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# The Behaviour of the Nominal Interest Rate in a World with Technological Innovations

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## Abstract

In this paper we look at the behaviour of the nominal interest rate in a world where there are technological innovations that are specific to the banking industry as well as technological innovations in nonbank firms. We find that in a world with technological shocks the nominal interest rate has to adjust so as to make the MPL in banks equal the MPL in nonbank firms. Because the banking sector is very small, the key real variables, including the real wage and the real demand for money, are mainly determined outside the banking industry. Given this behaviour, the nominal interest rate charged by the banks has to adjust so as to make the MPL in banks equal the real wage at the point that corresponds to the amount of hours of work that banks need in order to be able to supply the real amount of credit that is being demanded.

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# 1 Introduction

A key post-war U.S. business cycle fact is the negative correlation between the real interest rate and real output (King and Watson, 1996). However, in standard RBC models the technological innovations that drive fluctuations make output and the interest rate move in the same direction. For instance, a positive technological shock leads not only to an increase in real output but also, by raising the marginal productivity of capital, to higher interest rates. Since, on the other hand, a positive shock in technology is propagated into the following periods by the process that generates the shocks, output also rises in the following periods. As a consequence, RBC models generate positive correlations between the real interest rate and current and future values of real output - instead of the negative correlations observed in the U.S. business cycle.

King and Watson show that not only the standard RBC model but also two other benchmark models, the sticky price model and the liquidity effect model, miss the negative correlation between the real interest rate and real output.

To address this problem, we add a banking sector to a standard RBC model. The impulse response experiments we performed with our model show that whereas positive shocks in the nonbank firms technology make the real interest rate go up, positive shocks in the banks technology make the real interest rate fall. This implies that, if the technological shocks in banks are strong enough, stochastic simulations of our model generate negative correlations between the real interest rate and current and future values of real output - the key post-war business cycle fact difficult to reproduce in model economies.

This result is particularly relevant because technological innovations over the past few decades have affected the banking industry with special intensity. In fact, the diffusion of information and communication technologies (ICT) has been a prime vehicle of technological progress, and services in general and banking in particular are the main beneficiaries of investment in ICT. For instance, the category of Financial Institutions - of which banking is the major part - is by now the most ICT intensive industry in the U.S. as measured by the ratio of computer equipment and software to value added (Triplet and Bosworth, 2002, Table 2). Section 7.2.2. of the present paper presents empirical evidence which suggests that technological innovations in banks have been larger than in the rest of the economy.

The structure of the article is as follows. In section 2, we present the economic environment: economic flows, preferences, technology, resource constraints and market structure. In section 3, we describe the typical bank's behaviour and the typical firm's behaviour. The typical household's

behaviour is outlined in section 4. In section 5 we write down the market clearing conditions, and then we show the equations that describe the competitive equilibrium. The calibration of the model is explained in section 6. Section 7 looks at the impact of technological innovations in bank and nonbank firms, and then discusses the main results of the paper. The last section makes an overview and concludes.

## 2 The Economic Environment

We build a closed economy model with no government. There are  $H$  homogeneous households,  $F$  homogeneous firms (nonbank firms) and  $L$  homogeneous banks in operation. Firms and banks are owned by households. As a consequence, both the firms' and the banks' profits are distributed to households (the shareholders) at the end of each period. Only one physical good is produced (physical output), which can either be consumed or used for investment (i.e., used to increase the capital stock).

Bank loans are the only source of money to the economy. They are provided in real terms (i.e. in units of physical output). At the beginning of the period, banks make loans to households who then use the money to buy consumption goods from the firms. At the end of the period, households receive the money back from the firms as wage and dividend payments. Afterwards, households use this money to pay to the banks the principal and interest of the debt contracted at the beginning of the period. The structure of the model is such that, after all these payments, households are left with nothing, and must therefore borrow again from the banks at the start of the new period.

The reason why it seems important to work with this kind of credit structure is the fact that, in modern economies, most money has its origin in loans from commercial banks to households, firms or the government (to confirm this we only have to look at the aggregate balance sheet of the Monetary Sector).

It is important to emphasize that the essence of our extension of the RBC model is not money, but rather the addition of a banking sector which provides credit and charges interest in real terms. There is not a nominal money stock, nor a price level. Money is provided in real terms, and has no effect or function in the economy except that of a means of settling exchanges.

We next examine the typical household's preferences, the technology available in the economy, the resource constraints, and the market structure. The typical household seeks to maximize lifetime utility. Utility in period  $t$  is given by  $u(c_t, \ell_t)$ , where  $c_t$  is consumption and  $\ell_t$  is leisure. The function  $u(.,.)$  has the usual properties. At the beginning of period 0, the household maximizes

$U_0 = E_0 \left[ \sum_{t=0}^{t=\infty} \beta^t u(c_t, \ell_t) \right]$ , where  $\beta$  is a discount factor ( $0 < \beta < 1$ ).

The technology available in this economy is as follows. In each period, the typical firm's production function is described by  $y_t = A_t F(k_t, n_t^d)$ , where  $y_t$  is the physical output of the firm,  $A_t$  is a technological parameter,  $k_t$  is the firm's (pre-determined) capital stock, and  $n_t^d$  is the firm's labour demand. The capital accumulation equation of each firm is  $k_{t+1} = (1 - \delta)k_t + i_t$ , where  $i_t$  is investment and  $\delta$  is the per-period rate of depreciation of the capital stock.

For each bank, there is also a production function that tells how many hours of work the bank must hire to be able to supply a certain amount of credit in real terms. The technology available to all banks can be summarized by

$$b_t^s = D_t (n_t^b)^\gamma \tag{1}$$

where  $b_t^s$  is the bank's supply of credit in real terms,  $D_t$  is a technological parameter, and  $n_t^b$  is the number of work hours hired by the bank. Also note that we are implicitly assuming that physical capital in banks is fixed and does not depreciate; and that an improvement in the bank's technology can be modelled as an increase in  $D_t$ .

The resource constraints in this economy are as follows. Each firm starts period  $t$  with a capital stock,  $k_t$ , which was determined at the beginning of period  $(t - 1)$ . Hence, the stock of capital that enters the production function in period  $t$  is pre-determined and cannot be changed at the beginning or during period  $t$ . Each household has an endowment of time per-period which is normalized to *one* by an appropriate choice of units. This endowment can be used to work or to rest. Therefore, we can write  $n_t^s + \ell_t = 1$ , where  $n_t^s$  is the household's supply of labour during period  $t$ .

Finally, the market structure is as follows. There are five markets: the goods market, the labour market, the bank loans' market, the market for firms' shares and the market for banks' shares. Labour is homogeneous and perfectly mobile between the two industries of the economy - the physical output industry and the banking industry. All households, firms and banks are price-takers. Prices are perfectly flexible and adjust to clear all markets in every period.

