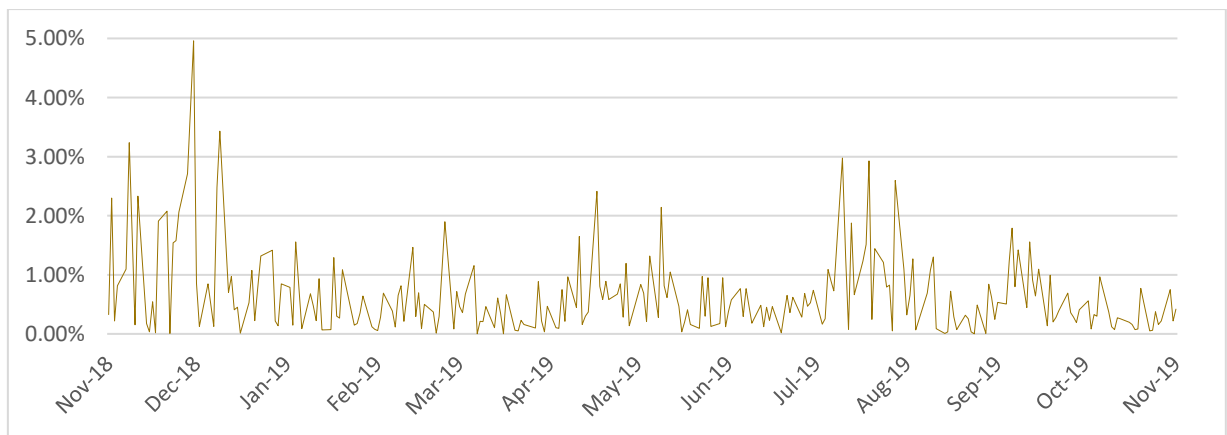


Figure A. 3: S&P 500 absolute daily 1-year volatility

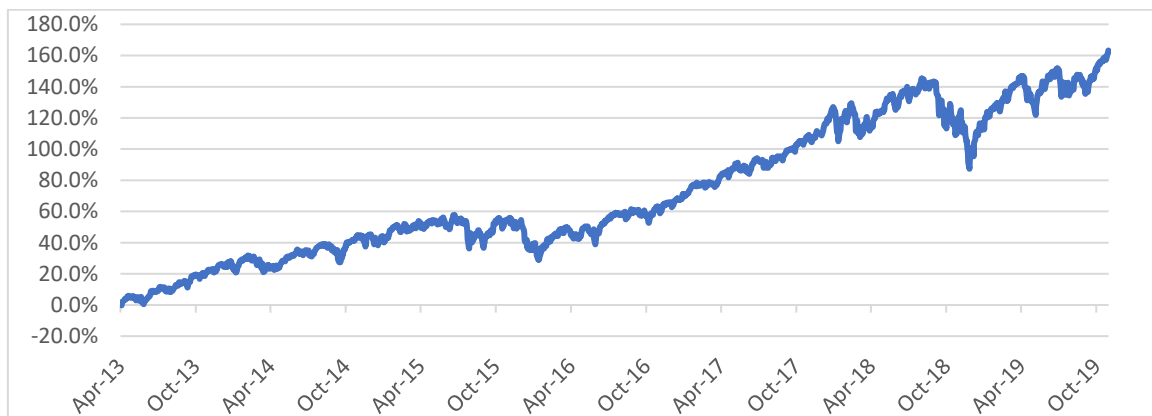
Source: Processed from Yahoo Finance data (2020)

As we can conclude by the figure above the S&P presents us with much more stable values, its peak absolute volatility for the year barely reached 5%, while most of the times it stood much lower, well below the 2% line.

## NASDAQ

Formally, the NASDAQ Composite is a stock market index composed by stocks that are listed on the NASDAQ stock exchange. Initially it stood as an acronym for National Association of Securities Dealers Automate Quotations, but the name has been more commonly used to refer to the NASDAQ 100 composite

While not showing the same levels of the S&P 500, the Nasdaq comes close in all fields. It is a testament to the strength of the American juggernaut economy the strength of its main markets. In times where the power of the East is emerging more and more with China, its dependence on the dollar is still great, and American markets are still seen as a show of the strength of the world's economy.



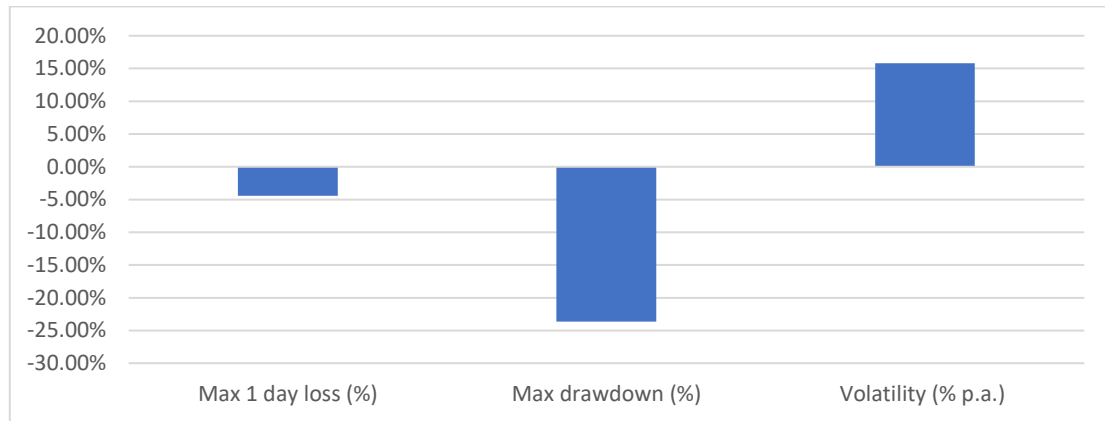
*Figure A. 4: NASDAQ Cumulative return from 2013 to 2019*

*Source: Processed from Yahoo Finance data (2020)*

As seen with the chart above, the trends on the NASDAQ seem to mirror those of the previous talked S&P point by point. We can see the same uptrend over the course of time and even the same highs and lows. Especially noticeable on the sharp decline correction that occurred at the end of 2018. This is, however, nothing unexpected as these two indices share many similarities.

Doing the same exercise as before we can see something in which the NASDAQ positively outperforms the S&P 500, with that being the cumulative return. With the same time frame as before (and the same ones from henceforth), an investor would have made

a total return of 163.2% of its portfolio by investing in this index. That is close to 65% more than one would make in the same period with the S&P 500. Taking into account the below figure, in which we see that the volatility on this index was of 15.8% , when compared with the previous 13.2% of the S&P it does seem like a good return to volatility ratio.



*Figure A. 5: NASDAQ drawdown and volatility characteristics*

*Source: Processed from Yahoo Finance data (2020)*

In matters of the drawdown ratio we can see that it is as one would expect higher than the previous index. NASDAQ saw its valuation fall some 23.6%, almost ¼ of its total value, before seeing a green day. Its maximum one-day loss also sees a higher value standing at 4.4%.