### 3 The Behaviour of Banks and Firms

Banks are profit maximizing firms that make loans to households. The loans imply costs of operation to the banks which in our case are only labour costs. In period  $t$ , the real profits of each

bank are given by real interest income minus real wage payments to the bank's employees:

$$\pi_t^{bank} = r_t b_t^s - w_t n_t^b \quad (2)$$

where  $r_t$  is the real interest rate charged for loans,  $b_t^s$  is the real amount of credit supplied at the beginning of period  $t$ , and  $w_t$  is the real wage rate. We assume that banks pay wages and dividends to households only at the end of each period. Using equation (1), equation (2) becomes

$$\pi_t^{bank} = r_t D_t (n_t^b)^\gamma - w_t n_t^b \quad (3)$$

Each bank maximizes the Value of its Assets (VA), i.e., the expected discounted value of its stream of present and future dividends. Therefore, at the beginning of period 0 the typical bank's optimization problem is

$$Max_{n_t^b} VA = E_0 \left[ \sum_{t=0}^{t=\infty} \frac{1}{(1+r_0)(1+r_1)\dots(1+r_t)} \pi_t^{bank} \right]$$

where  $\pi_t^{bank}$  is given by (3).

Real profits of each firm in period  $t$  are given by sales of output minus the wage bill minus investment expenditure (all variables in real terms):

$$\pi_t = A_t F(k_t, n_t^d) - w_t n_t^d - i_t \quad (4)$$

Like banks, firms pay wages and distribute dividends to households at the end of each period. Each firm maximizes the Value of its Assets (VA). Therefore, at the beginning of period 0 the typical firm's optimization problem is

$$Max_{n_t^d, k_{t+1}} VA = E_0 \left[ \sum_{t=0}^{t=\infty} \frac{1}{(1+r_0)(1+r_1)\dots(1+r_t)} \pi_t \right]$$

where  $\pi_t$  is given by (4). Note that  $i_t = k_{t+1} - (1 - \delta)k_t$ . There is also an initial condition for the capital stock, the standard transversality condition for the capital stock, and non-negativity constraints.

## 4 The Typical Household's Behaviour

To write the typical household's budget constraint, we have first to understand the way bank loans and shares work in our model. Bank loans work as follows. At the beginning of period  $t$ , each household borrows from banks  $\frac{b_{t+1}}{1+r_t}$  units in real terms, and then pays  $\frac{b_{t+1}}{1+r_t}(1+r_t) = b_{t+1}$  units at the end of period  $t$ .

The shares of firms work as follows.  $q_t^f$  is the real price that someone would have to pay to buy 100% of firm  $f$  at the beginning of period  $t$ .  $z_t^f$  is the percentage of firm  $f$  that the household *bought* at the beginning of period  $(t-1)$  and *sells* at the beginning of period  $t$ . The shares of banks work in the same way:  $q_t^{bank,l}$  is the real price of 100% of bank  $l$  at the beginning of period  $t$ ; and  $z_t^{bank,l}$  is the percentage of bank  $l$  that the household *bought* at the beginning of period  $(t-1)$  and *sells* at the beginning of period  $t$ .

In this setting, the typical household's budget constraint is

$$\begin{aligned}
w_{t-1}n_{t-1}^s + \sum_{f=1}^{f=F} z_t^f \pi_{t-1}^f + \sum_{l=1}^{l=L} z_t^{bank,l} \pi_{t-1}^{bank,l} + \sum_{f=1}^{f=F} z_t^f q_t^f + \sum_{l=1}^{l=L} z_t^{bank,l} q_t^{bank,l} + \frac{b_{t+1}}{1+r_t} = \\
= b_t + c_t + \sum_{f=1}^{f=F} z_{t+1}^f q_t^f + \sum_{l=1}^{l=L} z_{t+1}^{bank,l} q_t^{bank,l}
\end{aligned} \tag{5}$$

The left-hand side of this equation indicates the total amount of money the household obtains at the beginning of period  $t$  - wage earnings, dividend earnings from firms and banks, money received from selling the shares of firms and banks bought at the beginning of period  $(t-1)$ , and the amount of bank loans obtained at the beginning of period  $t$ . In turn, the right-hand side of the equation indicates the amount of money that the household spends at the beginning or during period  $t$  - payment of the debt contracted from the banks at the beginning of period  $(t-1)$ , consumption expenditure during period  $t$ , and the purchase of shares of firms and banks at the beginning of period  $t$ . Hence, equation 5 simply states that the total amount of money the household *obtains* at the beginning of period  $t$  must be equal to the amount that the household *spends* at the beginning or during period  $t$ .

We use the following initial condition to describe the household's debt position at the beginning of period 0

$$b_0 = w_{-1}n_{-1}^s + \sum_{f=1}^{f=F} z_0^f \pi_{-1}^f + \sum_{l=1}^{l=L} z_0^{bank,l} \pi_{-1}^{bank,l} \tag{6}$$

This initial condition states that the household begins period 0 with a debt which equals the sum of the wage and dividend earnings she receives at the beginning of that period 0 [she receives these earnings because of the hours she worked during period  $(-1)$  and because of the shares of firms and banks she bought at the beginning of period  $(-1)$ ]. Note that period 0 is not the period where the household's life starts but rather the period where our analysis of the economy begins (the household has been living for some periods and we catch her in period 0 and try to model



her behaviour). In the appendix, we show that this initial condition arises naturally when we think back to the initial moment of a closed economy without government and where firms don't borrow.

Consequently, at the beginning of period 0, the household is looking into the future and maximizes  $U_0 = E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t) \right]$  subject to (5), (6) and  $n_t^s + \ell_t = 1$ . The choice variables are  $c_t, \ell_t, n_t^s, b_{t+1}, z_{t+1}^f$  and  $z_{t+1}^{bank,l}$ . There are also initial conditions on holdings of shares [ $z_0^f = \frac{1}{H}$  and  $z_0^{bank,l} = \frac{1}{H}$ ], a standard transversality condition on the pattern of borrowing, and non-negativity constraints.

## 5 Market Clearing Conditions and the Competitive Equilibrium

With  $H$  homogeneous households,  $F$  homogeneous firms and  $L$  homogeneous banks, the market clearing conditions for period 0 are as follows:  $Hc_0 + Fi_0 = Fy_0$ , for the goods market;  $Fn_0^s = Fn_0^d + Ln_0^b$ , for the labour market; and  $H\frac{b_1}{1+r_0} = Lb_0^s$ , for the bank loans market. The market clearing condition in the shares market is that each firm and each bank should be completely held by the households. Since households are all alike, each household will hold an equal share of each firm and of each bank. Therefore, the market clearing conditions in the shares market are  $Hz_1^f = 1$  and  $Hz_1^{bank,l} = 1$ .

To obtain the system of equations that describes the competitive equilibrium, we put together in a system the typical household's first order conditions (which implicitly give the household's demand and supply functions), the typical firm's first order conditions (which give, in an implicit way, the firm's demand and supply functions), the typical bank's first order conditions (which implicitly give the bank's demand and supply functions), and the market clearing conditions. We then assume Rational Expectations and that the production function of each firm is homogeneous of degree one. After all these steps, and if we define the following new *per household* variables  $\bar{k}_t = \frac{F}{H} k_t, \bar{n}_t^d = \frac{F}{H} n_t^d, \bar{n}_t^b = \frac{L}{H} n_t^b, \bar{q}_t^f = \frac{F}{H} q_t^f, \bar{\pi}_t^f = \frac{F}{H} \pi_t^f, \bar{q}_t^{bank,l} = \frac{L}{H} q_t^{bank,l}$  and  $\bar{\pi}_t^{bank,l} = \frac{L}{H} \pi_t^{bank,l}$ , we can write the system describing the Competitive Equilibrium as

$$u_1(c_t, 1 - n_t^s) = \lambda_t \quad (7)$$

$$u_2(c_t, 1 - n_t^s) = E_t [\beta \lambda_{t+1} w_t] \quad (8)$$

$$\frac{\lambda_t}{1 + r_t} = E_t [\beta \lambda_{t+1}] \quad (9)$$

$$\lambda_t \bar{q}_t^f = E_t \left[ \beta \lambda_{t+1} \left( \bar{\pi}_t^f + \bar{q}_{t+1}^f \right) \right] \quad (10)$$

$$\lambda_t \bar{q}_t^{bank,l} = E_t \left[ \beta \lambda_{t+1} \left( \bar{\pi}_t^{bank,l} + \bar{q}_{t+1}^{bank,l} \right) \right] \quad (11)$$

$$\frac{b_{t+1}}{1+r_t} = c_t \quad (12)$$

$$A_t F_2(\bar{k}_t, \bar{n}_t^d) = w_t \quad (13)$$

$$E_t \left[ A_{t+1} F_1(\bar{k}_{t+1}, \bar{n}_{t+1}^d) + (1-\delta) \right] = 1 + r_t \quad (14)$$

$$r_t D_t \gamma \left( \frac{L}{H} \right)^{1-\gamma} (\bar{n}_t^b)^{\gamma-1} = w_t \quad (15)$$

$$c_t + [\bar{k}_{t+1} - (1-\delta)\bar{k}_t] = A_t F(\bar{k}_t, \bar{n}_t^d) \quad (16)$$

$$n_t^s = \bar{n}_t^d + \bar{n}_t^b \quad (17)$$

$$\frac{b_{t+1}}{1+r_t} = \left( \frac{L}{H} \right)^{1-\gamma} D_t (\bar{n}_t^b)^\gamma \quad (18)$$

$$z_{t+1}^f = \frac{1}{H} \quad (19)$$

$$z_{t+1}^{bank,l} = \frac{1}{H} \quad (20)$$

$$\bar{\pi}_t^f = A_t F(\bar{k}_t, \bar{n}_t^d) - w_t \bar{n}_t^d - [\bar{k}_{t+1} - (1-\delta)\bar{k}_t] \quad (21)$$

$$\bar{\pi}_t^{bank,l} = r_t D_t \left( \frac{L}{H} \right)^{1-\gamma} (\bar{n}_t^b)^\gamma - w_t \bar{n}_t^b \quad (22)$$

for  $t = 0, 1, 2, 3, \dots$

Equations 7-12 have their origin in the typical household's first order conditions. Equation 12 is the credit-in-advance constraint which results from combining the household's budget constraint

with the household's initial debt condition and then using the market clearing conditions from the shares' market. In the appendix of Leao (2003), it is explained in detail how this credit-in-advance constraint appears in period 0 and how, under Rational Expectations, it is propagated into future periods. Equations 13 and 14 result from the typical firm's first-order conditions. Equation 15 has its origin in the typical bank's first-order condition. Equations 16-20 result from the market clearing conditions. Equation 21 is obtained by multiplying the definition of firm profits by  $(F/H)$ . Equation 22 results from multiplying the definition of bank profits by  $(L/H)$ .

If we consider equation (15) in the context of the whole system, we can understand how this model binds the real supply of credit (which is given by  $b_t^s = D_t(n_t^b)^\gamma$ ). As long as the market determined ratio  $(w_t/r_t)$  is finite, equation (15) determines the variable  $\bar{n}_t^b$  which through the banks' production function determines the supply of credit.

In the simulations below we will use the following firm's production function and household's utility function:  $A_t F(k_t, n_t^d) = A_t (k_t)^{1-\alpha} (n_t^d)^\alpha$  and  $u(c_t, \ell_t) = \ln c_t + \phi \ln \ell_t$ , respectively.

## 6 Calibration

In order to study the dynamic properties of the model, we have log-linearized each of the equations in the system 7-22 around the steady-state values of its variables. The log-linearized system was then calibrated. To calibrate the log-linearized system we used the following parameters. With the specific utility function we are using, we obtain

Elasticity of the MU of consumption with respect to consumption	-1
Elasticity of the MU of consumption with respect to leisure	0
Elasticity of the MU of leisure with respect to consumption	0
Elasticity of the MU of leisure with respect to leisure	-1

where MU denotes "Marginal Utility". From the U.S. data, we obtain

Investment share of total expenditure in the s.s. $(\frac{\dot{y}}{y})$	0.167	Barro (1993)
Firm workers' share of the firm's income $(\alpha)$	0.58	King et al. (1988)
Labour supply in the steady-state $(n^s)$	0.2	King et al. (1988)
Real interest rate in the steady-state $(r)$	0.007060	FRED and Barro (1993)
Banks' share of total "hours of work" in s.s. $(\frac{Ln^b}{Hn^s})$	0.014	BLS
Bank workers' share of the bank's income $(\gamma)$	0.271	FDIC

In order to calibrate the steady-state real interest rate, we have computed the average quarterly value of the real interest rate for the period 1949-1986. To do this, we used the Federal Reserve

Economic Data (FRED) to obtain quarterly values for the Bank Prime Loan Rate for the period 1949-1986 and data from Barro (1993) to obtain quarterly values for the inflation rate in the same period. To calibrate the Banks' share of "total hours of work" in the steady-state, we have computed the average value of the ratio (total hours of work in U.S. commercial banks / total hours of work in the U.S. economy) for the period 1972:1-1986:12. To do this, we used data from the web site of the Bureau of Labour Statistics (BLS).<sup>1</sup>

The parameter  $\gamma$  is equal to the bank workers' share of the bank's income. In our model, the only income banks have is interest income. To calibrate  $\gamma$  we have used the "Historical Statistics on Banking" from the Federal Deposit Insurance Corporation (FDIC) to compute the average value of the ratio (commercial banks' wage payments in the U.S. / commercial banks' interest income in the U.S.) for the period 1966-1986. The values for the wage payments of commercial banks we used were the values for "Employee Salaries & Benefits" which were taken from the table "Noninterest Income and Noninterest Expense of Insured Commercial Banks" of the FDIC's web site<sup>2</sup>. The values for the commercial banks' interest income were the values for "Interest Income on Loans and Leases" which were obtained from the table "Interest Income of Insured Commercial Banks" in their web site. The values in the preceding tables of the present section imply

Consumption share of total expenditure in the s.s. ( $\frac{Hc}{Fy}$ )	0.833
Household's discount factor ( $\beta$ )	0.993
Per-quarter rate of depreciation of the firm's capital stock ( $\delta$ )	0.0047

With these parameter values, the model has steady-state values of physical output, consumption and investment which are 1.4% lower than in a zero growth version of the RBC model presented in King, Plosser and Rebelo (1988) calibrated with our parameters. This is what we would expect because in their model transactions occur without the need to use money whereas in our model households have to borrow from the banks to finance their consumption expenditure and the supply of credit by the banking industry involves spending some of the resources of the economy, namely "hours of work". As a consequence, the amount of "hours of work" left to be used by the physical output industry is lower (note that the total supply of "work effort" in the steady-state in this economy is still calibrated as  $n^s = 0.2$ , i.e., the same value that is used by King, Plosser and Rebelo).