Over the last year of data volatility for the NASDAQ performed much in line with the previous S&P, having the same characteristics, peaks, and lows. This is not surprising as both are American market indexes that overall tend to trend quite like one another. When we look at the figure 3.3.3 below, we can however see where they differ. That difference being the absolute values that it presents with the NASDAQ volatility being some decimal points higher on average than that of the S&P 500, which, while still rarely going over the 3% mark it does get through it more often than on the other American index.

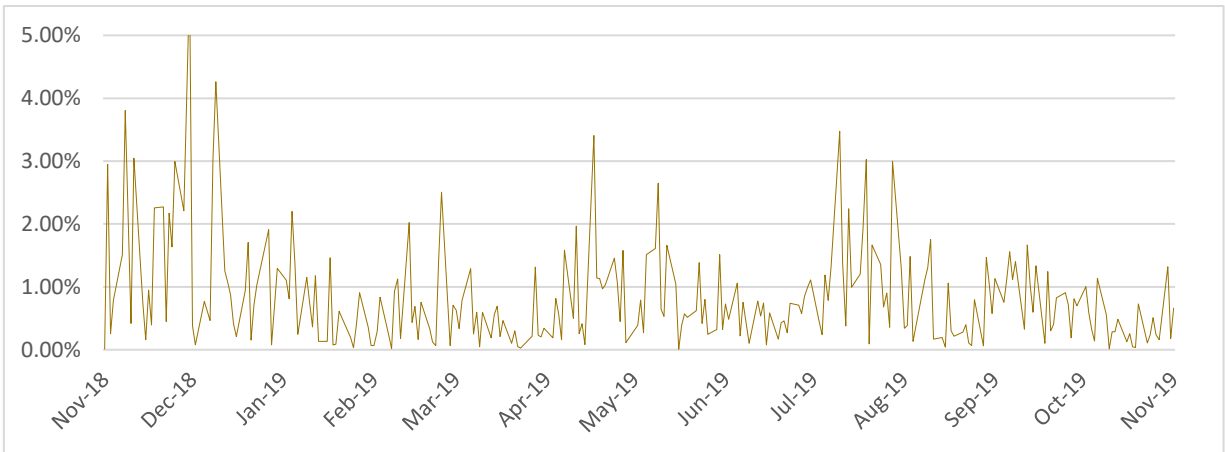


Figure A. 6: NASDAQ absolute daily 1-year volatility

Source: Processed from Yahoo Finance data (2020)

### MSCI

The MSCI World composite tracker differs from the previous markets as it is made up from stocks from all over the world. Its main difference, coupled with the above sentence is that it seeks to be a benchmark intended to represent a broader section of the entirety of the developed markets, excluding those from emerging and fringe economies.

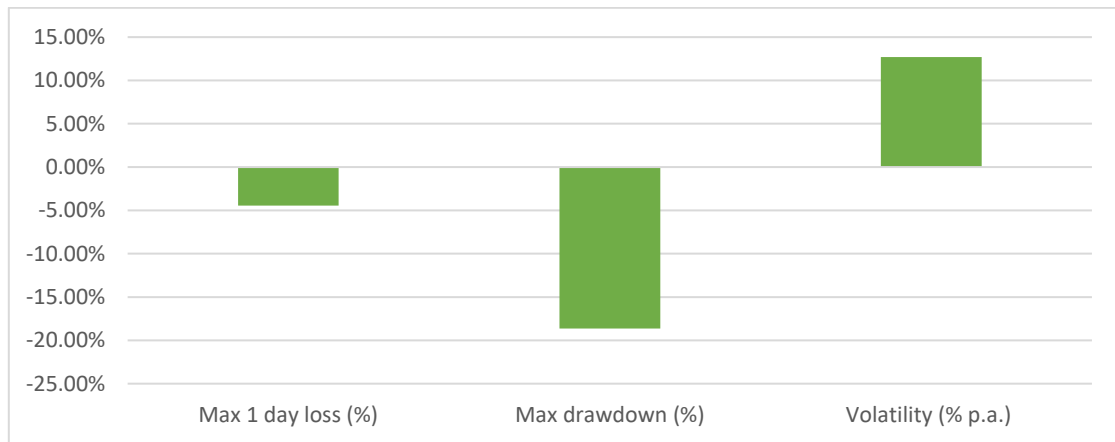


Figure A. 7: MSCI Composite Cumulative return from 2013 to 2019

Source: Processed from Yahoo Finance data (2020)

When we look at the returns, we see that those are below the higher S&P and NASDAQ numbers reported above, showing a more modest return of 79.4% for an investor over the period. This highlights the strength of the American economy during this period, which greatly carried the tracker. This index is mainly composed by American equities, totaling more than half of its portfolio. The other equities are the ones that lagged

the portfolio behind when compared to pure American equities portfolios that replicate the former indexes.



*Figure A. 8: MSCI Composite drawdown and volatility characteristics*

*Source: Processed from Yahoo Finance data (2020)*

Its nature as a more diversified portfolio is not without profit, however. Investors that took advantage of this strategy could boast of lower volatility over its assets, being at around 12.7% over the period. The maximum drawdown is also lower than any seen until now with a value of 18.6%, while the maximum loss that occurred in a trading day is hovering the 4.5%.

The absolute volatility for the past year does show us some interesting considerations. In it we can see that the MSCI presented much lower volatility numbers over the quarter than the 100% American markets. For instance, it never once appeared to surpass by much the 3% value, being most of the time hovering between 1% and 2%. This shows how much of a stable index the MSCI Composite was over the more recent periods and highlighted its importance for risk averse investors that seek a highly diversified alternative. The index performed quite reasonably well while managing a low volatility level as we can see by the figure 3.4.3 below.

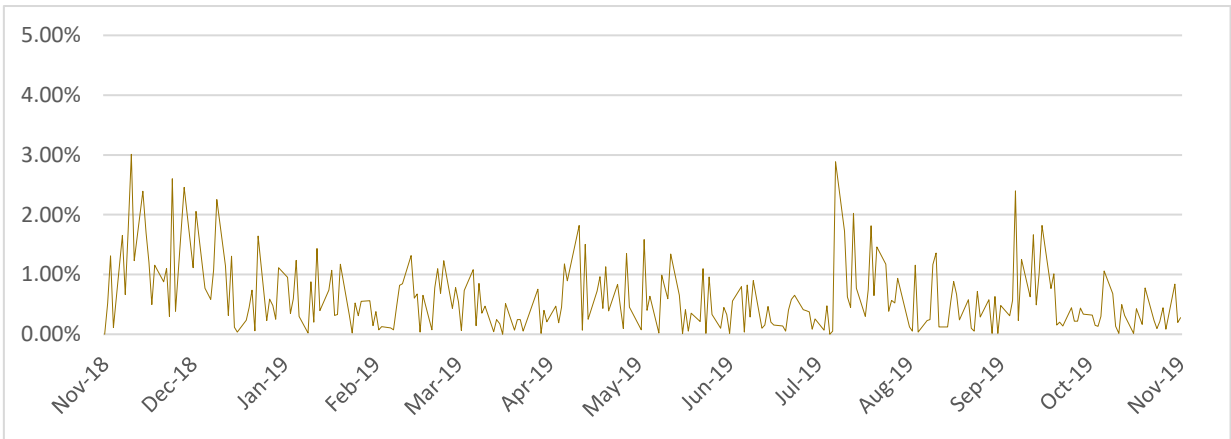


Figure A. 9: MSCI Composite absolute daily 1-year volatility

Source: Processed from Yahoo Finance data (2020)

## GOLD

At last we have gold, as previously mentioned in the literature and run-through over bitcoin; gold is probably the asset with the most similarities with bitcoin. One could even see bitcoin as digital gold, as both are intended as a limited currency. Unlike the dollar which the federal reserve can print more in case of need, one cannot print gold, there is a cap in its maximum number of units in circulation, just as there is a cap in how much bitcoin can be mined. For those similarities it proves interesting to see how does gold stack up against (and with) bitcoin in a portfolio.