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<sup>1</sup>The internet address is [www.bls.gov](http://www.bls.gov)

<sup>2</sup>The internet address is [www2.fdic.gov/hsob](http://www2.fdic.gov/hsob)

## 7 The Dynamic Properties of the Model

The response of the log-linearized model to shocks in the exogenous variables ( $A_t$  and  $D_t$ ) can be obtained using the King, Plosser and Rebelo (1988) method. First, we examine the impact of shocks in the firms' technology. Afterwards, we look at the impact of shocks in the banks' technology. Finally, we study the impact of shocks in both sectors.

### 7.1 Technological innovations in nonbank firms

In this section, we examine the results of two experiments that use shocks in the firms' technological parameter ( $A_t$ ): impulse response and stochastic simulation. In order to perform these two exercises, we assumed that the firms' technological parameter evolves according to the following process

$$\hat{A}_t = 0.9\hat{A}_{t-1} + \varepsilon_t \quad (23)$$

where  $\hat{A}_t$  denotes the % deviation of  $A_t$  from its steady-state value and  $\varepsilon_t$  is a white noise.

#### 7.1.1 Impulse Response

The impulse response experiment we carried out with the model was a 1% shock to the firms' technological parameter (assuming that there are no shocks to the banks' technology). This shock occurs at  $t = 2$ . The results are plotted in figures 1 to 8. Several conclusions can be drawn.

First, our model is capable of reproducing the key results that the King, Plosser and Rebelo (1988) RBC model is able to reproduce. On the one hand, consumption, investment and "hours of work" are pro-cyclical. On the other hand, consumption is less volatile than output and investment is more volatile than output. These are very well documented stylized facts about the United States economy [references on this include Kydland and Prescott (1990) and Backus and Kehoe (1992)]. Note also that the figures for  $\hat{y}_t$ ,  $\hat{c}_t$ ,  $\hat{i}_t$ ,  $\hat{n}_t^s$  and  $\hat{w}_t$  are very close to the figures we obtain with a zero growth version of the model in King, Plosser and Rebelo. The reason is that in our model the banking industry is very small (as can be seen in the values we obtained for the parameters that calibrate its weight - see Section 6).

Second, the response of consumption is slightly weaker in our model than in the zero growth version of the King, Plosser and Rebelo model (0.1169% instead of 0.122% deviation from the steady state). This happens for the same reason that explains why the steady-state value of

consumption in our model is 1.4% lower than in the zero growth version of their model (see end of Section 6).

Third, work hours in banks and their supply of credit respond positively to the technological shock (figure 5). The reason is as follows. A positive technological shock implies more physical output and higher wage and dividend payments in real terms. Knowing these payments (which will only be made at the end of the period) to be higher, households want to consume more and, therefore, borrow more. The banking industry responds to this increased demand for credit by hiring more people in order to be able to supply a higher amount of credit.

Finally, the real interest rate reacts positively to a shock in the firms technology (figure 8). This result can be explained as follows. When the positive shock to the firms technology occurs, the marginal product of labour (MPL) increases in firms. Because in equilibrium the value of the MPL in banks must be equal to the value of the MPL in firms (see equations 13 and 15 above), the interest rate charged by commercial banks has to increase in order to make the value of the MPL in banks rise as well. The real interest rate ( $r_t$ ) must necessarily rise because the increase in  $\bar{n}_t^b$ , which is needed to obtain a higher supply of credit in real terms, acts in the direction of reducing the MPL in banks.

### 7.1.2 Stochastic Simulation

Table 1 shows the results of a stochastic simulation exercise with shocks to the firms' technological parameter (with no shocks to the banks' technological parameter). These results are consistent with what we obtained in the previous impulse response experiment.

First, the results for the real variables  $\hat{y}_t$ ,  $\hat{c}_t$ ,  $\hat{u}_t$ ,  $\hat{n}_t^s$  and  $\hat{w}_t$  are again very close to the results we obtained with a zero growth version of the model in King, Plosser and Rebelo (1988).

Second, both  $\hat{n}_t^b$  and  $\hat{r}_t$  respond positively to improvements in the firms technology and, therefore, appear positively correlated with output. This stochastic simulation with shocks restricted to the firm's technology is thus unable to reproduce the negative correlation between the real interest rate and real output observed in the U.S. business cycle.

## 7.2 Technological innovations in both sectors

In this section, we first examine an impulse response experiment that considers shocks only in the banks' technological parameter (shocks in  $D_t$ ), and then we look at what happens when there are shocks in both sectors (shocks in  $A_t$  and in  $D_t$ ). In these experiments, we assumed that the banks' technological parameter evolves according to

$$\widehat{D}_t = 0.9\widehat{D}_{t-1} + \varphi_t \quad (24)$$

where  $\widehat{D}_t$  denotes the % deviation of  $D_t$  from its steady-state value and  $\varphi_t$  is a white noise.

### 7.2.1 Response to a shock in the banks' technology

Figures 9 to 16 plot the impulse response results of a 1% shock to the banks' technology (with no shock in the firms' technology). It is interesting to see how the economy uses the technological gift in the banking industry to improve the general welfare of the people that live in it. We can see that consumption and leisure, the variables that give utility to households, both increase slightly (figures 10 and 12). This happens because the banking industry now has a more powerful technology, and can therefore perform its role of supplier of credit using fewer hours of work. Work hours in banks are reduced by almost 4% (figure 13), and this reduction is used to increase work hours in firms (figure 14) and to increase leisure (figure 12).

On the other hand, it should be emphasized that the real interest rate falls as a result of the positive technological shock in the banking sector (figure 16). This can be explained as follows. The increase in labour demand by firms makes the MPL in firms fall (this can be seen in the fall of the real wage - figure 15). Since the positive shock in  $D_t$  and the ensuing decrease in work hours in banks both work in the direction of making the MPL rise in banks, the real interest rate must fall (figure 16) so that the value of the MPL in banks can follow the decrease of the value of the MPL in firms. This negative impact of the technological innovations in banks on the real interest rate is the result that made us think it might be possible to replicate the empirical findings of King and Watson (1996).

### 7.2.2 Technological innovations bigger in the banking sector

By looking at figures 8 and 16, we can easily see that if a 1% shock in  $A_t$  is combined with a shock in  $D_t$  which is strong enough, the result will be a fall in the real interest rate. As an example, figures 17 to 24 plot the impulse response results of a 1% shock in  $A_t$  combined with a 5% shock in  $D_t$ . We can see in Figure 24 that the real interest rate falls and in Figure 17 that real output rises. These *impulse response* results suggest that negative correlations between the real interest rate and real output may be obtained in *stochastic simulation* experiments where the technological shocks are stronger in banks than in firms.

A question that immediately arises is: does it make sense to use shocks in  $D_t$  bigger than the shocks in  $A_t$ ? There are strong reasons to believe that technological innovations have been

stronger in the banking sector than in the rest of the economy. In fact, and as mentioned in the introduction to the paper, the banking sector has been the main beneficiary of investment in ICT - the prime form technological progress has taken in the last few decades. In particular, the category of Financial Institutions - of which banking is the major part - is by now the most ICT intensive industry in the U.S. as measured by the ratio of computer equipment and software to value added (Triplet and Bosworth, 2002, Table 2).

The following facts illustrate the extent to which innovations in the banking industry have transformed the way it operates. (i) While in the early 1980s the number of ATMs was roughly equal to the number of bank branches (offices), by 2000 ATMs had outnumbered branches by more than four to one (Berger, 2003, Table 1). (ii) The ratio between the number of check payments (which require paper work) and the number of debit and credit card payments (which are almost entirely processed by electronic means) fell from four in 1995 to only two in 2000 (Gerdes and Walton, 2002, Table 2). (iii) By the end of 2001, banks with transactional internet websites already served the majority of bank customers in the U.S. (Furst, Lang and Nolle, 2002).

The effects of these transformations on the productivity statistics have been clear. Productivity growth has been stronger in the banking sector than in the rest of the economy. According to the data of the Bureau of Labour Statistics (BLS), between 1982 and 2000 labour productivity grew at an annual rate of 3.23% in the banking sector against only 1.95% in the private nonfarm business sector. A study by the Mckinsey Global Institute (2001) estimated an even stronger growth of labour productivity in the banking sector - an annual rate of 5.36% over the same period.

In our model, technological shocks are measured as percentage deviations from the steady-state technological trend. In order to obtain an indication about the relative magnitude of the shocks in  $D_t$  and  $A_t$ , we estimated that relationship using real world data. Technological shocks for the total private sector were estimated as follows. Using the BLS series for the total factor productivity (TFP) in the private sector - taken from their web site -, we applied the Hodrick-Prescott filter to estimate the trend, and then calculated the “Percentage deviations from trend of the TFP in the private sector”.

The estimates for the technological shocks in the banking sector were obtained in the following way. From the production function of the banking sector (equation 1), we obtained the formula  $D_t = \frac{b_t^s}{(n_t^b)^\gamma}$ . Values for  $b_t^s$  were obtained from the Federal Reserve Economic Data (series “Bank Credit of Commercial Banks” deflated by the “GDP Implicit Price Deflator” ). Values for  $n_t^b$  were computed with BLS data (using the series “Employees of Commercial Banks” and “Average



Weekly Hours of Production Workers of Commercial Banks”). For  $\gamma$ , we used the calibration value (see Section 6). We then used the Hodrick-Prescott filter to estimate the trend of the  $D_t$  series and, finally, computed the “Percentage deviations from trend of the technological levels in the banking sector”. (We used annual data for the 1980-2000 period).

Table 2 shows the estimates for  $\widehat{D}_t$  and  $\widehat{A}_t$  and the ratio between the two shocks for each year in the 1980-2000 period. We can see that the shocks in the two sectors were in the same direction in 16 of the 21 years. In these 16 years, the shocks in the banking industry were on average 5.63 times higher than in the total private sector.

### 7.2.3 Stochastic simulation with shocks in both sectors

King and Watson (1996) report the following cross-correlations between the real interest rate and real output in the U.S. data:

Variable	correlation with								
	$y_{t-4}$	$y_{t-3}$	$y_{t-2}$	$y_{t-1}$	$y_t$	$y_{t+1}$	$y_{t+2}$	$y_{t+3}$	$y_{t+4}$
$r_t$	0.19	0.19	0.10	-0.07	-0.27	-0.43	-0.52	-0.50	-0.41

We can see that, in the real world data, there are negative correlations between the real interest rate and current and future values of real output.

In our stochastic simulation experiments, we find that if in each period the technological shock in banks is at least 2.6 times bigger than the technological shock in firms, negative correlations arise between the real interest rate and current and future real output. Also, if the shock in banks is 2.8 times larger, the correlation values we obtain are similar to the values reported by King and Watson.

Table 3 shows the results of a stochastic simulation experiment with our model assuming  $\widehat{D}_t = 2.8\widehat{A}_t$  in each period. As shown in Table 3B, this simulation generates correlations between the real interest rate and current and future values of real output which are not far from the correlations reported in the King and Watson empirical study. The intuition for this result is as follows. In our model, a high level of real economic activity comes mainly from the positive shocks in  $A_t$ . Because each positive shock in  $A_t$  is propagated into the following periods by the process which generates the shocks (equation 23), so is a high level of real economic activity propagated into the following periods. On the other hand, in our simulation each positive shock in  $A_t$  is associated with a stronger positive shock in  $D_t$  which exerts downward pressure on the real interest rate. As a consequence, negative correlations arise between the real interest rate and the current and future values of real output.

Comparing Tables 1 and 3 we can conclude that the main differences are not only the results for  $\hat{r}_t$  but also those for  $\hat{n}_t^b$ . Let us compare the results we report in Table 3B for  $\hat{n}_t^b$  with the results for the U.S. data. Using quarterly data for the period 1972-1986 and using a Hodrick-Prescott filter to estimate the trends, we have obtained a contemporaneous correlation between “Percentage deviations from trend of work hours in U.S. commercial banks” and “Percentage deviations from trend of Real GDP in fixed 1992 dollars” equal to  $-0.158$ . The data were obtained from the BLS and from the FRED, respectively. While this value does not completely match the contemporaneous correlation between  $\hat{n}_t^b$  and  $\hat{y}_t$  that we obtained with our model (which is  $-0.93$  as can be seen in table 3B), it suggests a slightly negative correlation between the two variables in the U.S. economy. It is obvious from the impulse response exercises and stochastic simulation exercises we have examined that this negative correlation could only arise in our model if there are technological innovations in the banking industry. The whole set of correlations we obtain *from the data* is

Variable	Correlation with										
	$\hat{y}_{t-12}$	$\hat{y}_{t-8}$	$\hat{y}_{t-4}$	$\hat{y}_{t-2}$	$\hat{y}_{t-1}$	$\hat{y}_t$	$\hat{y}_{t+1}$	$\hat{y}_{t+2}$	$\hat{y}_{t+4}$	$\hat{y}_{t+8}$	$\hat{y}_{t+12}$
$\hat{n}_t^b$	0.37	0.53	0.53	0.22	0.07	-0.16	-0.39	-0.59	-0.77	-0.51	-0.013

It may be noted that our results in table 3B also show negative correlations between  $\hat{n}_t^b$  and the current and future values of real output. These negative correlations can be explained as follows. When  $A_t$  rises, the real income of each household rises which makes them wish to increase consumption. This causes an increase in the demand for credit in real terms. Since the rise in  $A_t$  is accompanied by a rise in  $D_t$ , the real supply of credit can rise without the need for an increase in  $n_t^b$ .