Gold has always been seen as a cornerstone in the portfolio for those more risk averse, especially during economic downturns or high inflation scenarios. In that field bitcoin greatly opposes it, as we have stated before the investors that seek to invest in the cryptocurrency generally show a different risk profile and characteristics.

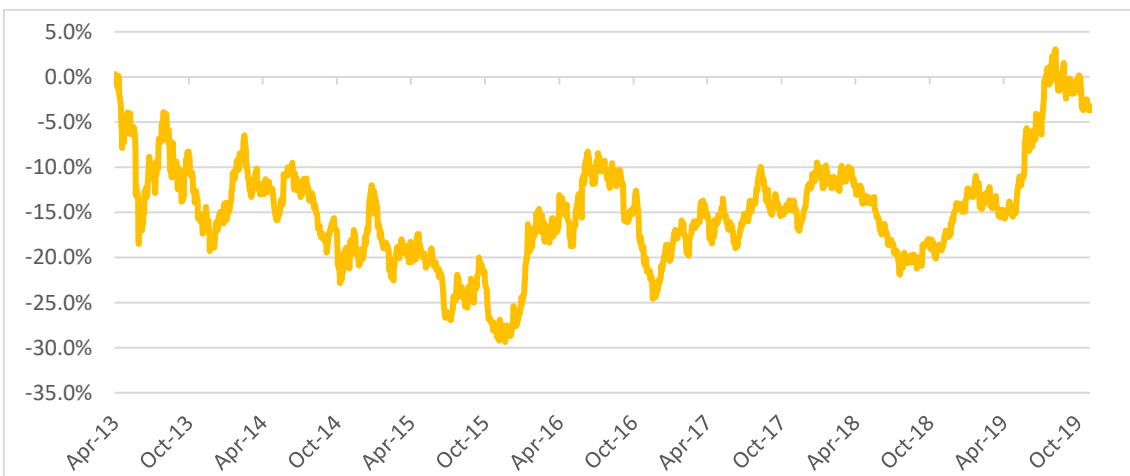


Figure A. 10: Gold Cumulative return from 2013 to 2019

Source: Processed from Yahoo Finance data (2020)

For the qualities seen above we can expect gold not to have many fluctuations in price over time. In truth, however we do know that it tends to be seen as a safe haven for investors during economic down cycles. With that said it is not surprising to see that during the opposite, that being the bull run we see here represented over most of the 10's, gold has been down from its initial price. Although it rebound at the end of the chart, we can see that the cumulative return for the asset ended negative at -3.7%.

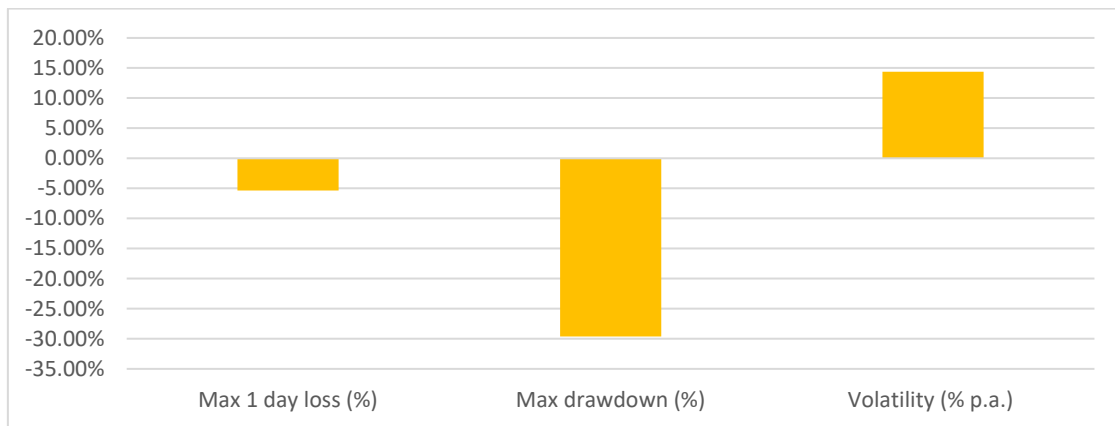


Figure A. 11: Gold drawdown and volatility characteristics

Source: Processed from Yahoo Finance data (2020)

The volatility appears to be exceptionally high, standing at 14.4%. Surprising as well is the amount that gold fell in one day, being 5.4% of its previous value. It is a fast descent in price for such a stable commodity. Its maximum drawdown also stands at a high value of -29.6%.

Finally, we can evaluate the volatility characteristics of the supposedly at first guess, more stable and less volatile of all the assets regarded here.

Gold as expected presents the lowest of all, being most of the time below the 1% line or between it and 2%. Not many times did the commodity surpass those levels of volatility being easy to single out the instances where that happen. Those numbers, as we can see on figure 3.5.3 below, are not surprising when taking into account all the characteristics associated with this commodity that have been highlighted before.

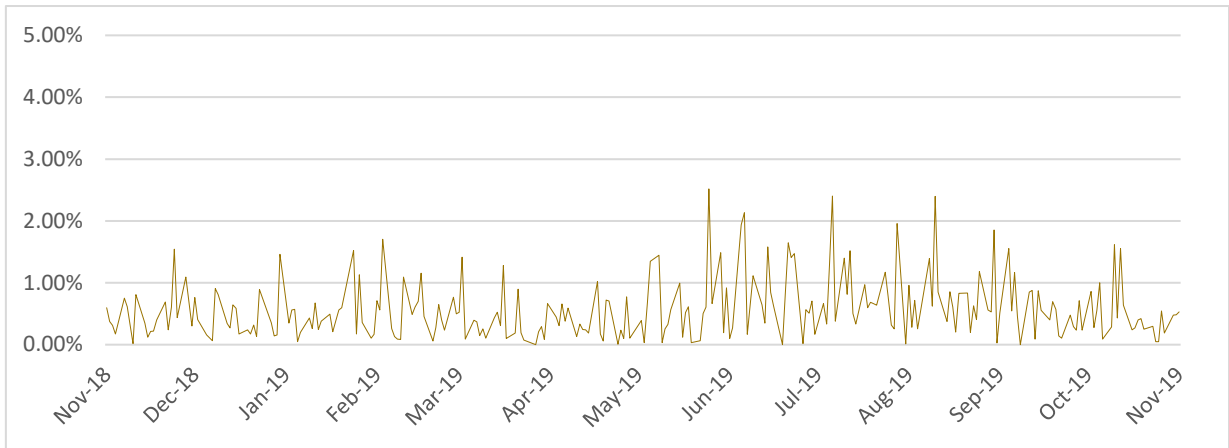


Figure A. 12: Gold absolute daily 1-year volatility

Source: Processed from Yahoo Finance data (2020)