We may summarize by saying that the results in Table 3 reproduce the usual results from RBC models and, at the same time, give us some good results in terms of  $\hat{r}_t$  and  $\hat{n}_t^b$ .

## 8 Conclusion

We have built a dynamic general equilibrium model that adds a banking sector to the standard RBC model. We have looked at the response of the model to innovations in both the banks' and the firms' technology. In a stochastic simulation exercise with technological shocks that affect both sectors, but are stronger in the banking sector, we not only reproduce the usual RBC results, but also obtain interesting results in terms of the cross-correlations between the real interest rate and real output, and cross-correlations between bank labour demand and real output. This is

particularly relevant because the U.S. data show that technological innovations have affected the banking industry with special intensity.

The main contribution of the paper is to show that technological innovations in banks may help explain the negative correlation between the real interest rate and current and future values of real output. This is especially significant because benchmark models have not been able to reproduce this key feature of the post-war U.S. data.

One avenue for further research is to see if technological shocks in banks can generate similar results in the context of sticky price models.

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## APPENDIX

(Not for publication if the article is considered too long; in this case, there would be a note in the main text saying that this appendix is available on request from the author)

In this appendix we show that the initial condition on the household's problem that we used in this paper (equation 6) is the initial condition which naturally arises when we think back to the initial moment of a closed economy without government and where firms don't borrow. We argue that when we start our analysis of the economy at a certain point in time that is in the middle of History (let us denote this point by period 0), the specific initial condition (describing the household's debt position at the beginning of period 0) that we ought to use is the initial condition we have used in this article.

Let us consider an economy where there are only households, firms and banks and where firms don't borrow. Let us imagine that we are at the moment in time where serious economic activity is going to begin and denote this "Beginning of the economy" period by  $t_B$ . Since production is only going to take place during period  $t_B$ , firms will only receive income from the sales of the final good during period  $t_B$  and hence will only pay wages and dividends at the end of period  $t_B$ . However, in order to be able to buy consumption goods from the firms during period  $t_B$  households must use money. The only possibility they have to obtain this money at the beginning of period  $t_B$  is to obtain a loan from a bank. Hence, when the household is at the beginning of period  $t_B$ , and considering choices for  $t_B$  and future periods, she faces the following set of budget constraints. For period  $t_B$  the budget constraint is

$$\frac{b_{t_B+1}}{1+r_{t_B}} = c_{t_B} + \sum_{f=1}^{f=F} q_{t_B}^f (z_{t_B+1}^f - z_{t_B}^f) + \sum_{l=1}^{l=L} q_{t_B}^{bank,l} (z_{t_B+1}^{bank,l} - z_{t_B}^{bank,l}) \quad (25)$$

This budget constraint for period  $t_B$  simply states that, at the beginning of period  $t_B$ , the household must borrow from the banks an amount enough to finance her consumption expenditure during period  $t_B$  and to finance her net purchase of shares of firms and of shares of banks.

Since the household is looking into the future, when she is at the beginning of period  $t_B$  she also takes into account the budget constraints for periods  $(t_B + 1), (t_B + 2), (t_B + 3), \dots$  which are given by

$$w_{t_B-1+i} n_{t_B-1+i}^s + \sum_{f=1}^{f=F} z_{t_B+i}^f \pi_{t_B-1+i}^f + \sum_{f=1}^{f=F} z_{t_B+i}^f q_{t_B+i}^f + \sum_{l=1}^{l=L} z_{t_B+i}^{bank,l} \pi_{t_B-1+i}^{bank,l} + \sum_{l=1}^{l=L} z_{t_B+i}^{bank,l} q_{t_B+i}^{bank,l} +$$

$$+\frac{b_{t_B+1+i}}{1+r_{t_B+i}} = b_{t_B+i} + c_{t_B+i} + \sum_{f=1}^{f=F} z_{t_B+1+i}^f q_{t_B+i}^f + \sum_{l=1}^{l=L} z_{t_B+1+i}^{bank,l} q_{t_B+i}^{bank,l}$$

for  $i=1,2,3,\dots$

These budget constraints for periods  $(t_B+1), (t_B+2), (t_B+3), \dots$  are identical to the household's budget constraints examined in section 4 and can be explained in the same way.

Note that when the household is at the beginning of period  $t_B$ , the budget constraint for  $t_B$  is different from the budget constraint for following periods (this happens because at  $t_B$  there is no previous period).

In order to derive the initial condition, let us start by writing again equation 25 (the budget constraint for period  $t_B$ )

$$\frac{b_{t_B+1}}{1+r_{t_B}} = c_{t_B} + \sum_{f=1}^{f=F} q_{t_B}^f (z_{t_B+1}^f - z_{t_B}^f) + \sum_{l=1}^{l=L} q_{t_B}^{bank,l} (z_{t_B+1}^{bank,l} - z_{t_B}^{bank,l})$$

Imposing here the market clearing conditions in the shares market in period  $t_B$  (which are  $z_{t_B+1}^f = \frac{1}{H}$  and  $z_{t_B+1}^{bank,l} = \frac{1}{H}$ ) and assuming that the world starts with every household owning the same share of each firm and each household owning the same share of each bank (so that we also have  $z_{t_B}^f = \frac{1}{H}$  and  $z_{t_B}^{bank,l} = \frac{1}{H}$ ), we obtain

$$\frac{b_{t_B+1}}{1+r_{t_B}} = c_{t_B} \tag{26}$$

We now write the following tautology

$$b_{t_B+1} = \frac{b_{t_B+1}}{1+r_{t_B}}(1+r_{t_B}) \Leftrightarrow b_{t_B+1} = \frac{b_{t_B+1}}{1+r_{t_B}} + \frac{b_{t_B+1}}{1+r_{t_B}}r_{t_B}$$

This tautology simply says that the household's debt at the end of period  $t_B$  is equal to the principal borrowed at the beginning of period  $t_B$  plus interest on it.

Using equation 26, this last equation becomes

$$b_{t_B+1} = c_{t_B} + \frac{b_{t_B+1}}{1+r_{t_B}}r_{t_B}$$

Using the market clearing condition in the goods market (see section 5), we obtain

$$b_{t_B+1} = \frac{F}{H} [A_{t_B} F(k_{t_B}, n_{t_B}^d) - i_{t_B}] + \frac{b_{t_B+1}}{1+r_{t_B}} r_{t_B}$$

Using the definition of profits of firm  $f$  in period  $t_B$  (see equation 4), we obtain

$$\begin{aligned} b_{t_B+1} &= \frac{F}{H} [\pi_{t_B}^f + w_{t_B} n_{t_B}^d] + \frac{b_{t_B+1}}{1+r_{t_B}} r_{t_B} \Leftrightarrow \\ \Leftrightarrow b_{t_B+1} &= w_{t_B} \frac{F}{H} n_{t_B}^d + \frac{F}{H} \pi_{t_B}^f + \frac{b_{t_B+1}}{1+r_{t_B}} r_{t_B} \end{aligned}$$

With the market clearing condition in the labour market (see section 5), this becomes

$$b_{t_B+1} = w_{t_B} (n_{t_B}^s - \frac{L}{H} n_{t_B}^b) + \frac{F}{H} \pi_{t_B}^f + \frac{b_{t_B+1}}{1+r_{t_B}} r_{t_B}$$

Using the market clearing condition from the bank loans market (see section 5), we obtain

$$b_{t_B+1} = w_{t_B} (n_{t_B}^s - \frac{L}{H} n_{t_B}^b) + \frac{F}{H} \pi_{t_B}^f + \frac{L}{H} b_{t_B}^s r_{t_B}$$

Rearranging, we obtain

$$b_{t_B+1} = w_{t_B} n_{t_B}^s + \frac{F}{H} \pi_{t_B}^f + \frac{L}{H} [r_{t_B} b_{t_B}^s - w_{t_B} n_{t_B}^b]$$

Using the definition of nominal profits of bank  $l$  in period  $t_B$  (see equation 2), we obtain

$$b_{t_B+1} = w_{t_B} n_{t_B}^s + \frac{F}{H} \pi_{t_B}^f + \frac{L}{H} \pi_{t_B}^{bank,l}$$

Finally, using the market clearing conditions in the shares market (see section 5), this can be written as

$$b_{t_B+1} = w_{t_B} n_{t_B}^s + \sum_{f=1}^{f=F} z_{t_B+1}^f \pi_{t_B}^f + \sum_{l=1}^{l=L} z_{t_B+1}^{bank,l} \pi_{t_B}^{bank,l}$$

Remember that  $b_{t_B+1}$  denotes the debt that the household has at the beginning of period  $(t_B + 1)$   $[\frac{b_{t_B+1}}{1+r_{t_B}}$  is the amount borrowed by the household at the beginning of period  $t_B$  and  $r_{t_B}$  is the interest rate between the beginning of period  $t_B$  and the beginning of period  $(t_B + 1)$ ]. Therefore, this last equation tells us that, in equilibrium, the amount the household borrows at the beginning of period  $t_B$  is such that it implies a debt at the beginning of period  $(t_B + 1)$  equal to

the sum of the wage earnings and dividend earnings that she will receive there [at the beginning of period  $(t_B + 1)$ ]. Hence, when we go on to consider the typical household's optimization problem at the beginning of period  $(t_B + 1)$  we should add this equation as an initial condition (describing the debt she carries from the previous period). Therefore, at the beginning of period  $(t_B + 1)$  the household faces the following budget constraints

$$w_{t_B+i}n_{t_B+i}^s + \sum_{f=1}^{f=F} z_{t_B+1+i}^f \pi_{t_B+i}^f + \sum_{f=1}^{f=F} z_{t_B+1+i}^f q_{t_B+1+i}^f + \sum_{l=1}^{l=L} z_{t_B+1+i}^{bank,l} \pi_{t_B+i}^{bank,l} + \sum_{l=1}^{l=L} z_{t_B+1+i}^{bank,l} q_{t_B+1+i}^{bank,l} + \frac{b_{t_B+2+i}}{1+r_{t_B+1+i}} = b_{t_B+1+i} + c_{t_B+1+i} + \sum_{f=1}^{f=F} z_{t_B+2+i}^f q_{t_B+1+i}^f + \sum_{l=1}^{l=L} z_{t_B+2+i}^{bank,l} q_{t_B+1+i}^{bank,l}$$

for  $i = 0, 1, 2, 3, \dots$

plus the initial condition

$$b_{t_B+1} = w_{t_B}n_{t_B}^s + \sum_{f=1}^{f=F} z_{t_B+1}^f \pi_{t_B}^f + \sum_{l=1}^{l=L} z_{t_B+1}^{bank,l} \pi_{t_B}^{bank,l}$$

Using the initial condition in the budget constraint written with  $i = 0$ , the budget constraint for  $i = 0$  becomes

$$\frac{b_{t_B+2}}{1+r_{t_B+1}} = c_{t_B+1} + \sum_{f=1}^{f=F} q_{t_B+1}^f (z_{t_B+2}^f - z_{t_B+1}^f) + \sum_{l=1}^{l=L} q_{t_B+1}^{bank,l} (z_{t_B+2}^{bank,l} - z_{t_B+1}^{bank,l})$$

Therefore the complete description of the budget constraints faced by the household at the beginning of period  $(t_B + 1)$  is

$$\frac{b_{t_B+2}}{1+r_{t_B+1}} = c_{t_B+1} + \sum_{f=1}^{f=F} q_{t_B+1}^f (z_{t_B+2}^f - z_{t_B+1}^f) + \sum_{l=1}^{l=L} q_{t_B+1}^{bank,l} (z_{t_B+2}^{bank,l} - z_{t_B+1}^{bank,l})$$

$$w_{t_B+i}n_{t_B+i}^s + \sum_{f=1}^{f=F} z_{t_B+1+i}^f \pi_{t_B+i}^f + \sum_{f=1}^{f=F} z_{t_B+1+i}^f q_{t_B+1+i}^f + \sum_{l=1}^{l=L} z_{t_B+1+i}^{bank,l} \pi_{t_B+i}^{bank,l} + \sum_{l=1}^{l=L} z_{t_B+1+i}^{bank,l} q_{t_B+1+i}^{bank,l} + \frac{b_{t_B+2+i}}{1+r_{t_B+1+i}} = b_{t_B+1+i} + c_{t_B+1+i} + \sum_{f=1}^{f=F} z_{t_B+2+i}^f q_{t_B+1+i}^f + \sum_{l=1}^{l=L} z_{t_B+2+i}^{bank,l} q_{t_B+1+i}^{bank,l}$$

for  $i = 1, 2, 3, \dots$



These two equations are identical in form to the two equations the household was facing at the beginning of period  $t_B$  but written one period ahead. Therefore we can repeat the reasoning and derive an initial condition for the household's optimization problem at the beginning of period  $(t_B + 2)$ . We will obviously obtain a condition which has the same form but written one period ahead: we obtain

$$b_{t_B+2} = w_{t_B+1} n_{t_B+1}^s + \sum_{f=1}^{f=F} z_{t_B+2}^f \pi_{t_B+1}^f + \sum_{l=1}^{l=L} z_{t_B+2}^{bank,l} \pi_{t_B+1}^{bank,l}$$

(and so on and so forth for all periods ahead). The conclusion to be drawn is that, when we start our analysis of the economy in the middle of History (in period 0, for example), we should add to the household's optimization problem the following initial condition

$$b_0 = w_{-1} n_{-1}^s + \sum_{f=1}^{f=F} z_0^f \pi_{-1}^f + \sum_{l=1}^{l=L} z_0^{bank,l} \pi_{-1}^{bank,l}$$

This is exactly the initial condition we have used in this paper. This initial condition was derived by thinking back to its initial moment a closed economy without government and where firms don't borrow. If instead we think back to its initial moment a closed economy without government where firms borrow the wages at the beginning of the period (in order to be able to pay wages in advance at the beginning of the period), the initial condition we obtain is  $b_0 = \sum_{f=1}^{f=F} z_0^f \pi_{-1}^f + \sum_{l=1}^{l=L} z_0^{bank,l} \pi_{-1}^{bank,l}$ . The results we obtained while working with this other initial condition were not very different from the results quoted in this article.