## Appendix B

### Portfolios returns and Sortino calculations

RpMVP	Rp50/50		Rp50/50		Rp50/50	
SP500	0.1049%	30.0%	SP500	0.2647%	93.7%	
NDQ	0.1324%	39.2%	NDQ	0.2730%	97.7%	
MSCI	0.0945% Annual ->	26.6%	MSCI	0.2636%	93.1%	
GOLD	0.0617%	16.7%	GOLD	0.2596%	91.2%	

N° Observations	237														
Sortino															
N° Downsides	Sp500	NDQ	MSCI	Gold	Bitcoin	MVP SP500	MVP NASDAQ	MVP MSCI	MVP Gold	50/50 SP500	50/50 NASDAQ	50/50 MSCI	50/50 Gold		
Semi Variance	119	114	122	120	116	112	117	116	121	116	114	115	117		
Semi Deviation	0.005%	0.007%	0.004%	0.003%	0.080%	0.004%	0.006%	0.003%	0.003%	0.020%	0.021%	0.021%	0.023%		
Semi Deviation Annualized	10.9%	13.2%	9.5%	8.7%	44.7%	10.0%	11.9%	8.9%	8.8%	22.5%	22.9%	22.8%	23.8%		
Sortino Ratio	1.82	1.90	2.03	1.92	4.63	2.84	3.16	2.81	1.72	4.09	4.20	4.01	3.77		

Figure B. 1: Portfolios returns and Sortino calculations

Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)

RpMVP	Rp50/50		Rp50/50		Rp50/50	
SP500	-0.0138%	-3.4%	SP500	-0.2840%	-50.9%	
NDQ	-0.0118%	-2.9%	NDQ	-0.2776%	-50.1%	
MSCI	-0.0214% Annual ->	-3.2%	MSCI	-0.2879%	-51.4%	
GOLD	-0.0125%	-3.1%	GOLD	-0.2834%	-50.8%	

N° Observations	201													
Sortino														
N° Downsides	Sp500	NDQ	MSCI	Gold	Bitcoin	MVP SP500	MVP NASDAQ	MVP MSCI	MVP Gold	50/50 SP500	50/50 NASDAQ	50/50 MSCI	50/50 Gold	
Semi Variance	113	115	109	109	136	108	117	114	111	137	116	114	114	123
Semi Deviation	0.007%	0.010%	0.006%	0.006%	0.003%	0.112%	0.007%	0.010%	0.006%	0.003%	0.031%	0.032%	0.033%	0.029%
Semi Deviation Annualized	0.8%	1.0%	0.8%	0.8%	0.5%	3.3%	0.8%	1.0%	0.8%	0.5%	1.8%	1.8%	1.8%	1.7%
Sortino Ratio	-0.34	-0.07	-0.51	-0.51	-0.49	-1.48	-0.48	-0.35	-0.64	-0.70	-1.92	-1.86	-1.89	-1.97

Figure B. 2: Portfolios returns and Sortino calculations on the bearish scenario

Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)



RpMVP		Rp50/50		
SP500	0.1049%	30.0%	SP500	0.2647%
NDQ	0.1324%	39.2%	NDQ	0.2730%
MSCI	0.0945%	26.6%	MSCI	0.2636%
GOLD	0.0617%	16.7%	GOLD	0.2596%
				93.7%
				97.7%
				93.1%
				91.2%

N° Observations 237

Sortino

	Sp500	NDQ	MSCI	Gold	Bitcoin	MVP SP500	MVP NASDAQ	MVP MSCI	MVP Gold	50/50 SP500	50/50 NASDAQ	50/50 MSCI	50/50 Gold
N° Downside	119	114	122	120	116	112	117	116	121	116	114	115	117
Semi Variance	0.005%	0.007%	0.004%	0.003%	0.080%	0.004%	0.006%	0.003%	0.003%	0.020%	0.021%	0.021%	0.023%
Semi Deviation	0.7%	0.8%	0.6%	0.6%	2.8%	0.6%	0.8%	0.6%	0.6%	1.4%	1.4%	1.4%	1.5%
Semi Deviation Annualized	10.9%	13.2%	9.5%	8.7%	44.7%	10.0%	11.9%	8.9%	8.8%	22.5%	22.9%	22.8%	23.8%
Sortino Ratio	1.82	1.90	2.03	1.92	4.63	2.84	3.16	2.81	1.72	4.09	4.20	4.01	3.77

Figure B. 3: Portfolios returns and Sortino calculations on the bullish scenario

Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)

## Appendix C

### MVP Portfolios Creation

DETERMINING THE PORTFOLIO FRONTIER EQUATION

Assets (Shares)	Bitcoin	SP500
Expected Return	157.6%	12.0%
Standard Deviation	83.1%	13.1%

A = Bitcoin  
B = SP500

Variances and Covariances Matrix		
	Bitcoin	SP500
Bitcoin	69.1%	-0.1%
SP500	-0.1%	1.7%

Coefficients of Correlation Matrix		
	Bitcoin	SP500
Bitcoin	1.00	-0.01
SP500	-0.01	1.00

50/50 Portfolio

ER(p) 69.9%  
SD(p) 48.1%

Sharpe Ratio

1.42

MVP Portfolio

ER(p) 14.4%  
SD(p) 14.9%

0.86

Using the ER(p) and SD(p) we found separately for each combination of assets. On the appendixes

$$V = 0.01184$$

$$D = 179.16414536$$

$$C = 60.041340266$$

$$B = 4.474571221$$

$$A = 9.460185405$$

PORTFOLIO FRONTIER EQUATION

$$VAR(P) = (1/D) * [C * E(Rp)^2 - 2 * A * E(Rp) + B]$$

$$a = 0.335119173 \quad E(Rp)^2$$

$$b = -0.105603556 \quad E(Rp)^1$$

$$c = 0.024974702 \quad E(Rp)^0$$

$$VAR(P) = \frac{0.3}{a} E(Rp)^2 - \frac{0.11}{b} E(Rp) + \frac{0.025}{c}$$

$$mvp \Rightarrow 0.670238346 E(Rp) - 0.1056 = 0$$

$$E(Rp) = 0.157561196 = A / C \quad 0.157561$$

$$VAR(mvp) = 0.016655191 = 1 / C \quad 0.016655$$

$$SD(mvp) = 0.129054993 = 1/\sqrt{C} \quad 0.129055$$

$$w^*A(mvp) = 2.6\%$$

$$w^*B(mvp) = 97.4\%$$

VAR (mvp)	Bitcoin	SP500
Bitcoin	0.0465%	-0.0033%
SP500	-0.0033%	1.6256%
	VAR (mvp) = 1.6655%	
	SD (mvp) = 12.9055%	

Control: mvp using BK&M (Bodie, Kane & Marcus) General 2 Asset Case Formula

$$wA(mvp) = [VAR(B) - COVAR(A,B)] / [VAR(A) + VAR(B) - 2 * COVAR(A,B)]$$

$$w^*A(mvp) = 0.025932827 \quad OK!$$

Figure C. 1: Bitcoin + SP500 MVP weights

Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)

**DETERMINING THE PORTFOLIO FRONTIER EQUATION**

Assets (Shares)	Bitcoin	NASDAQ
Expected Return	157.6%	17.4%
Standard Deviation	83.1%	15.7%

Variations and Covariances Matrix

	Bitcoin	NASDAQ
Bitcoin	69.1%	0.0%
NASDAQ	0.0%	2.5%

Coefficients of Correlation Matrix

	Bitcoin	NASDAQ
Bitcoin	1.00	0.00
NASDAQ	0.00	1.00

A = Bitcoin  
B = NASDAQ

V = 0.01705  
D = 115.41218871  
C = 42.041711640  
B = 4.830989910  
A = 9.364341728

**PORTFOLIO FRONTIER EQUATION**

$$\text{VAR}(P) = (1/D) * [ C * E(Rp)^2 - 2 * A * E(Rp) + B ]$$

a = 0.364274451 E(Rp)<sup>2</sup>  
b = -0.162276478 E(Rp)<sup>1</sup>  
c = 0.041858576 E(Rp)<sup>0</sup>

$$\text{VAR}(P) = \frac{0.4}{a} E(Rp)^2 - \frac{0.16}{b} E(Rp) + \frac{0.042}{c}$$

$$\text{mvp} \Rightarrow 0.728548901 E(Rp) - 0.16228 = 0$$

$$E(Rp) = 0.222739307 = A / C = 0.222739$$

$$\text{VAR}(\text{mvp}) = 0.023785901 = 1 / C = 0.023786$$

$$\text{SD}(\text{mvp}) = 0.154226785 = 1/\text{sqrt}(C) = 0.154227$$

w\*A (mvp) = 3.5%  
w\*B (mvp) = 96.5%

VAR (mvp)	Bitcoin	NASDAQ
Bitcoin	0.0845%	-0.0014%
NASDAQ	-0.0014%	2.2968%
	VAR (mvp) =	2.3786%
	SD (mvp) =	15.4227%

Control: mvp using BK&M (Bodie, Kane & Marcus) General 2 Asset Case Formula

$$wA(\text{mvp}) = [ \text{VAR}(B) - \text{COVAR}(A,B) ] / [ \text{VAR}(A) + \text{VAR}(B) - 2 * \text{COVAR}(A,B) ]$$

w\*A (mvp) = 0.034967052 OK!

**50/50 Portfolio**

ER(p) 74.0%  
SD(p) 49.4%

**Sharpe Ratio**

1.46

**MVP Portfolio**

ER(p) 20.6%  
SD(p) 18.1%

1.05

Using the ER(p) and SD(p) we found separately for each combination of assets.

1.6% US 1 Year

Figure C. 2: Bitcoin + NASDAQ MVP weights

Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)

**DETERMINING THE PORTFOLIO FRONTIER EQUATION**

Assets (Shares)	Bitcoin	MSCI
Expected Return	15.76%	10.2%
Standard Deviation	83.1%	12.6%

A = Bitcoin  
B = MSCI World

**50/50 Portfolio**

ER(p) 68.6%  
SD(p) 47.9%

**Sharpe Ratio**

1.40

Variances and Covariances Matrix		
	Bitcoin	MSCI
Bitcoin	69.1%	-0.1%
MSCI	-0.1%	1.6%

**MVP Portfolio**

ER(p) 12.5%  
SD(p) 14.3%

0.76

Using the ER(p) and SD(p) we found separately for each combination of assets. On the appendixes

Coefficients of Correlation Matrix		
	Bitcoin	MSCI
Bitcoin	1.00	-0.01
MSCI	-0.01	1.00

V = 0.01102  
D = 197.28402316  
C = 64.357941389  
B = 4.275941964  
A = 8.826482829

1.6% US 1 Year treasury Yit <https://www.treasury.gov>

**PORTFOLIO FRONTIER EQUATION**

$$VAR(P) = (1/D) * [ C^2 E(Rp)^2 - 2^*A^*E(Rp) + B ]$$

a = 0.326219733 E(Rp)^2  
b = -0.089479956 E(Rp)^1  
c = 0.021674041 E(Rp)^0

$$VAR(P) = \frac{0.3}{a} E(Rp)^2 - \frac{0.09}{b} E(Rp) + \frac{0.022}{c}$$

$$mvp \Rightarrow 0.652439466 E(Rp) - 0.08948 = 0$$

$$E(Rp) = 0.137146755 = A / C \quad 0.137147$$

$$VAR(mvp) = 0.015538098 = 1 / C \quad 0.015538$$

$$SD(mvp) = 0.124651907 = 1/\sqrt{C} \quad 0.124652$$

w\*A (mvp) = 2.4%  
w\*B (mvp) = 97.6%

VAR (mvp)	Bitcoin	MSCI
Bitcoin	0.0386%	-0.0019%
MSCI	-0.0019%	1.5190%
VAR (mvp) =	1.5538%	
SD (mvp) =	12.4652%	

Control: mvp using BK&M (Bodie, Kane & Marcus) General 2 Asset Case Formula

$$wA(mvp) = [ VAR(B) - COVAR(A,B) ] / [ VAR(A) + VAR(B) - 2^*COVAR(A,B) ]$$

w\*A (mvp) = 0.023623285 OK!

*Figure C. 3: Bitcoin + MSCI World MVP weights*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

**DETERMINING THE PORTFOLIO FRONTIER EQUATION**

Assets (Shares)	Bitcoin	Gold
Expected Return	157.6%	0.4%
Standard Deviation	83.1%	14.2%

A = Bitcoin  
B = Gold

Variances and Covariances Matrix		
	Bitcoin	Gold
Bitcoin	69.1%	0.0%
Gold	0.0%	2.0%

Coefficients of Correlation Matrix		
	Bitcoin	Gold
Bitcoin	1.00	0.00
Gold	0.00	1.00

V = 0.01397  
D = 177.09429067  
C = 50.895842000  
B = 3.595203528  
A = 2.426235778

**PORTFOLIO FRONTIER EQUATION**

$$VAR(P) = (1/D) * [ C^2 E(Rp)^2 - 2^*A^*E(Rp) + B ]$$

a = 0.287394031 E(Rp)^2  
b = -0.027400497 E(Rp)^1  
c = 0.02030107 E(Rp)^0

$$VAR(P) = \frac{0.3}{a} E(Rp)^2 - \frac{-0.03}{b} E(Rp) + \frac{0.020}{c}$$

$$mvp \Rightarrow 0.574788061 E(Rp) - 0.0274 = 0$$

$$E(Rp) = 0.047670609 = A / C \quad 0.047671$$

$$VAR(mvp) = 0.01964797 = 1 / C \quad 0.019648$$

$$SD(mvp) = 0.140171218 = 1/\sqrt{C} \quad 0.140171$$

w\*A (mvp) = 2.8%  
w\*B (mvp) = 97.2%

VAR (mvp)	Bitcoin	Gold
Bitcoin	0.0539%	0.0010%
Gold	0.0010%	1.9090%
	VAR (mvp) =	1.9648%
	SD (mvp) =	14.0171%

Control: mvp using BK&M (Bodie, Kane & Marcus) General 2 Asset Case Formula

$$wA(mvp) = [ VAR(B) - COVAR(A,B) ] / [ VAR(A) + VAR(B) - 2^*COVAR(A,B) ]$$

w\*A (mvp) = 0.027915057 OK!