**Table 1. Stochastic Simulation. Shocks in the nonbank firms' technological parameter only.**

**A. Standard deviations (s.d.)**

Variable	s.d.	s.d. of the variable divided by the s.d. of $\hat{y}_t$
$\hat{y}_t$	4.24	1.00
$\hat{c}_t$	1.73	0.41
$\hat{l}_t$	21.85	5.16
$\hat{n}_t^s$	2.83	0.67
$\hat{w}_t$	2.07	0.49
$\hat{n}_t^d$	2.86	0.67
$\hat{n}_t^b$	6.37	1.50
$\hat{r}_t$	5.64	1.33

**B. Cross-correlations**

Variable	Correlation with										
	$\hat{y}_{t-12}$	$\hat{y}_{t-8}$	$\hat{y}_{t-4}$	$\hat{y}_{t-2}$	$\hat{y}_{t-1}$	$\hat{y}_t$	$\hat{y}_{t+1}$	$\hat{y}_{t+2}$	$\hat{y}_{t+4}$	$\hat{y}_{t+8}$	$\hat{y}_{t+12}$
$\hat{y}_t$	0.35	0.49	0.70	0.84	0.91	1.00	0.91	0.84	0.70	0.49	0.31
$\hat{c}_t$	0.64	0.63	0.61	0.58	0.57	0.55	0.51	0.47	0.40	0.29	0.22
$\hat{l}_t$	0.15	0.32	0.58	0.74	0.84	0.94	0.86	0.79	0.66	0.46	0.32
$\hat{n}_t^s$	0.10	0.28	0.53	0.70	0.80	0.91	0.83	0.76	0.64	0.44	0.31
$\hat{w}_t$	0.60	0.66	0.72	0.76	0.78	0.80	0.74	0.68	0.57	0.41	0.30
$\hat{n}_t^d$	0.08	0.26	0.52	0.69	0.79	0.90	0.82	0.75	0.63	0.44	0.30
$\hat{n}_t^b$	0.64	0.63	0.61	0.58	0.57	0.55	0.51	0.47	0.40	0.29	0.22
$\hat{r}_t$	-0.06	0.11	0.38	0.55	0.66	0.78	0.70	0.64	0.54	0.37	0.25

**Table 2. Percentage deviations from trend of the technological levels in the banking sector and in the total private sector, U.S. 1980-2000**

Year	Estimated $\widehat{D}_t$ ( in %)	Estimated $\widehat{A}_t$ ( in %)	Estimated $\frac{\widehat{D}_t}{\widehat{A}_t}$
1980	4.4413	1.5806	2.81
1981	-1.585	0.8774	-1.81
1982	-4.1119	-3.385	1.21
1983	-2.4652	-1.617	1.52
1984	-2.4735	0.4364	-5.67
1985	2.3113	0.719	3.21
1986	5.1603	1.355	3.81
1987	3.6755	0.654	5.62
1988	3.7579	0.5554	6.77
1989	2.4072	0.2598	9.27
1990	0.4465	0.0903	4.95
1991	-1.01	-1.39	0.73
1992	-3.0781	0.3848	-8.00
1993	-4.1743	-0.3699	11.29
1994	-3.9902	0.1629	-24.49
1995	-2.4342	-1.041	2.34
1996	-3.8563	-0.166	23.20
1997	-0.6057	-0.27	2.24
1998	2.1994	-0.088	-25.06
1999	1.1326	0.2785	4.07
2000	4.4055	0.6316	6.98
		Average of Positive Values	5.63

Note:  $\widehat{D}_t$  and  $\widehat{A}_t$  denote the percentage deviations from trend of the technological levels in the banking sector and in the total private sector, respectively.

Source: authors' calculations, as explained in the main text (Section 7.2.2).

**Table 3. Stochastic simulation. Shocks in the banks' technology 2.8 times greater than in the firms' technology.**

**A. Standard deviations (s.d.)**

Variable	s.d.	s.d. of the variable divided by the s.d. of $\hat{y}_t$
$\hat{y}_t$	4.26	1.00
$\hat{c}_t$	1.78	0.42
$\hat{i}_t$	20.57	4.83
$\hat{n}_t^s$	2.66	0.62
$\hat{w}_t$	1.91	0.45
$\hat{n}_t^d$	2.98	0.70
$\hat{n}_t^b$	20.23	4.75
$\hat{r}_t$	2.84	0.67

**B. Cross-correlations**

Variable	Correlation with										
	$\hat{y}_{t-12}$	$\hat{y}_{t-8}$	$\hat{y}_{t-4}$	$\hat{y}_{t-2}$	$\hat{y}_{t-1}$	$\hat{y}_t$	$\hat{y}_{t+1}$	$\hat{y}_{t+2}$	$\hat{y}_{t+4}$	$\hat{y}_{t+8}$	$\hat{y}_{t+12}$
$\hat{y}_t$	0.30	0.44	0.67	0.82	0.91	1.00	0.91	0.82	0.67	0.44	0.30
$\hat{c}_t$	0.59	0.63	0.66	0.67	0.67	0.67	0.62	0.56	0.47	0.34	0.25
$\hat{i}_t$	0.11	0.28	0.55	0.73	0.83	0.95	0.86	0.77	0.63	0.41	0.26
$\hat{n}_t^s$	0.06	0.23	0.51	0.69	0.80	0.92	0.83	0.75	0.60	0.39	0.25
$\hat{w}_t$	0.56	0.62	0.70	0.74	0.77	0.79	0.72	0.66	0.55	0.38	0.27
$\hat{n}_t^d$	0.07	0.23	0.51	0.70	0.80	0.92	0.83	0.75	0.61	0.39	0.25
$\hat{n}_t^b$	-0.09	-0.25	-0.52	-0.71	-0.82	-0.93	-0.84	-0.76	-0.61	-0.40	-0.26
$\hat{r}_t$	-0.58	-0.52	-0.41	-0.32	-0.27	-0.20	-0.19	-0.18	-0.16	-0.14	-0.12