*Figure C. 4: Bitcoin + Gold MVP weights*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

**50/50 Portfolio**

ER(p) 60.9%  
SD(p) 48.7%

**Sharpe Ratio**

1.22

**MVP Portfolio**

ER(p) 3.1%  
SD(p) 16.1%

0.09

Using the ER(p) and SD(p) we found separately for each combination of assets. On the appendixes

1.6% US 1 Year treasury Yiehttps://www

**DETERMINING THE PORTFOLIO FRONTIER EQUATION**

Assets (Shares)	Bitcoin	SP500
Expected Return	-75.5%	-1.7%
Standard Deviation	78.7%	15.5%

A = Bitcoin  
B = SP500

**50/50 Portfolio**

ER(p) -50.9%  
SD(p) 47.1%

**Sharpe Ratio**

-1.14

**Variances and Covariances Matrix**

	Bitcoin	SP500
Bitcoin	62.0%	1.6%
SP500	1.6%	2.4%

**MVP Portfolio**

ER(p) -3.4%  
SD(p) 16.3%

-0.37

**Coefficients of Correlation Matrix**

	Bitcoin	SP500
Bitcoin	1.00	0.13
SP500	0.13	1.00

Using the ER(p) and SD(p) we found separately for each combination of assets. On the appendix

V = 0.01471  
D = 37.05275738  
C = 41.535716840  
B = 0.920755502  
A = -1.091550474

2.7% US 1 Year treasury Yield

**PORTFOLIO FRONTIER EQUATION**

$$VAR(P) = (1/D) * [ C^2 E(Rp)^2 - 2^*A^*E(Rp) + B ]$$

a = 1.120988552 E(Rp)^2  
b = 0.058918718 E(Rp)^1  
c = 0.024849851 E(Rp)^0

$$VAR(P) = \frac{1.1}{a} E(Rp)^2 + \frac{0.06}{b} E(Rp) + \frac{0.025}{c}$$

$$mvp \Rightarrow 2.241977104 E(Rp) + 0.058919 = 0$$

$$E(Rp) = -0.026279803 = A / C = -0.02628$$

$$VAR(mvp) = 0.024075665 = 1 / C = 0.024076$$

$$SD(mvp) = 0.155163349 = 1/sqrt(C) = 0.155163$$

w\*A (mvp) = 1.3%  
w\*B (mvp) = 98.7%

VAR (mvp)	Bitcoin	SP500
Bitcoin	0.0099%	0.0205%
SP500	0.0205%	2.3567%
VAR (mvp) =	2.4076%	
SD (mvp) =	15.5163%	

Control: mvp using BK&M (Bodie, Kane & Marcus) General 2 Asset Case Formula

$$wA(mvp) = [ VAR(B) - COVAR(A,B) ] / [ VAR(A) + VAR(B) - 2^*COVAR(A,B) ]$$

w\*A (mvp) = 0.012615465 OK!

*Figure C. 5: Bitcoin + SP500 MVP weights on the bearish scenario*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

**DETERMINING THE PORTFOLIO FRONTIER EQUATION**

Assets (Shares)	Bitcoin	NASDAQ
Expected Return	-75.5%	1.5%
Standard Deviation	78.7%	19.4%

Variances and Covariances Matrix		
	Bitcoin	NASDAQ
Bitcoin	62.0%	1.8%
NASDAQ	1.8%	3.8%

Coefficients of Correlation Matrix		
	Bitcoin	NASDAQ
Bitcoin	1.00	0.12
NASDAQ	0.12	1.00

A = Bitcoin  
B = NASDAQ

V = 0.02298  
D = 25.82299642  
C = 27.023242729  
B = 0.957780126  
A = -0.243574225

**PORTFOLIO FRONTIER EQUATION**

$$\text{VAR}(P) = (1/D) * [ C^2 E(Rp)^2 - 2^*A^*E(Rp) + B ]$$

a = 1.046479746 E(Rp)^2  
b = 0.018864908 E(Rp)^1  
c = 0.037090201 E(Rp)^0

$$\text{VAR}(P) = \frac{1.0}{a} E(Rp)^2 + \frac{0.02}{b} E(Rp) + \frac{0.037}{c}$$

$$\text{mvp} \Rightarrow 2.092959491 E(Rp) + 0.018865 = 0$$

$$E(Rp) = -0.009013508 = A / C = -0.00901$$

$$\text{VAR}(\text{mvp}) = 0.037005181 = 1 / C = 0.037005$$

$$\text{SD}(\text{mvp}) = 0.192367309 = 1/\text{sqrt}(C) = 0.192367$$

w^A (mvp) = 3.1%  
w^B (mvp) = 96.9%

VAR (mvp)	Bitcoin	NASDAQ
Bitcoin	0.0610%	0.0551%
NASDAQ	0.0551%	3.5293%
VAR (mvp) =	3.7005%	
SD (mvp) =	19.2367%	

Control: mvp using BK&M (Bodie, Kane & Marcus) General 2 Asset Case Formula

$$wA(\text{mvp}) = [ \text{VAR}(B) - \text{COVAR}(A,B) ] / [ \text{VAR}(A) + \text{VAR}(B) - 2^*\text{COVAR}(A,B) ]$$

w^A (mvp) = 0.03136667 OK!

**50/50 Portfolio**

ER(p) -50.1%  
SD(p) 49.1%

**Sharpe Ratio**

-1.08

**MVP Portfolio**

ER(p) -2.9%  
SD(p) 21.3%

-0.26

Using the ER(p) and SD(p) we found separately for each combination of assets.

2.7% US 1 Year

Figure C. 6: Bitcoin + NASDAQ MVP weights on the bearish scenario

Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)

**DETERMINING THE PORTFOLIO FRONTIER EQUATION**

Assets (Shares)	Bitcoin	MSCI
Expected Return	-75.5%	-3.6%
Standard Deviation	78.7%	14.4%

A = Bitcoin  
B = MSCI World

Variances and Covariances Matrix		
	Bitcoin	MSCI
Bitcoin	62.0%	0.7%
MSCI	0.7%	2.1%

Coefficients of Correlation Matrix		
	Bitcoin	MSCI
Bitcoin	1.00	0.06
MSCI	0.06	1.00

V = 0.01283  
D = 40.35473204  
C = 48.880249822  
B = 0.956942072  
A = -2.533936757

**PORTFOLIO FRONTIER EQUATION**

$$VAR(P) = (1/D) * [ C^2 * E(Rp)^2 - 2 * A * E(Rp) + B ]$$

a = 1.211264388 E(Rp)^2  
b = 0.125583129 E(Rp)^1  
c = 0.023713256 E(Rp)^0

$$VAR(P) = \frac{1.2}{a} E(Rp)^2 + \frac{0.13}{b} E(Rp) + \frac{0.024}{c}$$

$$mvp \Rightarrow 2.422528777 E(Rp) + 0.125583 = 0$$

$$E(Rp) = -0.051839685 = A / C = -0.05184$$

$$VAR(mvp) = 0.020458161 = 1 / C = 0.020458$$

$$SD(mvp) = 0.143032026 = 1 / \sqrt{C} = 0.143032$$

w\*A (mvp) = 2.3%  
w\*B (mvp) = 97.7%

VAR (mvp)	Bitcoin	MSCI
Bitcoin	0.0314%	0.0147%
MSCI	0.0147%	1.9851%
	VAR (mvp) =	2.0458%
	SD (mvp) =	14.3032%

Control: mvp using BK&M (Bodie, Kane & Marcus) General 2 Asset Case Formula

$$wA(mvp) = [ VAR(B) - COVAR(A,B) ] / [ VAR(A) + VAR(B) - 2 * COVAR(A,B) ]$$

w\*A (mvp) = 0.022508633 OK!

**50/50 Portfolio**

ER(p) -51.4%  
SD(p) 46.6%

**Sharpe Ratio**

-1.16

**MVP Portfolio**

ER(p) -5.2%  
SD(p) 15.9%

-0.50

Using the ER(p) and SD(p) we found separately for each combination of assets. On the app

2.7% US 1 Year treasury Yie

Figure C. 7: Bitcoin + MSCI World MVP weights on the bearish scenario

Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)

**DETERMINING THE PORTFOLIO FRONTIER EQUATION**

Assets (Shares)	Bitcoin	Gold
Expected Return	-75.5%	-1.4%
Standard Deviation	78.7%	9.5%

A = Bitcoin  
B = Gold

Variances and Covariances Matrix		
	Bitcoin	Gold
Bitcoin	62.0%	-0.5%
Gold	-0.5%	0.9%

Coefficients of Correlation Matrix		
	Bitcoin	Gold
Bitcoin	1.00	-0.07
Gold	-0.07	1.00

V = 0.00559  
D = 98.36579202  
C = #####  
B = 0.965099782  
A = -3.465029511

**PORTFOLIO FRONTIER EQUATION**

$$\text{VAR}(P) = (1/D) * [ C * E(Rp)^2 - 2 * A * E(Rp) + B ]$$

a = 1.162635215 E(Rp)^2  
b = 0.070451921 E(Rp)^1  
c = 0.009811335 E(Rp)^0

$$\text{VAR}(P) = \frac{1.2}{a} E(Rp)^2 + \frac{0.07}{b} E(Rp) + \frac{0.010}{c}$$

$$\text{mvp} \Rightarrow 2.32527043 E(Rp) + 0.070452 = 0$$

$$E(Rp) = -0.030298377 = A / C = -0.0303$$

$$\text{VAR}(\text{mvp}) = 0.008744046 = 1 / C = 0.008744$$

$$\text{SD}(\text{mvp}) = 0.093509604 = 1/\text{sqrt}(C) = 0.09351$$

w\*A (mvp) = 2.2%  
w\*B (mvp) = 97.8%

VAR (mvp)	Bitcoin	Gold
Bitcoin	0.0313%	-0.0116%
Gold	-0.0116%	0.8664%
	VAR (mvp) =	0.8744%
	SD (mvp) =	9.3510%

Control: mvp using BK&M (Bodie, Kane & Marcus) General 2 Asset Case Formula

$$wA(\text{mvp}) = [ \text{VAR}(B) - \text{COVAR}(A,B) ] / [ \text{VAR}(A) + \text{VAR}(B) - 2 * \text{COVAR}(A,B) ]$$

w\*A (mvp) = 0.02247248 OK!

**50/50 Portfolio**

ER(p) -50.8%  
SD(p) 44.1%

**Sharpe Ratio**

-1.21

**MVP Portfolio**

ER(p) -3.1%  
SD(p) 11.1%

-0.52

Using the ER(p) and SD(p) we found separately for each combination of assets. On the app

2.7% US 1 Year treasury Yie

Figure C. 8: Bitcoin + Gold MVP weights on the bearish scenario

Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)



**DETERMINING THE PORTFOLIO FRONTIER EQUATION**

Assets (Shares)	Bitcoin	SP500
Expected Return	208.5%	21.5%
Standard Deviation	73.4%	15.0%

A = Bitcoin  
B = SP500

Variances and Covariances Matrix		
	Bitcoin	SP500
Bitcoin	53.9%	-2.1%
SP500	-2.1%	2.2%

Coefficients of Correlation Matrix		
	Bitcoin	SP500
Bitcoin	1.00	-0.19
SP500	-0.19	1.00

V = 0.01161  
D = 301.39547109  
C = 52.025304186  
B = 12.174258023  
A = 18.220153836

**PORTFOLIO FRONTIER EQUATION**

$$\text{VAR}(P) = (1/D) * [ C * E(Rp)^2 - 2 * A * E(Rp) + B ]$$

a = 0.172614751 E(Rp)^2  
b = -0.120905293 E(Rp)^1  
c = 0.040392969 E(Rp)^0

$$\text{VAR}(P) = \frac{0.2}{a} E(Rp)^2 - \frac{0.12}{b} E(Rp) + \frac{0.040}{c}$$

$$\text{mvp} \Rightarrow 0.345229502 E(Rp) - 0.12091 = 0$$

$$E(Rp) = 0.350217151 = A / C \quad 0.350217$$

$$\text{VAR}(\text{mvp}) = 0.019221416 = 1 / C \quad 0.019221$$

$$\text{SD}(\text{mvp}) = 0.13864132 = 1/\text{sqrt}(C) \quad 0.138641$$

w\*A (mvp) = 7.2%  
w\*B (mvp) = 92.8%

VAR (mvp)	Bitcoin	SP500
Bitcoin	0.2831%	-0.1438%
SP500	-0.1438%	1.9266%
	VAR (mvp) =	1.9221%
	SD (mvp) =	13.8641%

Control: mvp using BK&M (Bodie, Kane & Marcus) General 2 Asset Case Formula

$$wA(\text{mvp}) = [ \text{VAR}(B) - \text{COVAR}(A,B) ] / [ \text{VAR}(A) + \text{VAR}(B) - 2 * \text{COVAR}(A,B) ]$$

w\*A (mvp) = 0.072480002 OK!

**50/50 Portfolio**

ER(p) 93.7%  
SD(p) 44.2%

**Sharpe Ratio**

2.08

**MVP Portfolio**

ER(p) 30.0%  
SD(p) 19.2%

1.48

Using the ER(p) and SD(p) we found separately for each combination of assets. On 1

1.6% US 1 Year treas

Figure C. 9: Bitcoin + SP500 MVP weights on the bullish scenario

Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)

**DETERMINING THE PORTFOLIO FRONTIER EQUATION**

Assets (Shares)	Bitcoin	NASDAQ
Expected Return	206.5%	26.6%
Standard Deviation	73.4%	20.5%

Variances and Covariances Matrix		
	Bitcoin	NASDAQ
Bitcoin	53.9%	-2.5%
NASDAQ	-2.5%	4.2%

Coefficients of Correlation Matrix		
	Bitcoin	NASDAQ
Bitcoin	1.00	-0.17
NASDAQ	-0.17	1.00

A = Bitcoin  
B = NASDAQ

V = 0.02208  
D = 149.93497803  
C = 28.583248943  
B = 11.280837003  
A = 13.134229871

**PORTFOLIO FRONTIER EQUATION**

$$\text{VAR}(P) = (1/D) * [ C * E(Rp)^2 - 2 * A * E(Rp) + B ]$$

a = 0.190637631 E(Rp)^2  
b = -0.17519901 E(Rp)^1  
c = 0.075238194 E(Rp)^0

$$\text{VAR}(P) = \frac{0.2}{a} E(Rp)^2 - \frac{0.18}{b} E(Rp) + \frac{0.075}{c}$$

$$\text{mvp} \Rightarrow 0.381275261 E(Rp) - 0.1752 = 0$$

$$E(Rp) = 0.459507941 = A / C = 0.459508$$

$$\text{VAR}(\text{mvp}) = 0.034985526 = 1 / C = 0.034986$$

$$\text{SD}(\text{mvp}) = 0.187044182 = 1/\text{sqrt}(C) = 0.187044$$

w\*A (mvp) = 10.6%  
w\*B (mvp) = 89.4%

VAR (mvp)		
	Bitcoin	NASDAQ
Bitcoin	0.6098%	-0.2376%
NASDAQ	-0.2376%	3.3640%
	VAR (mvp) = 3.4986%	
	SD (mvp) = 18.7044%	

Control: mvp using BK&M (Bodie, Kane & Marcus) General 2 Asset Case Formula

$$wA(\text{mvp}) = [ \text{VAR}(B) - \text{COVAR}(A,B) ] / [ \text{VAR}(A) + \text{VAR}(B) - 2 * \text{COVAR}(A,B) ]$$

w\*A (mvp) = 0.106372032 OK!

*Figure C. 10: Bitcoin + NASDAQ MVP weights on the bullish scenario*

*Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)*

**50/50 Portfolio**

ER(p) 97.7%  
SD(p) 47.0%

**Sharpe Ratio**

2.05

**MVP Portfolio**

ER(p) 39.2%  
SD(p) 26.2%

1.44

Using the ER(p) and SD(p) we found separately for each combination of assets.

1.6% US 1 Year

**DETERMINING THE PORTFOLIO FRONTIER EQUATION**

Assets (Shares)	Bitcoin	MSCI
Expected Return	208.5%	20.8%
Standard Deviation	73.4%	14.0%

A = Bitcoin  
B = MSCI World

Variances and Covariances Matrix		
	Bitcoin	MSCI
Bitcoin	53.9%	-0.9%
MSCI	-0.9%	2.0%

Coefficients of Correlation Matrix		
	Bitcoin	MSCI
Bitcoin	1.00	-0.09
MSCI	-0.09	1.00

V = 0.01048  
D = 336.33307313  
C = 55.094953821  
B = 11.141641698  
A = 16.658786321

**PORTFOLIO FRONTIER EQUATION**

$$VAR(P) = (1/D) * [ C^2 E(Rp)^2 - 2^*A^*E(Rp) + B ]$$

a = 0.163810693 E(Rp)^2  
b = -0.099061244 E(Rp)^1  
c = 0.03312681 E(Rp)^0

$$VAR(P) = \frac{0.2}{a} E(Rp)^2 - \frac{-0.10}{b} E(Rp) + \frac{0.033}{c}$$

$$mvp \Rightarrow 0.327621386 E(Rp) - 0.09906 = 0$$

$$E(Rp) = 0.302365011 = A / C \quad 0.302365$$

$$VAR(mvp) = 0.018150483 = 1 / C \quad 0.01815$$

$$SD(mvp) = 0.134723727 = 1/\sqrt{C} \quad 0.134724$$

w\*A (mvp) = 5.0%  
w\*B (mvp) = 95.0%

VAR (mvp)	Bitcoin	MSCI
Bitcoin	0.1362%	-0.0449%
MSCI	-0.0449%	1.7687%
	VAR (mvp) =	1.8150%
	SD (mvp) =	13.4724%

Control: mvp using BK&M (Bodie, Kane & Marcus) General 2 Asset Case Formula

$$wA(mvp) = [ VAR(B) - COVAR(A,B) ] / [ VAR(A) + VAR(B) - 2^*COVAR(A,B) ]$$

w\*A (mvp) = 0.050268188 OK!

**50/50 Portfolio**

ER(p) 93.1%  
SD(p) 43.7%

**Sharpe Ratio**

2.09

**MVP Portfolio**

ER(p) 26.6%  
SD(p) 17.0%

1.47

Using the ER(p) and SD(p) we found separately for each combination of assets. On the app

1.6% US 1 Year treasury Yie

Figure C. 11: Bitcoin + MSCI World MVP weights on the bullish scenario

Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)

**DETERMINING THE PORTFOLIO FRONTIER EQUATION**

Assets (Shares)	Bitcoin	Gold
Expected Return	208.5%	18.4%
Standard Deviation	73.4%	11.6%

A = Bitcoin  
B = Gold

Variances and Covariances Matrix		
	Bitcoin	Gold
Bitcoin	53.9%	2.1%
Gold	2.1%	1.3%

Coefficients of Correlation Matrix		
	Bitcoin	Gold
Bitcoin	1.00	0.25
Gold	0.25	1.00

V = 0.00681  
D = 530.91516950  
C = 74.876828318  
B = 8.888166130  
A = 11.601832609

**PORTFOLIO FRONTIER EQUATION**

$$VAR(P) = (1/D) * [ C^2 E(Rp)^2 - 2^*A^*E(Rp) + B ]$$

a = 0.141033507 E(Rp)^2  
b = -0.043705033 E(Rp)^1  
c = 0.016741217 E(Rp)^0

$$VAR(P) = \frac{0.1}{a} E(Rp)^2 - \frac{0.04}{b} E(Rp) + \frac{0.017}{c}$$

$$mvp \Rightarrow 0.282067014 E(Rp) - 0.04371 = 0$$

$$E(Rp) = 0.154945567 = A / C \quad 0.154946$$

$$VAR(mvp) = 0.013355267 = 1 / C \quad 0.013355$$

$$SD(mvp) = 0.115564988 = 1/\sqrt{C} \quad 0.115565$$

w\*A (mvp) = -1.5%  
w\*B (mvp) = 101.5%

VAR (mvp)	Bitcoin	Gold
Bitcoin	0.0126%	-0.0330%
Gold	-0.0330%	1.3890%
	VAR (mvp) =	1.3355%
	SD (mvp) =	11.5565%

Control: mvp using BK&M (Bodie, Kane & Marcus) General 2 Asset Case Formula

$$wA(mvp) = [ VAR(B) - COVAR(A,B) ] / [ VAR(A) + VAR(B) - 2^*COVAR(A,B) ]$$

w\*A (mvp) = -0.015298001 OK!

**50/50 Portfolio**

ER(p) 91.2%  
SD(p) 42.5%

**Sharpe Ratio**

2.11

**MVP Portfolio**

ER(p) 16.7%  
SD(p) 10.7%

1.41

Using the ER(p) and SD(p) we found separately for each combination of assets. On the appendixes

1.6% US 1 Year treasury Yield <https://www>

Figure C. 12: Bitcoin +Gold MVP weights on the bullish scenario

Source: Processed from Yahoo Finance data (2020) and CoinMarketCap data (2020)